Optimal Nuclear Waste Burying Policy under Uncertainty

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Abstract

The aim of this paper is to study the optimal nuclear waste storage policy under an uncertainty: the possibility that an accident, might occur in the future. The framework is an optimal growth model with pollution disutility. The main result of the paper in the deterministic case is that, if the relative disutility of the radioactive stock is high enough, vis a vis the elasticities of marginal utility with respect to consumption and to the radioactive stock, then optimal choice of the rate of waste burying consists in burying all the stock at the beginning of the time horizon, until a date which is analytically founded, and then letting the rate decrease. Under uncertainty, the rate of waste burying will be scaled down at the time the accident occurs.

Keywords: Waste Management; Pollution; Growth; Optimal Public Policy; Uncertainty.

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1 Introduction

Most of the OECD countries have chosen nuclear technology as a source of long-term energy, the main reason being to protect themselves against the permanent international fluctuations of carbon energy prices, at minimum cost in terms of economic competitiveness. Furthermore, it is a good tool for the struggle against global warming. For example, 75% of the electricity production in France is nuclear energy, and several countries are involved in vast programs of nuclear energy development (in particular the U.S.A., China and Japan). Choosing nuclear technologies is a long-term economic commitment and it commits future generations for the very long term, since we bequeath to them an inheritance of radioactive waste, harmful for some thousands of years. Nuclear activities create radioactive waste and there is no real solution concerning their management: "In the half century of the nuclear age, the U.S. has accumulated some 30 000 metric tons of spent fuel rods from power reactors and another 380 000 cubic meters of high-level radioactive waste, a by-product of producing plutonium for nuclear weapons. None of these materials have found anything more than interim accommodation, despite decades of study and expenditures in the billions of dollars on research, development and storage" (Whipple (1996)). Currently, two techniques of treatment are operative: Temporary Storage in spent fuel pools and in dry cask storage facilities (France, UK) or Final Storage in deep geological repositories (USA, Sweden). The decision rule between temporary or final storage results from a trade-off between the deterioration of waste protections and the decrease of the harmfulness of the radioactivity (delay means decay). But these two solutions are not similar. In the case of a temporary storage, contemporaneous generations suffer the consequences of the nuclear waste proximity, whereas in the case of final burying, waste disappears underground for several thousand years. Hence, choosing deep geological storage can be analyzed as a NIMBY behaviour[1], but in an intertemporal framework.

The aim of this paper is to determine if there exists an optimal policy for waste burying and therefore an optimal justification for the projection of externalities in the future. The existence of these optimality conditions would justify the implementation of intertemporal NIMBY behaviour. A spatial resolution of conflicts misled by the NIMBY behaviours has been introduced by Feinerman *et alii* (2004). They show that the government choice of localization between two cities of a public bad will depend on the social costs and the social profits of every site, but depends also on the lobbies' power and on the degree of corruption of the government. Groothuis and Miller (1994) analyze the influence of the NIMBY syndrome in economic decision making concerning the location of hazardous waste facilities. They show that according to some characteristics of the NIMBY behaviours, the acceptance of a hazardous waste disposal facility in one's neighbourhood is likely to occur when compensation is offered. In an intertemporal case, the one introduced in this paper, the future generations cannot be defended by any lobby group and cannot negotiate their level of the reasonable compensation for accepting our nuclear waste.

This paper studies more generally the optimal growth path of an economy facing a dilemma of consumption vs pollution. Consumption, which is a source of welfare, is permitted by the increase of pollution stock but this deterioration in turn diminishes welfare. Consumption and pollution enter in a non-separable way into the utility function. Besides, a social planner can decrease the waste stock by burying a part of the remaining waste stock in deep geological repositories. We then ask ourselves about the optimal growth path of the waste burying rate.

The framework introduced in this paper is symmetrical to the problem of

[1] NIMBY: Not In My Back Yard.

the optimal use of exhaustible resources[2] (Hotteling (1931), Dasgupta and Heal (1974), Hartwick (1977), Johnson *et alii* (1980)). These papers show that the government or the social planner must determine the optimal rules for the resource exploitation and the preservation of natural environments. In this paper, the social planner has to determine the optimal rule of waste accumulation to fill up a nuclear waste repository. Moreover, our study is in the Ramsay framework and thus is quite similar to studies dealing with optimal pollution control (Van der Ploeg and Withagen (1991), Gradus and Smulders (1996), Ayong Le Kama (2001), Ayong Le Kama and Schubert (2004 and 2006)) but the approach is different since we determine endogenously the rate of waste burying (which can be interpreted as a natural rate of absorption in an exogenous case).

We consider an economy in which there is only one good: nuclear electricity. Its production generates radioactive solid waste. For simplification, we assume that the production of the nuclear electricity is proportional to the final demand, which is the level of consumption. We also assume that the flow of radioactive waste is proportional to the production. We therefore obtain that, at each period t the flow of radioactive waste is proportional to the level of consumption. Besides, we assume that at each date t the social planner can decide to bury a part γ_t of the remaining stock of the radioactive waste in some appropriately deep final geological repositories. Furthermore, we assume that at time t when the representative consumer chooses his optimal consumption, he knows the level of γ_t. The time t optimal consumption, as well as the remaining stock, can therefore be defined as a function of the rate of waste burying. On the other hand, we assume that the social planner chooses the optimal waste burying policy knowing the optimal consumption behaviour and the optimal level of the stock for any given profile of γ_t, $t \in [0, +\infty[$.

The main result of the paper in this deterministic case is that, if the rela-

[2] See for example Heal (1993) for a survey on these topics.

tive disutility of the radioactive stock is high enough, vis a vis the elasticities of marginal utility with respect to consumption and to the radioactive stock, then optimal choice of the rate of waste burying consists in burying all the stock at the beginning of the time horizon, until a date which is analytically founded, and then letting the rate decrease. But if the relative disutility of the radioactive stock is low enough, the policy consists of a zero waste burying at the beginning and an increasing rate after.

This paper also introduces an uncertainty. We assume that an accident can occur in the future and implies the destocking of all the previous buried stock of waste. We also assume that the destocking is a once and for all phenomenon. Besides, we assume that the social planner, facing this uncertainty will try to smooth the consumption of the household at the time the accident occurs. Therefore, at this time, he will select an optimal rate of waste burying in a way to maintain the two current levels of consumption (the one just before the accident and the one just after) equal.

The main result in this case with uncertainty is that, if the relative disutility of the radioactive stock is low enough, vis a vis the elasticities of marginal utility with respect to consumption and to the radioactive stock, then the rate of waste burying will scale down at the time the accident occurs. This rate will scale up otherwise.

2 The model

We consider an economy in which there is only one good: nuclear electricity. This good is consumed at a level C_t. Its production generates radioactive solid waste. For simplification, we assume that the production of the nuclear electricity is proportional to the final demand, which is the level of consumption C_t. We also assume that the flow of radioactive waste is proportional to the production. We therefore obtain that, at each period t, the flow of radioactive waste is proportional to the level of consumption: βC_t, with a constant marginal rate $\beta > 0$.

Besides, we assume that at each date t, the social planner can decide to bury a part γ_t of the remaining stock S_t of the radioactive waste in some appropriate deep final geological repositories. Thus, the rate of change of the stock is given by:

$$\dot{S} = \beta C_t - \gamma_t S_t \tag{1}$$

where $\gamma_t \in [0, 1]$ measures the time t rate of waste burying.

We assume that at time t when the representative consumer chooses his optimal consumption, he knows the level of γ_t. The time t optimal consumption can therefore be define as a function of the rate of waste burying: $C_t = C_t(\gamma_t)$, with γ_t given. The same properties apply for the remaining stock $S_t = S_t(\gamma_t)$.

In the other hand, we assume that the social planner chooses the optimal waste burying policy knowing the optimal consumption behavior and the optimal level of the stock for any given profile of γ_t, $t \in [0, +\infty[$.

2.1 The optimal consumption path

At time t the representative consumer derives utility from the consumption of electricity at a rate C_t, but his utility is depleted by the stock of radioactive waste S_t. The utility function $U(C, S)$ is assumed to be strictly concave, twice continuously differentiable and to possess the following properties.

Assumption 1: $U'_C(C, S) > 0$[3]; $U''_{CC}(C, S) < 0$; $U'_S(C, S) < 0$; $U''_{SS}(C, S) > 0$; and also $U''_{CS}(C, S) < 0$[4].

Let us denote: $\varepsilon = \frac{SU'_S}{CU'_C} < 0$, which stands for the "relative disutility of the radioactive stock"; $\eta_1 = \frac{CU''_{CC}}{U'_C} < 0$, the elasticity of the marginal utility

[3] U'_C and U'_S are the first partial derivatives of the function $U(.)$ with respect to its arguments C and S. U''_{CC} is likewise the second partial derivative, using obvious notation.

[4] We assume that the marginal utility of consumption decreases when the stock of radioactive waste increases: utility exhibits a distate effect, in the terminology of Michel and Rotillon (1996).

with respect to consumption; $\eta_2 = \frac{SU''_{CS}}{U'_C} \leq 0$, the elasticity of the marginal utility with respect to the stock of radioactive waste; and $\eta_3 = \frac{SU''_{SS}}{U'_S} > 0$, the elasticity of the marginal disutility of the waste stock.

Thus, the feasibility of a constant growth path (CGP), that is the case where the rate of growth of the consumption and the stock is constant for a given γ (see below), yields the following:

Assumption 2. *(i) η_1, η_2, η_3 are constant; (ii) the relative disutility of the radioactive stock ε is constant*[5].

For the concavity of the utility function, we also need:

Assumption 3. $\varepsilon \leq \frac{\eta_2^2}{\eta_1 \eta_3}$.

Under these 3 assumptions, the problem of the representative consumer can be formulated as:

$$P(0) \quad \begin{cases} W(S_0) = \max \int_0^\infty U(C_t, S_t)\, e^{-\delta t} dt \\ s.t. \quad \left| \begin{array}{c} \dot{S} = \beta C_t - \gamma_t S_t \\ S_0 \text{ and } 0 \leq \gamma_t \leq 1 : \text{ given } ; S_t, C_t \geqslant 0 \quad \forall t \end{array} \right. \end{cases} \tag{2}$$

where $\delta > 0$ is the discount rate[6].

The current value Hamiltonian is:

$$H = U(C, S) + \lambda (\beta C - \gamma S)$$

where $\lambda \leq 0$ is the shadow cost of waste stock.

[5] Smulders and Gradus (1996) or Michel and Rotillon (1996) show that *(ii)* is a necessary condition for the existence of a balanced growth path when respectively the stock of environmental quality is a source of utility or the one of pollution affects the utility.

[6] For a discussion about the choice of exogenous or endogenous discount rate, see Ayong Le Kama and Schubert (2007).

The first order necessary conditions then give us:

$$U'_C = -\lambda\beta \tag{3}$$
$$\frac{\dot{\lambda}}{\lambda} = (\delta + \gamma) + \frac{\beta U'_S}{U'_C} \tag{4}$$

Then, by differentiating the first optimality condition (3) , we find:

$$\frac{\dot{C}}{C} = \frac{1}{\eta_1}\left[\frac{\dot{\lambda}}{\lambda} - \frac{SU''_{CS}}{U'_C}\frac{\dot{S}}{S}\right] \tag{5}$$

Moreover, using these two conditions ((3) and (4)), we obtain

$$\frac{\dot{\lambda}}{\lambda} = \delta + \gamma + \varepsilon\frac{C}{S}\beta$$

Then, from eq. (5) we can easily find the growth rate of consumption g, for a given γ:

$$g(\gamma) = \frac{\dot{C}}{C} = \frac{1}{\eta_1}\left[\delta + (1+\eta_2)\gamma + \frac{\beta}{r}\frac{C}{S}(\varepsilon - \eta_2)\right] \tag{6}$$

Let us define the stationary variable $x(\gamma) = \frac{C(\gamma)}{S(\gamma)}$. Thus the dynamic system characterizing the evolution of the economy and the stock of radioactive nuclear waste reduce to a single equation in x, for a given γ. It writes:

$$\frac{\dot{x}}{x} = \frac{1}{\eta_1}\left[\delta + \gamma(1+\eta_1+\eta_2) + \beta x(\varepsilon - \eta_1 - \eta_2)\right] \tag{7}$$

2.2 The stationary solution

A stationary solution x^* of the one dimension dynamic system is such that $\dot{x} = 0$ for any given value of γ. This implies that for a given level of the waste burying rate γ, the growth rate of the consumption $g(\gamma)$ and the one of the waste stock are equal. The stationary solution of equation (7) is then given by:

$$x^*(\gamma) = \frac{1}{\beta}\frac{\delta + (1+\eta_1+\eta_2)\gamma}{\eta_1 + \eta_2 - \varepsilon} \tag{8}$$

Using this, it is easy to rewrite the dynamic equation (7) as follow:

$$\frac{\dot{x}}{x} = -\frac{\beta}{\eta_1}\left(1+\eta_1+\eta_2\right)\left(x\left(\gamma\right)-x^*\left(\gamma\right)\right) \tag{9}$$

For any given γ, this dynamic equation is unstable. The x ratio then takes from time zero its stationary value $x^*\left(\gamma\right)$, as a function of the rate of waste burying. Thus the initial consumption will be $C_0 = x^*\left(\gamma_0\right)S_0$, depending on the choice of the time 0 rate of burying of the social planer.

We now have to find the conditions which ensure the non-negativity of the stationary solution $x^*\left(\gamma\right)$.

Before, let us introduce these two parameters:

$$\bar{\gamma} = -\frac{\delta}{1+\eta_1+\eta_2} \tag{10}$$

$$\gamma^g = -\frac{\delta}{1+\varepsilon} \tag{11}$$

with $\bar{\gamma}$ (as we will see below) which corresponds to the long run value of the optimal rate of waste burying and γ^g which is the value of the rate of waste burying at which the sign of the growth rate of consumption changes (that is, if at a given date t, $\gamma_t < \gamma^g$, then $g\left(\gamma\right) > 0$ and < 0 otherwise).

Given that $\bar{\gamma}$ and γ^g are specific values of γ, they also have to be such that : $0 \leq \bar{\gamma}, \gamma^g \leq 1$. For $0 \leq \gamma^g \leq 1$, we need the following condition on the "relative disutility of the radioactive stock": $\varepsilon < -1-\delta$; and to satisfy $0 \leq \bar{\gamma} \leq 1$ yields also: $\eta_1+\eta_2 < -1-\delta$. We therefore introduce the following assumption.

Assumption 4. (i) $\varepsilon < -1-\delta$; and (ii) $\eta_1+\eta_2 < -1-\delta$.

We can now examine two different cases for the non-negativity of the stationary solution $x^*\left(\gamma\right)$, depending on the relative values of the elasticities.

Case 1 $\eta_1+\eta_2 > \varepsilon : x^*\left(\gamma\right) > 0$ *iff* $\gamma < \bar{\gamma}$.

Indeed, as we need $\delta+(1+\eta_1+\eta_2)\,\gamma>0$ which implies $\delta>-(1+\eta_1+\eta_2)\,\gamma$ and gives $-\frac{\delta}{1+\eta_1+\eta_2}>\gamma$.

Case 2 $\eta_1+\eta_2<\varepsilon: x^*(\gamma)>0$ *iff* $\gamma>\bar{\gamma}$.

We must have $\delta+(1+\eta_1+\eta_2)\,\gamma<0 \Longleftrightarrow -\frac{\delta}{(1+\eta_1+\eta_2)}<\gamma$.

These positivity conditions can be summarized as follow:

$$x^*(\gamma)>0 \Longleftrightarrow \begin{cases} \eta_1+\eta_2>\varepsilon & \Longleftrightarrow & \gamma<\bar{\gamma} \\ \eta_1+\eta_2<\varepsilon & \Longleftrightarrow & \gamma>\bar{\gamma} \end{cases}$$

Let us examine the marginal effect of the waste burying rate on the positive stationary solution. We can easily show that,

$$x^{*\prime}=\frac{1}{\beta}\frac{1+\eta_1+\eta_2}{\eta_1+\eta_2-\varepsilon} \gtreqqless 0 \tag{12}$$

This marginal rate of waste burying is constant. The rate of waste burying has an ambiguous impact on the stationary solution and the final effect will depend on the values of the preference elasticities $(\eta_1,\eta_2,\varepsilon)$. Using Assumption 4, we have only two different cases, which are summarized by:

$$\begin{cases} x^{*\prime}(\gamma)<0 & \Longleftrightarrow & \eta_1+\eta_2>\varepsilon \\ x^{*\prime}(\gamma)>0 & \Longleftrightarrow & \eta_1+\eta_2<\varepsilon \end{cases}$$

We then have an economy in which the $\frac{C}{Q}$ ratio is constant at the level x^* and C and Q grow at the same rate $g(\gamma)$, with (using (eq. (1) and (8)):

$$g^*(\gamma)=\frac{\delta+(1+\eta_1+\eta_2)\,\gamma}{\eta_1+\eta_2-\varepsilon}=\beta x^*(\gamma)-\gamma$$

This growth rate can be positive or negative. Hence, this rate is negative if:

$$x^*(\gamma)<\frac{\gamma}{\beta} \Longleftrightarrow \frac{\delta+\gamma\,(1+\varepsilon)}{\eta_1+\eta_2-\varepsilon}<0$$

the level of consumption along the optimal path is relatively high *vis-à-vis* the level of waste stock, and both decrease. This happens when the household is very patient, or waste burying rate is high, or also the waste production rate of the consumption is low. In the opposite case, consumption and waste stock increase along the optimal path.

We can now examine the effects of the waste burying rate on the growth rate:

$$\begin{cases} g^*(\gamma) > 0 \Longleftrightarrow \left| \begin{array}{lll} (\eta_1 + \eta_2 > \varepsilon) & \cap & (\gamma < \gamma^g) \\ (\eta_1 + \eta_2 < \varepsilon) & \cap & (\gamma > \gamma^g) \end{array} \right. \\ g^*(\gamma) < 0 \quad \text{otherwise} \end{cases}$$

with γ^g defined in (11). We can notice that the thresholds $\bar{\gamma}$ and γ^g are linked by the following condition.

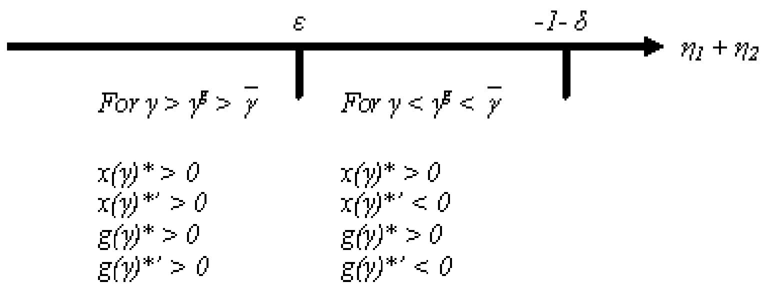
Proposition 3 *If $\eta_1 + \eta_2 > \varepsilon$ then we have $\gamma^g < \bar{\gamma}$. $\gamma^g > \bar{\gamma}$ otherwise.*

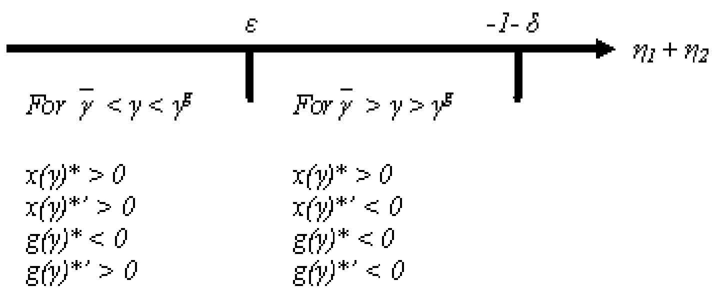
The marginal impact of waste burying rate on the growth rate is also ambiguous and is given by:

$$g^{*\prime}(\gamma) = \beta x^{*\prime} - 1 = \frac{1+\varepsilon}{\eta_1 + \eta_2 - \varepsilon} \gtreqless 0 \tag{13}$$

The sign of this derivative is also ambiguous but does not depend on the waste burying rate. We have $g^{*\prime}(\gamma) < 0$ iff $\eta_1 + \eta_2 > \varepsilon$ (i.e. $\gamma^g < \bar{\gamma}$); $g^{*\prime}(\gamma) > 0$ otherwise.

We can summarize the economic consequences of the elasticity values with the figures 1 and 2.





3 The deterministic optimal waste burying policy

The central planner objective is to maximize the household indirect utility function with respect to the waste burying rate γ. The problem writes:

$$P(1)\left\{\begin{array}{ll} \max\limits_{\gamma} W\left(S_0\right)=\int_0^{\infty} U\left(C_t^*; S_t^*\right) e^{-\delta t} dt & \\ \quad s.t. & \left|\begin{array}{l} S_t^*=S_0 e^{g^*(\gamma) t} \\ C_t^*=x^* S_0 e^{g^*(\gamma) t} \\ S_t^*, C_t^* \geqslant 0 \quad \forall t \\ \gamma_0, S_0 \text{ given} \end{array}\right. \end{array}\right. \tag{14}$$

The first order necessary condition $\left(\frac{\partial W(S_0)}{\partial \gamma}=0\right)$ gives:

$$x_t^*=-\frac{1}{t\left(1+\varepsilon\right)} \frac{\frac{\partial x^*}{\partial \gamma}}{\frac{\partial g^*}{\partial \gamma}} \geqslant 0$$

which implies (with *Assumption 1*) that the sign of $\frac{\partial x^*}{\partial \gamma}$ must be the same than the sign of $\frac{\partial g^*}{\partial \gamma}$ in order to ensure the non-negativity of the stationary solution. Using eq. (13) and (12), one obtain

$$\gamma_t^* = \left(\frac{\gamma^g}{\bar{\gamma}} - 1\right) \frac{\gamma^g}{\delta} \frac{1}{t} + \bar{\gamma} \quad \forall t > 0 \tag{15}$$

Hence, we can notice that:

- $\frac{\partial \gamma_t^*}{\partial t} \gtreqqless 0$ and $\frac{\partial^2 \gamma_t^*}{\partial t^2} \gtreqqless 0$ depending on the sign of $\frac{\gamma^g}{\bar{\gamma}} - 1$;
- $\lim_{t\to\infty} \gamma_t^* = \bar{\gamma}$. This long-term property implies that $\lim_{t\to\infty} x^* \left(\gamma_t^*\right) = 0$.

Furthermore, we must study the conditions that ensure that $\gamma_t^* \in [0, 1] \quad \forall t > 0$. In the long term, a necessary condition is $\bar{\gamma} \leq 1$, which is verified under *Assumption 2*.

If we define $X = \frac{\gamma^g}{\bar{\gamma}} > 0$, we can then write,

$$\gamma_t^* = -\underbrace{\left(\frac{1}{1+\eta_1+\eta_2}\right)}_{+} \left(\frac{1}{t}(X-1)X + \delta\right)$$

Let us define period t^1 such that

$$t^1 = \frac{(1-X)X}{\delta + 1 + \eta_1 + \eta_2} \gtreqqless 0$$

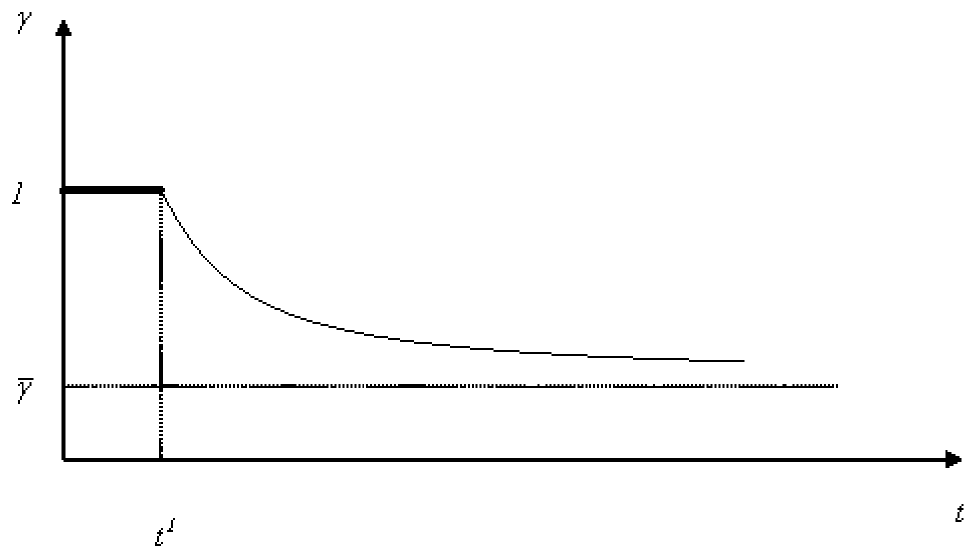
and period t^0 such that

$$t^0 = \frac{(1-X)X}{\delta} \gtreqqless 0$$

Under *Assumption 4*, we can easily show that $t^1 > 0$ *iff* $X > 1$ and $t^0 > 0$ *iff* $X < 1$.

Proposition 4 *Under Assumption 4, if* $X > 1 (\Leftrightarrow \eta_1 + \eta_2 < \varepsilon)$, *then, we obtain,* $\forall t > 0$*:* $\frac{\partial \gamma_t^*}{\partial t} < 0$; $\frac{\partial^2 \gamma_t^*}{\partial t^2} > 0$; $\gamma_t^* \geqslant 0$; *and finally* $\gamma_t^* \leq 1$ *iff* $t \geqslant t^1$.

Hence, the evolution of the optimal waste burying rate is represented in Figure 3.

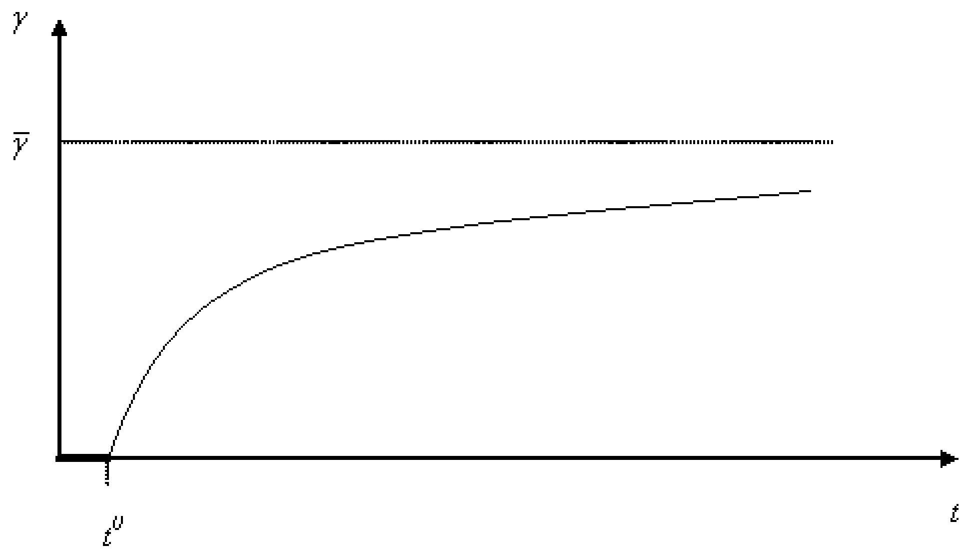


Proposition 5 *Under Assumption 2, if* $X < 1 \Leftrightarrow \varepsilon < \eta_1 + \eta_2 < -1 - \delta < -1$, *then we obtain,* $\forall t > 0$*:*$\frac{\partial \gamma_t^*}{\partial t} > 0$; $\frac{\partial^2 \gamma_t^*}{\partial t^2} < 0$; $\gamma_t^* \leq 1$; *and finally* $\gamma_t^* \geqslant 0$ *iff* $t \geqslant t^0$.

Hence, the evolution of the optimal waste burying rate is represented in Figure 4.

4 The optimal waste burying policy under uncertainty

We assume that an accident can occur in the future and implies the destocking of all the previous buried stock of waste. We also assume that the destocking is a once and for all phenomenon. Besides, we assume that the social planner, facing this uncertainty will try to smooth the consumption of the household at the time the accident occurs. Therefore, at this time, he will select an optimal rate of burying in a way to maintain the two current levels of consumption (the one just before the accident and the one just



after) equal.

Let T be the given date at which the accident occurs. We assume that the date at which the accidental destocking occurs is higher than the minimum level t_i $\left(T > t^i\right)$. The other case $(T < t_i)$ has no interesting properties for this work. Indeed, if $T < t_0$, the accident has no consequence on the economy as the government did not stock before $(\gamma_0 = 0)$. Moreover, if $T > t_1$, the accident increases the stock of waste but the government cannot adjust its environmental policy as $\gamma = 1$ already. Then, the program of the social planner writes:

$$P(3) \quad \begin{cases} \underset{\gamma_t}{Max} & \int_0^T U\left(C_t^*\left(\gamma_t\right); S_t^*\left(\gamma_t\right)\right) e^{-\delta t} dt + e^{-\delta t} EW\left(S_T\right) \\ s.t. & \Big| \; C_T^{(3)} = x\left(\gamma_T^{(3)}\right) S_T = E\left(C_T\left(\tilde{\gamma}_T\right)\right) = E\left(x\left(\tilde{\gamma}_T\right) \tilde{S}_T\right) \end{cases}$$

with (3) denoting the date just before that the accident occurs. From the previous results, we have,

$$\left| \begin{array}{l} \gamma_T^{(3)} = \frac{\gamma^g}{\delta g'} \frac{1}{T} + \bar{\gamma} \\ x\left(\gamma_T^{(3)}\right) = -\frac{1}{T} \frac{1}{1+\varepsilon} \frac{1}{\beta} \frac{\gamma^g}{\bar{\gamma}} = \frac{\gamma^g}{\bar{\gamma}} g' \left(\gamma_T^{(3)} - \bar{\gamma}\right) \frac{1}{\beta} \end{array} \right. \tag{16}$$

which gives the following law of evolution for the waste stock,

$$\begin{aligned} \tilde{S}_T &= p\left(S_T^{(3)} + \tilde{\theta} \int_0^T \gamma_t^{(3)} S_t^{(3)} dt\right) + (1-p) S_T^{(3)} \\ &= S_T^{(3)} + p\tilde{\theta} \int_0^T \gamma_t^{(3)} S_t^{(3)} dt \end{aligned}$$

with $p \in [0,1]$ a measure of the probability of occurrence of the accident and $\tilde{\theta} \in [0,1]$ the share of the previously buried waste stock that reappears above ground. We obtain,

$$\left\{ \begin{array}{l} \tilde{S}_T = S_T^{(3)} + p\tilde{\theta} \left(\int_0^{t^i} \gamma_i S_t^{(3)} dt + \int_{t^i}^T \gamma_t^{(3)} S_t^{(3)} dt\right) \quad \forall i = \{0;1\} \\ s.t. \left| \begin{array}{l} for\ i = 0 : \gamma_0 = 0;\ t^0 = \frac{\left(1 - \frac{\gamma^g}{\bar{\gamma}}\right) \frac{\gamma^g}{\bar{\gamma}}}{\delta} \\ for\ i = 1 : \gamma_1 = 1;\ t^1 = \frac{\left(1 - \frac{\gamma^g}{\bar{\gamma}}\right) \frac{\gamma^g}{\bar{\gamma}}}{\delta + 1 + \eta_1 + \eta_2} \end{array} \right. \end{array} \right.$$

$\forall T > t^i$, $\forall i = \{0;1\}$, the social planner must determine the optimal level of the burying rate $\tilde{\gamma}_T$ such that:

$$C_T^{(3)} = x\left(\gamma_T^{(3)}\right) S_T = x\left(\tilde{\gamma}_T\right) \tilde{S}_T = x\left(\tilde{\gamma}_T\right) \left(S_T^{(3)} + p\tilde{\theta} \left(\int_0^{t^i} \gamma_i S_t^{(3)} dt + \int_{t^i}^T \gamma_t^{(3)} S_t^{(3)} dt\right)\right)$$

this marginal condition yields to:

$$x\left(\tilde{\gamma}_T\right) = -\frac{\frac{1}{T} \frac{1}{1+\varepsilon} \frac{1}{\beta} \frac{\gamma^g}{\bar{\gamma}}}{1 + e^{-g\left(\gamma_t^{(3)}\right)T} p\tilde{\theta} \left(\int_0^{t^i} \gamma_i e^{g(\gamma_i)t} dt + \int_{t^i}^T \gamma_t^{(3)} e^{g\left(\gamma_t^{(3)}\right)t} dt\right)}$$

Therefore, we remark that if there is no risk of catastrophic (*i.e.* $\tilde{\theta} = 0$ and/or $p = 0$), we have:

$$x\left(\tilde{\gamma}_T\right) = -\frac{1}{T} \frac{1}{1+\varepsilon} \frac{1}{\beta} \frac{\gamma^g}{\bar{\gamma}} = x\left(\gamma_T^{(3)}\right)$$

that represents a continuity condition.

The optimal rate of waste storage must verifies the following condition (using $eq.$ (16)):

$$x\left(\tilde{\gamma}_T\right)=\frac{\gamma^g}{\bar{\gamma}}g'\left(\tilde{\gamma}_T-\bar{\gamma}\right)\frac{1}{\beta}$$
$$\Leftrightarrow \tilde{\gamma}_T=\bar{\gamma}-\frac{1}{g'}\frac{1}{T}\frac{1}{1+\varepsilon}\frac{1}{1+e^{-g\left(\gamma_t^{(3)}\right)T}p\tilde{\theta}\left(\int_0^{t^i}\gamma_i e^{g(\gamma_i)t}dt+\int_{t^i}^T\left(\frac{\gamma^g}{\delta g'}\frac{1}{t}+\bar{\gamma}\right)e^{g\left(\gamma_t^{(3)}\right)t}dt\right)}$$

Besides, we can easily show that $g\left(\gamma_t^{(3)}\right)=\frac{\gamma^g}{\delta}\frac{1}{t}-\bar{\gamma}$ and $g\left(\gamma_i\right)=\frac{\gamma^g}{\delta}\frac{1}{t^i}-\bar{\gamma}$. Finally, we obtain $\forall i=\{0;1\}$:

$$\tilde{\gamma}_T=\bar{\gamma}-\frac{1}{g'}\frac{1}{T}\frac{1}{1+\varepsilon}\frac{1}{1+p\tilde{\theta}\left(\int_0^{t^i}\gamma_i e^{g(\gamma_i)t}dt+\int_{t^i}^T\bar{\gamma}e^{\left(\frac{\gamma^g}{\delta}-\bar{\gamma}t\right)}dt+\int_{t^i}^T\frac{\gamma^g}{\delta g'}\frac{1}{t}e^{\left(\frac{\gamma^g}{\delta}-\bar{\gamma}t\right)}dt\right)}$$

Let, $\forall i=\{0;1\}$

$$\begin{aligned}\tilde{\gamma}_T &= \bar{\gamma}-\frac{1}{g'}\frac{1}{T}\frac{1}{1+\varepsilon}\frac{1}{1+p\tilde{\theta}\left(A_3+A_1+A_2\right)}\\ &= \gamma_T^{(3)}+\frac{1}{g'}\frac{1}{T}\frac{1}{1+\varepsilon}\left(1-\frac{1}{1+p\tilde{\theta}\left(A_3+A_1+A_2\right)}\right)\end{aligned}$$

with:

$$A_1=\int_{t^i}^T\bar{\gamma}e^{\left(\frac{\gamma^g}{\delta}-\bar{\gamma}t\right)}dt=\bar{\gamma}e^{\frac{\gamma^g}{\delta}}\int_{t^i}^T e^{-\bar{\gamma}t}dt=e^{\frac{\gamma^g}{\delta}}\left[e^{-\bar{\gamma}t^i}-e^{-\bar{\gamma}T}\right]>0$$
$$A_2=\int_{t^i}^T\frac{\gamma^g}{\delta g'}\frac{1}{t}e^{\left(\frac{\gamma^g}{\delta}-\bar{\gamma}t\right)}dt=\frac{\gamma^g}{\delta g'}e^{\left(\frac{\gamma^g}{\delta}\right)}\int_{t^i}^T\frac{1}{t}e^{(-\bar{\gamma}t)}dt>0$$
$$A_3^i=\int_0^{t^i}\gamma_i e^{g(\gamma_i)t}dt$$

For $i=0$, we have $\gamma_0=0$, then $A_3^0=0$. Otherwise, if $i=1$, we obtain $A_3^1=\int_0^{t^1}e^{g(\gamma_1)t}dt$ with $g\left(\gamma_1\right)=g\left(1\right)=\frac{\delta+1+\varepsilon}{\eta_1+\eta_2-\varepsilon}$ and $t^1=\frac{\left(1-\frac{\gamma^g}{\bar{\gamma}}\right)\frac{\gamma^g}{\bar{\gamma}}}{\delta+1+\eta_1+\eta_2}$.

Hence,

$$A_3^i=\begin{cases}0 & if \quad i=0\\ \int_0^{t^1}e^{\left(\frac{\delta+1+\varepsilon}{\eta_1+\eta_2-\varepsilon}\right)t}dt=\left(\frac{\eta_1+\eta_2-\varepsilon}{\delta+1+\varepsilon}\right)\left[e^{\left(\frac{\delta+1+\varepsilon}{\eta_1+\eta_2-\varepsilon}\right)t^1}-1\right] & if \quad i=1\end{cases}$$

Proposition 6 *Under assumption 4,*
(i) If $i = 0$, we have $\tilde{\gamma}_T \leq \gamma_T^{(3)}$. That is, if $\eta_1 + \eta_2 > \varepsilon$ the rate of waste burying will scale down at the time the accident occurs.
(ii) If $i = 1$, we have $\tilde{\gamma}_T \geq \gamma_T^{(3)}$. That is, if $\eta_1 + \eta_2 < \varepsilon$ the optimal choice is to increase the rate of radioactive waste burying at the time the accident happens.

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