International coordination over emissions and R&D expenditures: What does oil scarcity change? A differential game approach with two asymetric players (Preliminary Draft please don't quote)

Antoine Belgodere*
Dominique Prunetti†

CREShS
Université de Corse - Campus Caraman - 7, avenue Jean Nicoli
20250 CORTE
FRANCE

March 2007

1 Introduction

In a recent rapport (IEA (2006)), the International Energy Agency concludes that there is a need for a rapid improvement in energy efficiency and low-carbon technology, because of the historically hight increase in oil prices and in CO2 concentration in the last decade (more than 20 percent). The rapport also advocates an expansion of the R&D budgets in order to achieve technological progress in areas like hydrogen and fuel cells, advanced renewable

*belgodere@univ-corse.fr
†prunetti@univ-corse.fr

energies, next-generation biofuels and energy storage.

The interaction between Climate policy and endogenous technological change has been recently studied in several papers (see Golombek and Hoel (2005) and Golombek and Hoel (2006) for overviews), but none of these studies, as far as we know, focused on the interaction between the oil price, technological progress and Greenhouse gas emissions.

We study this problem in this paper using a differential game model that includes three stocks, each of which is relevant to understand the negotiation process about global warming: the pollution, the marginal extraction cost of the resource and the level of knowledge in the renewable non polluting resource sector. We don't focus, in this paper, on the choice of policy instruments that can be implemented in order to reach the first best optimum. We adopt the same point of view as van der Ploeg and de Zeeuw (1994), implicitly assuming that taxation is an appropriate instrument of environmental policy, as opposed to other instruments such as standards or marketable permits. Our approach is close to van der Ploeg and de Zeeuw (1994), concerning the comparison of centralized and decentralized solution in a global pollution problem with investment in clean technology by the means of differential games model. This authors compare the outcome under international coordination of environmental policies with the case of the adoption of "open-loop" strategies in a N-symmetric players game. As noted by the authors, it is well known, in the differential games literature, that this kind of strategies, by comparison with the "closed-loop" strategies, give smaller payoffs. The principal reason is that "closed-loop" strategies, like linear Markov perfect strategies, ensure subgame perfection contrary to open-loop strategies (Cf., e.g., Fudenberg and Tirole (1992), p.74-77). That's why we choose to adopt this kind of strategies in your model. Another specificity of our model is that we consider two asymmetrical players, as in List and Mason (2001), who can be thought of as two groups of nations: rich and poor countries. This assumption is more realistic than the symmetry assumption in climate change problems.

Unfortunately, a three stocks, two asymmetric countries differential game in Markov-linear strategies just cannot be solved analytically. To counter this difficulty, we use numerical techniques to get an insight into the behavior of the model.

The paper is organized as follows. Section 2 develops the model. Section 3 derives cooperative and non-cooperative equilibria.Section 4 presents a Monte Carlo procedure that enables to numerically solve the model. Results are analyzed in Section 5. Section 6 concludes.

2 The model

We consider a world with 2 players; indexed by i=1,2; corresponding to two heteregenous countries in terms of sentitivity to the environment and of wealth. Four factors are important for understand the structure of the model and are described hereafter.

2.1 Scarcity and pollution

Oil extraction has two harmful effects:

First, it lowers the oil stock for the future, because oil is a non-renewable resource. In this paper, we don't model oil as a finite-size non-renewable stock. We assume the stock is infinite, but the marginal extraction cost is an increasing function of the cumulated extractions. For simplicity, no scarcity rent is assumed. It follows that the resource price, P, equates its marginal cost expressed in term of the aggregate output. P follows:

$$\dot{P} = \sum_{i=1}^{2} \zeta E_i \tag{1}$$

Where E_i is the rate of resource extraction by country i, and ζ is a parameter. ζ denotes the importance of scarcity.

Second, burning oil pollutes. Oil pollution is cumulative. The stock of pollution follows:

$$\dot{M} = \sum_{i=1}^{2} E_i - \delta M \tag{2}$$

where δ is the constant rate of decay of pollution.

Pollution generates an external cost given by $\alpha_i M^2$, where $\alpha_i > 0$ is a parameter. The sensitivity to pollution is different between both countries.

2.2 The resource sector

The resource is used as an input to produce an aggregate good Q_i, together with a renewable non polluting energy. *Ceteris Paribus*, an improvement of the backstop technology makes profitable a shift from hydrocarbon to clean energy in a number of economic sectors. Thus, for a given E_i, this improvement generates an increase in the opportunity cost of E_i. This effect is represented in figure 1.[1]

Let A be a renewable non polluting resource and X a coefficient denoting the level of knowledge in the renewable energy sector. For a given X, the optimal level of production is Q, and the optimal combination of oil and clean energy is given by (E, A). Now, what happens if X increases, inducing a fall in the cost of the clean energy? First, the prices ratio between both inputs is changed. If this change in the prices ratio did not affect the quantity of output produced, then the new optimal combination of input would be (E', A'), with more clean energy and less oil than in the previous equilibrium. But this fall in the clean energy price results in a fall of the aggregate energy price, which implies a greater level of output production. Assume this new optimal output level is given by Q'. In this situation, the new optimal combination of input would be (E'', A''), with both more clean energy and more oil than in (E', A'). A crucial question is to know whether E'' is greater or smaller than E. It depends both on the elasticity of the production with respect to the energy price and on the elasticity of substitution between oil and clean energy. Since different energy sources are heavily substitutable[2], it seems realistic to assume that a fall in the clean energy price results in a fall in the

[1] Technological changes are presented, for illustrative purpose, at a point of time to be able to represented the effects in a two-dimensional figure. Obviously, in a dynamic framework, the effects of changes are, in fact, integrated over time.

[2] Because joules that come from oil are identical to joules that come from any other energy source. Only storage and transportation differ from one source to another.

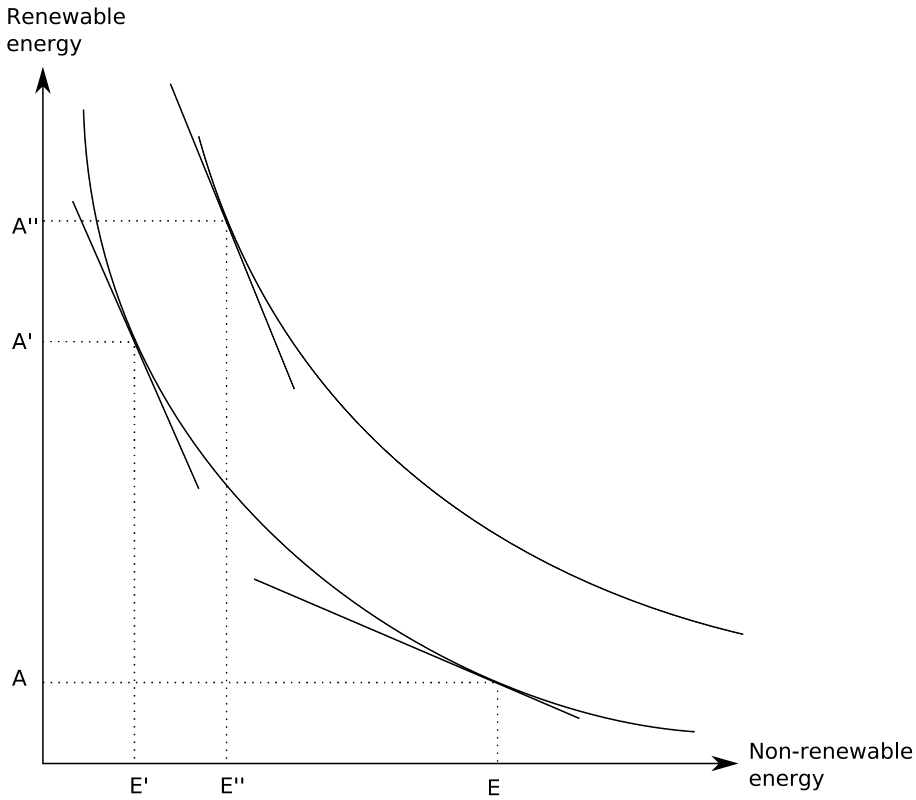

Figure 1: substitution between oil and clean energy

oil consumption.

The most relevant way to model these effects would be to introduce the clean energy A as a control variable into the model. However, by seek of simplicity, we propose a simpler modeling. The two consequences of an increase in X are an increase in the production and a fall in the use of oil. The net production function of country i is[3]:

$$Q_i = (\beta_{i,1} - X) E_i - \beta_2 E_i^2 + \eta_{i,1} X - \eta_2 X^2$$

where $(\beta_{i,1}, \beta_2, \eta_{i,1}, \eta_2) > 0$ are parameters[4].

Figure 2 presents the production net of the oil cost.

[3]The main technical problem with this formulation is that it does not respect the Innada Conditions. For Monte Carlo procedure of section 4, parameters are chosen such as the non negativity condition over E_i is respected for a relevant future.

[4]The interpretation of this parameters is explain below, see 2.4.

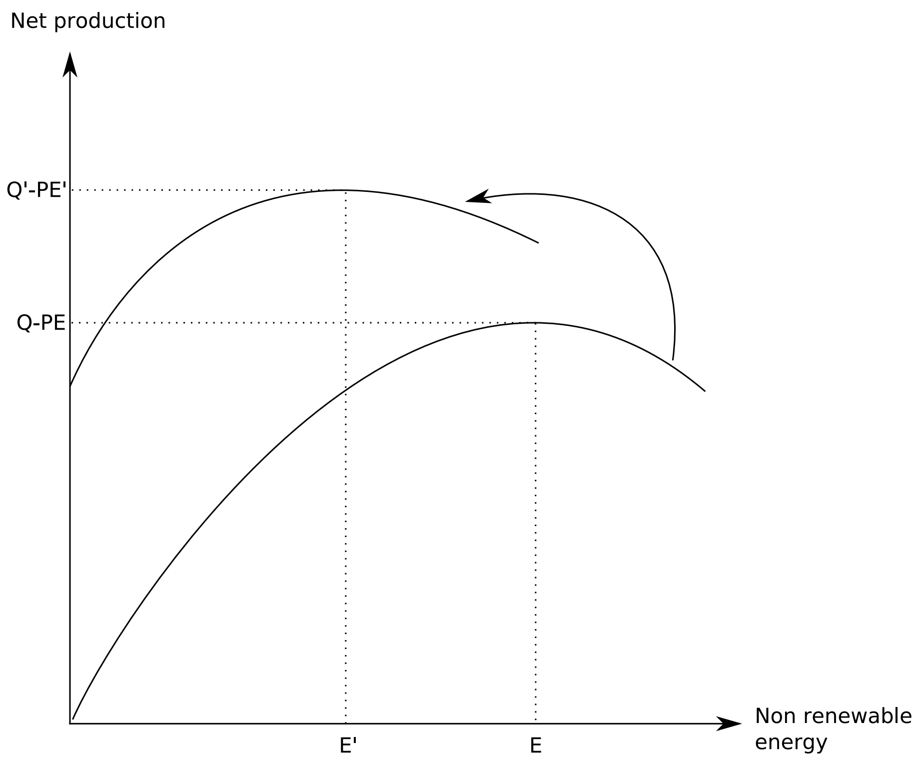

Figure 2: Impact of a technology improvement in the renewable energy sector

The curve at the south-east represents the net production as a function of the oil use before an increase in the knowledge stock. when X increases, the net production switches to the second curve. As one can see, the optimal level of production is higher after the switch, and the associated optimal level of oil is lower.

2.3 The research sector

The output of the research sector is an increase in X. Notice that X is a pure public good. The motivation to invest in research is twofold: first, it enables to lower the economic impact of the increasing scarcity of the nonrenewable resource. Second, it lowers the abatement cost of an environmental policy that would incite to substitute non-polluting to polluting energies. The country i invests I_i in research. X follows:

$$\dot{X} = \sigma (I_1 + I_2) - \epsilon X \tag{3}$$

where σ and ϵ are positive parameters.[5]

Country i faces an investment cost given by γI_i^2, where γ reflects the investment cost.

2.4 Welfare functions

The welfare function of country i is given by:

$$W_i = \int_0^\infty e^{-\rho t} \left[(\beta_{i,1} - P - X) E_i - \beta_2 E_i^2 + \eta_{i,1} X - \eta_2 X^2 - \alpha_i M^2 - \gamma I_i^2 \right] dt$$

where ρ is the discount rate.

Both countries differ from each other with respect to their sensitivity to the environment and to their wealth. The higher α_i, the higher the sensitivity to the environment. The higher $\beta_{i,1}$ and $\eta_{i,1}$, the richer. Indeed, the wealth of a country comes from it's capital accumulation. The capital accumulation makes the energy (both clean and polluting energy) more productive. It is assumed that:

[5] Two features of this knowledge production function must be discussed. First, we don't make the *giants' shoulders* assumption. That is to say that the research productivity does not depend on the current level of knowledge. This modeling differs from the one usually made in endogenous growth literature, such as in Romer (1990), where the research productivity is linear with respect to the knowledge stock. However, it should be noted that modeling the evolution of an aggregate stock of knowledge has to differ from the modeling of the evolution of a sectoral stock of knowledge. Indeed, (as pointed out by Aghion and Howitt (1998)) the number of new ideas that remain undiscovered in one particular sector should not be thought of as an infinite stock. The linear modeling in macroeconomic models accounts both for the knowledge increase in each sector (quality innovations) and for the increase of the number of sectors (variety innovations). Within a given sector, the best way to model the innovation would probably be a logistic function. It would account for the giants' shoulders effect when the stock of knowledge is low, and then for the rarefaction of the remaining undiscovered ideas when the stock of knowledge is high. To keep in touch with the linear quadratic formulation, the best modeling is the constant productivity assumption made in equation 3. Second, if no research investment is made, X decreases by ϵX per unit of time. This feature accounts for the necessity to maintain a research sector to transmit the knowledge from old generations to new generations.

$$\beta_{1,1} > \beta_{2,1}$$

and

$$\eta_{1,1} > \eta_{2,1}$$

In other words, country 1 is the rich one.

3 Cooperative and non-cooperative equilibria

3.1 Cooperative equilibrium

In this section, we characterize the optimal path that should be followed by the system in order to maximize a sum (W) of both objectives.

$$W \equiv W_1 + W_2$$

subject to 1, 2 and 3. In other words, this section characterizes the shape of an international agreement between both countries.

The cooperative problem can be restated as the minimization of

$$W = \int_0^\infty (y'Q^c y + v'R^c v)\, dt$$

subject to

$$\dot{y} = Ay + B^c v$$

$$y(t) \equiv e^{-\frac{1}{2}\rho t} \begin{pmatrix} P \\ M \\ X \\ 1 \end{pmatrix} ;\ v(t) \equiv e^{-\frac{1}{2}\rho t} \begin{pmatrix} \frac{X}{2\sqrt{\beta_2}} - \frac{\beta_{1,1}}{2\sqrt{\beta_2}} + \frac{P}{2\sqrt{\beta_2}} + \sqrt{\beta_2}E_1 \\ \frac{X}{2\sqrt{\beta_2}} - \frac{\beta_{2,1}}{2\sqrt{\beta_2}} + \frac{P}{2\sqrt{\beta_2}} + \sqrt{\beta_2}E_2 \\ I_1 \\ I_2 \end{pmatrix}$$

$$A \equiv \begin{pmatrix} -\frac{\rho}{2} - \frac{\zeta}{\beta_2} & 0 & -\frac{\zeta}{\beta_2} & \frac{\beta_{1,1}\,\zeta + \beta_{2,1}\,\zeta}{2\beta_2} \\ -\frac{1}{\beta_2} & -\frac{\rho}{2} - \delta & -\frac{1}{\beta_2} & \frac{\beta_{1,1} + \beta_{2,1}}{2\beta_2} \\ 0 & 0 & \frac{-\rho}{2} - \epsilon & 0 \\ 0 & 0 & 0 & -\frac{\rho}{2} \end{pmatrix}$$

$$B^c \equiv \begin{pmatrix} \frac{\zeta}{\sqrt{\beta_2}} & \frac{\zeta}{\sqrt{\beta_2}} & 0 & 0 \\ \frac{1}{\sqrt{\beta_2}} & \frac{1}{\sqrt{\beta_2}} & 0 & 0 \\ 0 & 0 & \sigma & \sigma \\ 0 & 0 & 0 & 0 \end{pmatrix} ;\ R^c \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & \gamma \end{pmatrix}$$

$$Q^c \equiv \begin{pmatrix} -\frac{1}{2\beta_2} & 0 & -\frac{1}{2\beta_2} & \frac{\beta_{1,1}}{4\beta_2} + \frac{\beta_{2,1}}{4\beta_2} \\ 0 & \alpha_1 + \alpha_2 & 0 & 0 \\ -\frac{1}{2\beta_2} & 0 & -\frac{1}{2\beta_2} + 2\eta_2 & \frac{\beta_{1,1} + \beta_{2,1}}{4\beta_2} - \frac{1}{\eta_2} \\ \frac{\beta_{1,1}}{4\beta_2} + \frac{\beta_{2,1}}{4\beta_2} & 0 & \frac{\beta_{1,1} + \beta_{2,1}}{4\beta_2} - \frac{1}{\eta_2} & -\frac{\beta_{1,1}^2}{4\beta_2} - \frac{\beta_{2,1}^2}{4\beta_2} \end{pmatrix}$$

$$S^c \equiv B^c R^{c\,-1} B^{c'}$$

The optimal linear strategy is given by

$$v^c = C^c y$$

where $C^c = -R^{c\,-1} B^{c'} K^c$ and K^c is the symmetric stabilizing solution of the following algebraic Riccati equation:

$$A' K^c + K^c A - K^c S^c K^c + Q^c = 0$$

One of the purposes of this paper is to analyze the equilibrium strategies of the players. However, C^c can not be analyzed in itself, because of the variable transformations we did in order to solve the model. This is why we define the following two transformation matrices:

$$TR_1 = \begin{pmatrix} \frac{0.5}{\sqrt{\beta_2}} & 0 & \frac{0.5}{\sqrt{\beta_2}} & -\frac{0.5\beta_{1,1}}{\sqrt{\beta_2}} \\ \frac{0.5}{\sqrt{\beta_{2,2}}} & 0 & \frac{0.5}{\sqrt{\beta_{2,2}}} & -\frac{0.5\beta_{2,1}}{\sqrt{\beta_2}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}; TR_2 = \begin{pmatrix} \frac{0.5}{\sqrt{\beta_2}} & 0 & 0 & 0 \\ 0 & \frac{0.5}{\sqrt{\beta_2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

They enable to compute the following matrix Z^c:

$$Z^c = TR_2 \left(C^c - TR_1 \right) = \begin{pmatrix} z^c_{1,1} & \cdots & \cdots & z^c_{1,4} \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ z^c_{4,1} & \cdots & \cdots & z^c_{4,4} \end{pmatrix}$$

such as the optimal strategy is given by:

$$\begin{pmatrix} E^c_1 \\ E^c_2 \\ I^c_1 \\ I^c_2 \end{pmatrix} = Z^c \begin{pmatrix} P \\ M \\ X \\ 1 \end{pmatrix}$$

3.2 Closed-loop differential game

In this section, we are looking for a Nash closed loop equilibrium. We define the following vectors and matrices:

$$v_i(t) \equiv e^{-\frac{1}{2}\rho t} \begin{pmatrix} \frac{X}{2\sqrt{\beta_2}} - \frac{\beta_{i,1}}{2\sqrt{\beta_2}} + \frac{P}{2\sqrt{\beta_2}} + \sqrt{\beta_2} E_i \\ I_i \end{pmatrix}$$

$$B_i^m \equiv \begin{pmatrix} \frac{\zeta}{\sqrt{\beta_{i,2}}} & 0 \\ \frac{1}{\sqrt{\beta_{i,2}}} & 0 \\ 0 & \sigma \\ 0 & 0 \end{pmatrix} ; R^m \equiv \begin{pmatrix} 1 & 0 \\ 0 & \gamma \end{pmatrix} ; S_i^m \equiv B_i^m R^{m\,-1} B_i^{m\prime}$$

$$Q_i^m \equiv \begin{pmatrix} -\frac{1}{4\beta_{i,2}} & 0 & -\frac{1}{4\beta_{i,2}} & \frac{\beta_{i,1}}{4\beta_{i,2}} \\ 0 & \alpha_i & 0 & 0 \\ -\frac{1}{4\beta_{i,2}} & 0 & -\frac{1}{4\beta_{i,2}} + \eta_{i,2} & \frac{\beta_{i,1}}{4\beta_{i,2}} - \frac{1}{2\eta_{i,2}} \\ \frac{\beta_{i,1}}{4\beta_{i,2}} & 0 & \frac{\beta_{i,1}}{4\beta_{i,2}} - \frac{1}{2\eta_{i,2}} & -\frac{\beta_{i,1}^2}{4\beta_{i,2}} \end{pmatrix}$$

Each country seeks to minimize:

$$\int_0^\infty (y' Q_i y + v_i' R v_i) dt$$

subject to:

$$\dot{y} = Ay + B_i^m v_i + B_j^m v_j \; j \neq i$$

In other words, this section characterizes the behavior of both countries when they act with no cooperation[6]. As shown in Engwerda (2005), the markovian linear strategy, for player i, is given by:

$$v_i^m = C_i^m y$$

where $C_i^m = -R^{m\,-1} B_i^{m\prime} K_i^m = \begin{pmatrix} c_i^m(1,1) & \cdots & \cdots & c_i^m(1,4) \\ c_i^m(2,1) & \cdots & \cdots & c_i^m(2,4) \end{pmatrix}$ and K_i^m, $i = 1,2$ are the symmetric stabilizing solutions of the following system of algebraic Riccati equations[7]:

[6]The only cooperation assumed here is about the choice of stabilizing strategies.

[7]The algorithm used to solve for this system is described in (ANNEX 1)

$$
\begin{aligned}
(A - S_2^m K_2^m)' K_1^m + K_1^m (A - S_2^m K_2^m) - K_1^m S_1^m K_1^m + Q_1^m + K_2^m S_2^m K_2^m = 0 \\
(A - S_1^m K_1^m)' K_2^m + K_2^m (A - S_1^m K_1^m) - K_2^m S_2^m K_2^m + Q_2^m + K_1^m S_1^m K_1^m = 0
\end{aligned}
\tag{4}
$$

Let's define C^m and v^m as:

$$
C^m = \begin{pmatrix} c_1^m(1,1) & \cdots & \cdots & c_1^m(1,4) \\ c_2^m(1,1) & \cdots & \cdots & c_2^m(1,4) \\ c_1^m(2,1) & \cdots & \cdots & c_1^m(2,4) \\ c_2^m(2,1) & \cdots & \cdots & c_2^m(2,4) \end{pmatrix} ; \mathrm{v^m} = C^m y
$$

Like in the previous section, a transformation has to be done in order to interpret the results. Z^m is defined as:

$$
Z^m = TR_2 \left(C^m - TR_1 \right) = \begin{pmatrix} z_{1,1}^m & \cdots & \cdots & z_{1,4}^m \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ z_{4,1}^m & \cdots & \cdots & z_{4,4}^m \end{pmatrix}
$$

such as the markovian strategies are given by:

$$
\begin{pmatrix} E_1^m \\ E_2^m \\ I_1^m \\ I_2^m \end{pmatrix} = Z^m \begin{pmatrix} P \\ M \\ X \\ 1 \end{pmatrix}
$$

4 Monte Carlo procedure

A complete solution of the model would express each endogenous variable of the model as a function of the set of parameters. Let $f_i(\pi)$ be the function that gives the endogenous variable ϕ_i and π the set of the N exogenous parameters indexed by k. Unfortunately, such a solution is very hard, if possible, to compute. However, a Monte Carlo procedure enables to give a Taylor approximation of f_i for a range of parameters values. Indeed, for a given π, simple algorithms compute the particular solution of the model. We run 1000 simulations, with, at each iteration j, a randomly chosen π_j. Let's Call $\bar{\phi}_i$ the average value of ϕ_i in the sample. Then, we compute the ols estimators $\hat{\psi}$ of the set of parameters ψ of the following function:

$$\phi_i = \psi_{1,i} + \sum_{k=1}^{N} \psi_{(k+1,i)} \pi_k + \sum_{k=1}^{N} \sum_{k' \geq k} \psi_{(k,k',i)} \pi_k \pi_{k'} + e_i$$

Where e is an error term. This error term is treated as random, even if it's not: it is the difference between the genuine deterministic function and its Taylor approximation.

At each iteration, the parameters are chosen via an homogeneously distributed density function defined between $0.5\mu_{\pi_i}$ and $1.5\mu_{\pi_i}$ where μ_{π_i} is the mean value of π_i.

5 Results

5 and 6 represent the equilibrium strategies, respectively for the cooperative game and for the markovian game. In each matrix is indicated the sign of the corresponding component as it appears in the simulations, followed by the percentage of occurrence of this sign when it is less than 100%. In the matrix representing the markovian strategy a comparision symbol indicate how is the absolute value of the corresponding component compared to the same component in the cooperative equilibrium. Again, between brackets is the percentage of occurence of this sense of comparision when it is less than 100%.

$$\begin{pmatrix} E_1^c \\ E_2^c \\ I_1^c \\ I_2^c \end{pmatrix} = \begin{pmatrix} - & - & - & + \\ - & - & - & + \\ + & + & - & + \\ + & + & - & + \end{pmatrix} \begin{pmatrix} P \\ M \\ X \\ 1 \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} E_1^m \\ E_2^m \\ I_1^m \\ I_2^m \end{pmatrix} = \begin{pmatrix} - > & - < & - > & + > \\ - > & - < & - > (0.92) & + > \\ + < (0.97) & +/- < & - < & + < (0.98) \\ +(0.99) < & + > (0.73) & - < & + < (0.97) \end{pmatrix} \begin{pmatrix} P \\ M \\ X \\ 1 \end{pmatrix} \tag{6}$$

At this point, rather obvious observations can be done. In both games, the oil price, the pollution stock and the stock of knowledge incite to emit less when they increase. This is true in 100% of the simulations. Indeed, these three stocks increase the cost of using oil: the direct private cost (price), the external cost (pollution) and the opportunity cost (knowledge) The negative impact of pollution over the emissions is, as could be expected, stronger in the cooperative game than in the markovian game. Indeed, in the cooperative case, each player takes into account the negative impact of pollution not only on his own situation, but also on the one of the other player. It can be added that the difference is rather important for the emissions of the rich player. Indeed, $\frac{\bar{z}^c_{1,2}}{\bar{z}^m_{1,2}} \simeq 3$

Surprisingly, the same does not apply for the negative impact of oil price on oil consumption: this impact is stronger in the Markovian game than in the cooperative game. In order to understand this result, we look for the coefficients that are the most affected by the change from the Markovian equilibrium to the cooperative equilibrium in the regressions of $\bar{z}^c_{1,1}$ and of $\bar{z}^m_{1,1}$. We also look for the coefficients the signs of which change between both situations.

The coefficient associated to γ is significantly negative in the Markovian equilibrium, whereas it is significantly positive in the cooperative equilibrium. That is to say that the higher the cost of research, the more the rich country reacts to high oil prices by lowering its oil consumption when both countries don't cooperate. When they cooperate, the higher the cost of research, the less the rich country reacts to high oil prices.

In order to go deeper in the analysis, we have to study the coefficients of the regressions explained in section 4. The great number of coefficients don't allow to study each of them.

5.1 On Chinese growth

The non-inclusion of emerging countries such as China is an important critic addressed to the Kyoto protocol by the United States.

(To be completed)

5.2 The paradox of knowledge

Maybe on of the most striking result of this model is that an agreement based both on R&D and on emission cutting reduces the aggregate cumulative R&D expenditure. Of course, the public good feature of the knowledge implies that, *ceteris paribus*, the aggregate R&D expenditure is higher in the cooperative case than in the non cooperative case. But, in the cooperative case, every thing else is not equal. In particular, in the cooperative case, both the stock of pollution and the oil prices are smaller than in the non cooperative case. This results from the limitation of the emissions which is a consequence of the agreement. In this situation, the private incentives to invest in research is higher, since knowledge is a way to offset higher prices and a higher level of pollution. Of course, the cooperative outcome remains optimal and better than the Markovian Outcome, even if it implies less research. But this finding is a counter-argument to the view that technology should be the key of a future agreement. This is however a generalization of a result van der Ploeg and de Zeeuw (1994), establishing that in the absence of international coordination for pollution control, levels of clean technology stocks are too excessive.

5.3 The carbon leakage revisited

As shown in cooperative game literature [References to be completed], the emission reduction by the coalition members incites the non-member countries to increase their own emissions through a fall in oil prices. Even if this paper is not in the same context, we show results close to this one, in a dynamic framework

(To be completed)

5.4 The impact of an exogenous economic boom in the poor countries

We study the impact of an exogenous economic boom in the poor countries in terms of aggregate pollution emission rates, R&D investment, oil prices...

(To be completed)

6 Conclusion

We study in this paper the problem of international coordination in Climate policy using a three stocks, two asymmetric countries differential game in Markov-linear strategies. *(To be completed)*

A Algorithm for the system of coupled algebraic Riccati equations

The algorithm used to compute the solutions of system 4 is taken from Freiling et al. (1996).

1. We compute $K_1^m(0)$ and $K_2^m(0)$ the stabilizing symmetric solutions of the following autonomous algebraic Riccati equations:

$$\begin{aligned} A'K_1^m + K_1^m A + Q_1 - K_1^m S_1^m K_1^m = 0 \\ A'K_2^m + K_2^m A + Q_2 - K_2^m S_2^m K_2^m = 0 \end{aligned}$$

2. We compute the following discrete dynamical system, by taking $K_1^m(0)$ and $K_2^m(0)$ as initial conditions:

$$\begin{aligned} & K_1^m(i+1)\left[A - S_2^m K_2^m(i)\right] + \left[A - S_2^m K_2^m(i)\right]' K_1^m(i+1) + Q_1 \\ & -K_1^m(i+1) S_1^m K_1^m(i+1) = 0 \end{aligned}$$

$$\begin{aligned} & K_2^m(i+1)\left[A - S_1^m K_1^m(i)\right] + \left[A - S_1^m K_1^m(i)\right]' K_2^m(i+1) + Q_2 \\ & -K_2^m(i+1) S_2^m K_2^m(i+1) = 0 \end{aligned}$$

   Where i is the number of iterations

3. We stop after i^*, where i^* is such as:

$$\begin{aligned} & \left|K_1^m(i^*)\left[A - S_2^m K_2^m(i^*)\right] + \left[A - S_2^m K_2^m(i^*)\right]' K_1^m(i^*) + Q_1 - K_1^m(i^*) S_1^m K_1^m(i^*)\right| \\ & + \left|K_2^m(i^*)\left[A - S_1^m K_1^m(i^*)\right] + \left[A - S_1^m K_1^m(i^*)\right]' K_2^m(i^*) + Q_2 - K_2^m(i^*) S_2^m K_2^m(i^*)\right| < \varepsilon \end{aligned}$$

   Where ε is a small number, set equal to 10^{-8} in the current simulations.

4. $K_1^m(i^*)$ and $K_2^m(i^*)$ are the solutions of system 4.

Notice that there exist no proof of convergence for this algorithm. However, in the simulations made for this paper, it always converged.

References

Aghion, P., Howitt, P., 1998. Endogenous Growth Theory. MA: MIT Press.

Engwerda, J. C., 2005. LQ Dynamic Optimization and Differential Games. Wiley.

Freiling, G., Jank, G., Abou-Kandil, H., 1996. On global existence of solutions to coupled matrix riccati equations in closed loop nash games. IEEE Trans. Automatic Conctrol 41, 264–269.

Fudenberg, D., Tirole, J., 1992. Game Theory (second printing).

Golombek, R., Hoel, M., 06 2005. Climate policy under technology spillovers. Environmental & Resource Economics 31 (2), 201–227.

Golombek, R., Hoel, M., 2006. Second-best climate agreements and technology policy. Advances in Economic Analysis & Policy 6 (1), 1472–1472.

IEA, 2006. Energy Technology Perspectives – Scenarios & Strategies to 2050. The Online Bookstore, IEA.

List, J. A., Mason, C. F., November 2001. Optimal institutional arrangements for transboundary pollutants in a second-best world: Evidence from a differential game with asymmetric players. Journal of Environmental Economics and Management 42 (3), 277–296.

Romer, P. M., October 1990. Endogenous technological change. Journal of Political Economy 98 (5), 71–102.

van der Ploeg, F., de Zeeuw, A., 1994. "Sustainable growth and renewable resources in the global economy" in Carraro, C. (ed.) Trade, Innovation, Environment. Kluwer, Dordrecht, Ch. 2.3, pp. 229–240.