The classical theory of value and exhaustible resources

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Abstract

Several post-Sraffian attempts have been made to build a theory of exhaustible resources of classical inspiration. We examine the formalisations successively elaborated by Parrinello, Schefold and Kurz & Salvadori, and show that all of them suffer from logical inconsistencies. We also respond to the critiques addressed to our own approach, based on the study of a simple model with one commodity and one exhaustible resource, called the corn-guano model.

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1 Introduction

The classical theory has the ambition to determine the long-term values of commodities. Although it recognizes the influence on market prices of temporary fluctuations and adaptations of supply and demand, it considers these phenomena to be of secondary importance and theferore largely ignores them. Instead the classical theory emphasizes the importance of objective factors in the determination of values (or fundamental prices). With given techniques of production and a given distribution, and in circumstances of constant returns to scale or constant levels of production, it considers values to be constant. This implies that exchange ratios, or relative prices, are constant too. By contrast, modern theories of prices give prominent roles to expectations, which are governed by subjective phenomena. In this paper we deal with a case where expectations are objective, in the sense they are based on a future real event which is supposed to be perfectly known by all economic agents: the exhaustion of a natural resource. We will show that the integration of exhaustible natural resources entails a modification of the classical theory, and in particular that it leads to a situation in which, with given techniques of production, relative prices are not constant. The attempts which have been undertaken, for some 25 years now, to develop a theory of exhaustible resources in a classical perspective, have revealed a great amount of theoretical divergence among the participants. In this note we first of all draw attention to some of the crucial methodological concepts involved (real rate of profit, corn-guano model) before we critically examine the main theoretical approaches which have been explored by various authors.

2 The Corn Model

It might be appropriate to start with a brief reminder of a simple economic model which is very familiar to those who have studied the classical theory: the corn model (see Bidard, 2004, ch. 1 for a more extensive presentation). Let us assume that there exists only one commodity, corn, which is produced by means of itself and labour. Let us furthermore assume that the same production process is used year in, year out, and that it can be described schematically as follows:

$$a \text{ (corn)} \oplus l \text{ (labour)} \longrightarrow 1 \text{ (corn)} \quad (1)$$

The long-term equilibrium price equation which corresponds to this process is equal to:

$$(1+r)(ap+lw) = p \quad (2)$$

with r the rate of profit, p the price of corn, and w the wage.

Taking corn as numeraire ($p = 1$), equation (2) determines the relation between the rate of profit and the wage expressed in units of corn (i.e. w/p). If the rate of profit is treated as the independent variable, it should not exceed the limit value of $(1 - a)/a$, otherwise the wage will be negative. Alternatively, if the wage is treated as the independent variable, it should not exceed the limit value of $(1 - a)/l$, otherwise the rate of profit will be negative.

Instead of taking corn as numeraire, we could also take labour ($w = 1$). We then obtain a relation between the rate of profit and the price of corn expressed in labour (i.e. p/w, the inverse of the wage expressed in corn). It is easy to show that the relation is equivalent to the one obtained with corn as numeraire.[1] In other words, whatever the numeraire - a mixed corn/labour numeraire would yield exactly the same result - we always find the same relation between the "real rate of profit" and the "real wage".

3 The Real Rate of Interest

If we leave the familiar framework of the corn model, the notion of the real rate of profit becomes somewhat more complicated. One element of disturbance concerns the presence of money. In macroeconomic theory it is customary to make a distinction between the nominal rate of interest i and the real rate of interest r. The two rates are different if the purchasing power of money changes in the period under consideration, i.e. if the rate of inflation π is different from zero. One often uses the approximation $r \approx i - \pi$, but the exact formula is:

$$r = \frac{1 + i}{1 + \pi} - 1 \tag{3}$$

The rate of inflation is defined as the change in the overall price level, and usually the implicit assumption is made that prices are always changing in the same proportion.

The point we want to make has nothing to do with the existence of money, and for simplicity we therefore assume there is no money. What we are interested in is the effect of a change in *relative* prices upon the rate of interest. The cause of the change need not occupy us at this stage; all that matters is that for some reason relative prices are not constant.

Let us consider a one-year period beginning at time t and ending at time $t + 1$. Suppose that at time t we possess a basket of goods equal to $a(t)$

[1] In the first case we have $\frac{w}{p} = \frac{1 - (1 + r)a}{(1 + r)l}$, and in the second $\frac{p}{w} = \frac{(1 + r)l}{1 - (1 + r)a}$.

which we invest in a one-year project. At the end of the year we reap the benefits of our investment in the form of a basket of goods equal to $b(t+1)$. Except when $b(t+1)$ is proportional to $a(t)$, we cannot calculate the rate of return of the project unless we know the values of the goods at t and $t+1$. In a static equilibrium only relative values are needed to determine the rate of return, but in an intertemporal context the numeraire comes into play. Three distinct rates of return can be defined.

We begin by assuming that the same numeraire is used at times t and $t+1$; this means that we use a constant yardstick to measure absolute values. Let the numeraire (i.e. a good or basket of goods) be represented by the semipositive vector n; then the absolute values at times t and $t+1$, i.e. $v(t)$ and $v(t+1)$, are such that $nv(t) = 1$ and $nv(t+1) = 1$. The *apparent* rate of return r^n, i.e. the one that appears when using the given numeraire n for purposes of valuation, compares the *apparent* value of the investment $a(t)$ to the *apparent* value of the outcome $b(t+1)$:

$$r^n = \frac{b(t+1)v(t+1) - a(t)v(t)}{a(t)v(t)} = \frac{b(t+1)v(t+1)}{a(t)v(t)} - 1 \tag{4}$$

A second possibility is that a different numeraire is used at times t and $t+1$; this implies that we are using a variable yardstick. Let the numeraire at time t be $n(t)$ and at time $t+1$ $n(t+1)$, with $n(t) \neq n(t+1)$; then the absolute values at times t and $t+1$, $\widetilde{v}(t)$ and $\widetilde{v}(t+1)$, are such that $n(t)\widetilde{v}(t) = 1$ and $n(t+1)\widetilde{v}(t+1) = 1$. The *fake* rate of return $\widetilde{r}$, i.e. the one that appears when using a non-constant numeraire, is equal to:

$$\widetilde{r} = \frac{b(t+1)\widetilde{v}(t+1) - a(t)\widetilde{v}(t)}{a(t)\widetilde{v}(t)} = \frac{b(t+1)\widetilde{v}(t+1)}{a(t)\widetilde{v}(t)} - 1 \tag{5}$$

With a flexible yardstick virtually any result can be produced.[2] In particular the sign of the rate of return $\widetilde{r}$ can be positive, negative or zero, even in the case where $b(t+1) > a(t)$, which unambiguously indicates a positive return. For this reason we reject variable numeraires, and confine ourselves to calculations made by a constant yardstick.

At first sight there seems to be no reason why one numeraire would be better than another - the choice of numeraire is generally considered to be arbitrary. But suppose that we would like to know the rate of return in terms of "what really counts for us". If we trade coal but consume only corn, then it would certainly interest us to compare the units of corn we sacrifice at time t by buying coal to the units of corn we can earn at time $t+1$ by

[2] Let $n(t)\widetilde{v}(t+1) = \gamma$. Then: $\widetilde{r} = \gamma(1 + r^n) - 1$.

selling coal. For this both the investment and the outcome must be expressed in corn, which in this case represents what matters for us. More generally, let us assume there exists a *standard of value* s which captures what really counts for us. At time t, the investment $a(t)$ is worth $a(t)v(t)/sv(t)$ units of the standard of value; and at time $t+1$, the outcome $b(t+1)$ is worth $b(t+1)v(t+1)/sv(t+1)$ units of the standard of value. The *real* rate of return r^*, based upon a comparison of the *real* investment and the *real* outcome, where real means that they are measured in units of the standard of value, can therefore be defined as:

$$r^* = \frac{\frac{b(t+1)v(t+1)}{sv(t+1)} - \frac{a(t)v(t)}{sv(t)}}{\frac{a(t)v(t)}{sv(t)}} = \frac{b(t+1)v(t+1)}{a(t)v(t)} \frac{sv(t)}{sv(t+1)} - 1 \tag{6}$$

If we denote by θ^n the rate of appreciation of the standard of value s in terms of the prices defined by numeraire n, i.e.

$$\theta^n = \frac{sv(t+1)}{sv(t)} - 1 \tag{7}$$

we can rewrite (6) in a way which brings out the analogy with (3):

$$r^* = \frac{1 + r^n}{1 + \theta^n} - 1 \tag{8}$$

It is easy to see that the real rate of return is equal to the apparent rate of return only if $\theta^n = 0$.

The following results are straigthforward.

Proposition 1 *Except by fluke, the real rate of profit is equal to the apparent rate of profit in only two cases: (i) when relative values are constant, since for a given numeraire this implies that* $v(t) = v(t+1)$*; (ii) when the standard of value* s *happens to be proportional to the numeraire* n*, say* $s = \alpha n$*, since then* $sv(t) = sv(t+1) = \alpha$*.*

Proposition 2 *In a situation of changing relative values, consider two different numeraires,* n *and* n'*, and one standard of value,* s*. Then the real rate of profit is the same whether numeraire* n *or* n' *is used.*

Proposition 3 *In a situation of changing relative values, consider one numeraire,* n*, and two standards of value,* s *and* s'*. Then the real rate of profit is in general different whether standard* s *or* s' *is used.*

The main result is that to each standard of value a different real rate of return corresponds. This reminds one of the statement made long ago by Irving Fisher, "that the rate of interest is always relative to the standard in which it is expressed" (Fisher, 1930: 41). If all investors have the same standard of value - which is not be taken for granted, as Keynes observed in (1936: 224-5) - then the analysis can be simplified considerably by assuming that the numeraire is proportional to the standard of value, since then the apparent rate of return will automatically be equal to the real rate of return. We make this assumption in the paper, but without forgetting that the numeraire now plays a double role: it is both numeraire *and* standard of value.

In a framework such as the classical theory of value, where equilibrium prices are defined on the basis of a uniform rate of profit, the determination of the rate of profit may be of crucial importance. It is largely irrelevant if the notion of equilibrium entails that relative prices are always constant (but observe that the issue cannot be avoided if one wants to extend the analysis to out-of-equilibrium behaviour). If, however, equilibrium prices are not necessarily constant, then the issue becomes relevant even if the analysis is limited to the study of equilibrium positions. What we would like to show is that the presence of exhaustible resources confronts us with a case in which the equilibrium relative prices *are* changing.

4 Exhaustible Resources

The previous analysis can be applied to the case of an exhaustible resource owner. Basically, the owner faces a choice between selling his resource immediately, or letting it lie idle and sell it in the future. Nothing is lost if we reduce his problem to the choice of selling one unit at time t or waiting for one year and selling it at time $t+1$. In the first case the sale gives him an immediate revenue of $z(t)$, i.e. the price of one unit of the resource at time t, which he can then invest. In the second case he simply waits and obtains a revenue of $z(t+1)$ at time $t+1$. If one of the two options were more profitable than the other, all resource owners would follow the most profitable course of action, which would mean that either the *whole* supply of the resource would be exploited at time t, or that *none* of it would. In general this is impossible: in any period of time, except the one in which the resource is depleted, part of the supply is exploited and part of it is conserved. This means that in equilibrium the two options must be equally profitable. If the first option allows you to obtain a rate of return r when you invest the proceeds of selling the resource, then so must the second. This leads to the following equilibrium condition:

$$r = \frac{z(t+1)}{z(t)} - 1 \tag{9}$$

from which it easily follows that:

$$(1+r)z(t) = z(t+1) \tag{10}$$

Equation (10) implies that the price of the exhaustible resource, i.e. its royalty, rises at a rate equal to the ruling rate of profit. This is the result known as the *Hotelling rule*, after the seminal contribution of Harold Hotelling (1931). Since exhaustible resources are typically goods which are used for the production of other goods, the Hotelling rule implies that one can expect that the prices of *all* goods will change over time.

5 The Corn-Guano Model

The classical theory of prices is often identified with the theory of long-term or normal prices, which are assumed to be constant as long as there are no changes in the methods of production. On this interpretation, the classical theory seems to be at odds with the Hotelling rule. A representative of this view is Bertram Schefold, who starts from the adage that "the classical approach relies on the conception of normal prices and is inseparable from it" (Schefold, 2001: 320), and therefore accepts only with the greatest reluctance the possibility of changing relative prices. We do not share Schefold's view, and we reject his way of dealing with the issue (see further). Instead we maintain that the classical theory cannot ignore changing relative prices and must find a way of integrating them. In two previous contributions (2001a, 2001b) we have shown, by means of the corn-guano model, that this is indeed possible.

The corn-guano model has been conceived as a methodological tool: its analytical simplicity allows us to shed light on the original economic features linked to the introduction of exhaustible resources. However, its structure is rich enough to initiate the reader to the study of the dynamics of models with exhaustible resources. There is only one produced commodity, called corn, and one exhaustible resource, called guano. Corn can be produced in a one-period time either by means of the "guano method":

$$a_1 \text{ (corn)} \ \oplus \ l_1 \text{ (labour)} \ \oplus \ 1 \text{ (guano)} \ \longrightarrow \ 1 \text{ (corn)} \tag{11}$$

or by means of the "backstop method" (which will be necessarily used after the exhaustion of the stock of guano):

$$a_2 \text{ (corn)} \oplus l_2 \text{ (labour)} \longrightarrow 1 \text{ (corn)} \tag{12}$$

The quantities of guano not used up to date t are available at date $t+1$:

$$1 \text{ (guano)} \longrightarrow 1 \text{ (guano)} \tag{13}$$

The two production processes and the guano conservation process admit constant returns. During any period, the operated methods yield the same rate of profit whereas the non-operated method(s) do not yield more. For simplicity, it is assumed that the date T when the stock of guano becomes exhausted is known. The underlying hypotheses may be that the initial stock and the demand for corn are exogenously given and that the guano method is continuously used until exhaustion.[3]

As is usual in Sraffian models, we treat the rate of profit as exogenously given. What this means in real terms depends upon the standard of value adopted by investors. We recall that in our case the numeraire acts as a standard of value. For period t, which begins at date t and ends at date $t+1$, the price system is such that:

$$p(t+1) \leq (1+r)\left[a_1 p(t) + z(t) + l_1 w(t)\right] \qquad [y_1(t)] \tag{14}$$

$$p(t+1) \leq (1+r)\left[a_2 p(t) + l_2 w(t)\right] \qquad [y_2(t)] \tag{15}$$

$$z(t+1) = (1+r)z(t) \tag{16}$$

$$dp(t) + fw(t) = 1 \tag{17}$$

where $p(t)$ is the price of corn, $z(t)$ the royalty of guano, $w(t)$ the wage, and $y_1(t)$ and $y_2(t)$ the activity levels of the two corn production processes, all at date t. Equation (17) is the numeraire equation.

In the first version of the model (Bidard and Erreygers, 2001a), corn is chosen as numeraire ($d > 0$, $f = 0$). The rate of profit cannot be too high; we assume it is such that:

$$r < \frac{1-a_1}{a_1}, \quad r < \frac{1-a_2}{a_2} \tag{18}$$

(these inequalities are required in order that processes (11) and (12) can sustain that rate of profit: the idea comes from the Ricardian corn model).

[3]Ex post, one must check that the last assumption is consistent with the analysis of prices, i.e. one must check that, up to date T, the guano method is cheaper that than the backstop method.

As long as guano is not exhausted, the preservation process (13) is operated, therefore the price of guano, or royalty rises at a rate equal to the rate of profit (the Hotelling rule). Since the speed of evolution of the royalty is determined, it suffices to know its level at some date. A simple economic argument is:

> At the moment of exhaustion (...) we expect the backstop method to be used alongside the guano-method. Only by fluke would the then remaining supply of guano be sufficient to satisfy the whole demand for corn by means of the guano process: normally the remaining quantity will be too low, and the backstop process must be operated to fill the gap. (Bidard and Erreygers, 2001a: 249)

The co-existence of the two processes in the period of exhaustion requires that they are equally costly at that time. This condition determines the royalty at the date T of exhaustion, when it is equal to the differential rent between the two processes to produce corn. Thanks to the Hotelling rule, the royalties in the preceding periods can be calculated by backward induction. The last unknown, viz. the real wage, is then obtained. As a consequence of the increasing royalties, the real wage decreases as time passes, a phenomenon at variance with the behaviour of Ricardian models without exhaustible resources. One may alternatively assume that the real wage is given and show that the current rate of profit declines up to the exhaustion period.

In a second version of the model (Bidard and Erreygers, 2001b), the numeraire is a given combination of d units of corn and f units of labour, with both d and f positive. During the exhaustion period T, these two processes are operated simultaneously. After the exhaustion of guano, only the second process can be operated, and expressions (14) and (16) become irrelevant. Since the relative price of corn and labour changes with time, the profitability of a given process now depends on the composition of the numeraire, in the same way as the profitability of an international firm depends on whether it is calculated in dollars or euros. The dynamics depend on the numeraire and become complex. The study of system (14) to (17) shows that the price equations admit one degree of freedom, say the level of $p(T)$. Once $p(T)$ is known, the system can be solved by means of backward induction. This procedure, however, leads to a negative price $p(T-\tau)$ for τ great enough, except for one specific choice of $p(T)$. That choice defines what we have called the 'natural path'. In fact, there exists an infinite number of paths with positive prices for T periods, but for large T all these paths are close to the natural path. Comparable complications occur in multisector models (several produced commodities) even when the numeraire is made of a unique commodity, but their resolution is similar when the formulae obtained for the corn-guano model are conveniently re-interpreted.

6 Parrinello's Alternative: Abandoning (Perfect) Foresight

Exhaustible resources would not present a major obstacle to post-Sraffian theory if their exhaustion were of no concern to economic agents. This would be the case if they foresaw that the supply of these resources would be forever sufficient to cover demand (e.g. because the resource becomes obsolete after a certain date). When exhaustion is not an issue, there is no reason why the price of exhaustible resources should change over time. Even stronger: their price would be zero because they are 'free goods'.

So the interesting case arises when economic agents do worry about exhaustion. Two points of view may be adopted here. Either one assumes that agents acknowledge that exhaustion will be on the agenda some time in the future, but do not have a clue about the date at which exhaustion will occur. Or, alternatively, one assumes that agents know the date of exhaustion exactly. Let us designate these hypotheses as those of 'imperfect' and 'perfect foresight', respectively. There is no doubt that the hypothesis of imperfect foresight is more realistic. From a theoretical point of view, however, it has the disadvantage of making the determination of prices subject to uncertainty and fragile hypotheses on expectations. The hypothesis of perfect foresight, on the other hand, is certainly heroic, but it allows us to calculate prices with certainty.

Following Hotelling (1931) and a large part of the literature on exhaustible resources, we have assumed perfect foresight in our corn-guano model. Parrinello (2001), however, explicitly rejects this hypothesis and assumes that the date of exhaustion is unknown. In order to close the model he must come up with an alternative assumption. The trouble is that Parrinello's assumption – the *rank condition* – is of a purely mathematical character, and may be in conflict with other assumptions of the model.

Parrinello's oil-corn model is very similar to the corn-guano model. As in our model, Parrinello assumes that the rate of profit is given, that corn serves as the numeraire, and that before the exhaustion of guano, two processes are available for the production of corn. (In Parrinello's model the processes can change over time, but this is a non-essential variant.) It is easy to show that, in order to arrive at a determinate solution, in exactly one period two processes must be operated simultaneously while in all others only one process (the cheapest of the two available) is used. If two processes were operated in more than one period, the system of prices would be over-determined; and if in no period two processes were used, it would be under-determined. In our corn-guano model a simple economic criterion is invoked to state that the two-process period must be the period of exhaustion (cf. the quotation in the previous section). Parrinello does not address this economic argument and does not assume that the two-process period is necessarily the period of exhaustion. In his oil-corn model, the period of exhaustion is unknown and the period of coincidence may be any period before exhaustion. Towards the end of the article, Parrinello seems to opt for the solution that the two-process period must be the initial period, on the grounds that "*The future cannot affect the past.*" (Parrinello, 2001: 311). Right, but what matters here is that (expectations about) the future can affect the *present*. The difficulty with Parrinello's procedure is that, except by fluke, the price of the exhaustible resource will be either too high or too low. This 'razor edge' problem was already clearly present in Hotelling's original formulation:

- If the initial price of the resource is too high, the resource will be priced out of the market before it is exhausted; but since no resource owner wants to end up with an unsold stock of his resource, there will be a downward pressure on the resource price.

- On the other hand, if the initial price is too low, resource owners will realise that they can increase their prices without running the risk of remaining stuck with an unsold stock, and so there will be an upward pressure on the resource price.

Hence both a too high and a too low price are incompatible with the notion of long-term equilibrium, which implies the notion that no agent wants to change her behaviour. Given that the price of an exhaustible resource increases at the rate of interest, its right initial level is the one which ensures that its level at the time of exhaustion equalises the cost of the resource-using process and that of the backstop process. Nothing guarantees that in Parrinello's model the price of the exhaustible resource is at this level.

Independently of these economic considerations, a strong argument against Parrinello's proposal is that it is self-contradictory and time inconsistent. Parrinello rightly stresses the fact that the price of the exhaustible resource cannot be permanently equal to the differential rent between the resource-using method and the backstop method, and his problem is to identify the period when the coincidence occurs. Since the period referred to as "today" moves when time passes, Parrinello's rule (coincidence in the present period) would be wrong tomorrow if it were true today. This contradiction does not occur when the period of coincidence is defined independently of the origin of time as that of exhaustion.

7 Schefold's Nostalgia for Long Period Analysis

The corn-guano model is a theoretical tool that sheds light on the treatment of exhaustible resources in a classical approach. It proceeds by building a bridge between the corn model, which belongs to the Ricardian tradition, and Hotelling's seminal model on exhaustible resources. Like its basic bricks, it is an economic abstraction and its ambition is methodological. Its main feature is to proceed by mixing the simplest characteristics of two models: three equations are sufficient and their treatment is transparent. Since there are substantial differences between the solution of the corn-guano model and that of the standard corn model, these differences can be attributed unambiguously to the presence of an exhaustible resource. For instance, in the corn-guano model the relationship between the wage and the rate of profit is not time invariant, despite the fact that the same production process remains in use as long as the stock of guano is not exhausted. This result is at odds with the 'objective' point of view defended by the classical economists and Sraffa, according to which the knowledge of the operated methods and the real wage suffices to infer the level of the rate of profit.

Once it is acknowledged that the introduction of exhaustible resources leads to qualitatively different results, a second step consists of examining the degree of generality and the robustness of the laws derived from the simple model (for instance: is the exhaustible resource always used continuously until exhaustion?), and of questioning key concepts (how is the notion of rate of profit defined in the presence of changing prices?). This justifies the analysis of more complicated multisector models. In our minds, models of exhaustible resources are simple cases of models characterised by time-varying prices, with the cause of changing relative prices lying in production (as opposed to psychological motives, such as the consumer's impatience). Therefore, the study of the corn-guano model is the first step in the elaboration of a research program of classical inspiration. It is not at all meant as an attempt to describe a "Peruvian" economy. When Schefold criticises our model for its unrealistic features, he is obviously right; yet, since we did not aim at empirical accuracy but at theoretical consistency, at least that part of his critique is ill-oriented.

Let us now turn to Schefold's own theoretical model. Hotelling's rule implies that the price of guano *in situ* increases at a rate equal to the rate of interest. Schefold criticises the lack of distinction between guano *in situ* and guano extracted in our model. We assumed – for reasons of simplicity – that guano *in situ* can be used without further processing or effort in the production of corn. According to Schefold this is nonsense; guano can be used as a fertiliser in the production of corn only after it has been extracted and transformed. Hence he stresses the need to make a distinction between exhaustible resources in the ground and exhaustible resources above the ground. It should be noted that for an unknown reason he shifts terminology and considers the extracted resource ('above the ground') rather than the *in situ* one ('in the ground') to be "the" exhaustible resource of his model – we will not follow this peculiarity and stick to the usual terminology. The issue at stake is whether the distinction makes a significant difference. It does not: a simple extension of the corn-guano model with an additional process describing the extraction of guano is basically all that is needed. It can be shown formally that the dynamics of this extended model are similar to those of the corn-guano model with a corn-labour numeraire. The price of *in situ* guano of course still follows the Hotelling rule, whereas the price of extracted guano - at least in the 'natural path' - follows a slightly modified Hotelling rule.

A more disturbing aspect of Schefold formalisation concerns the way in which prices change. In Schefold's view, production by means of exhaustible resources is comparable to production by means of lands of different fertility, in the sense that the normal prices of produced commodities "will rise and fall in steps, as in Sraffa's rendering of Ricardo's theory of rent" (Schefold, 2001: 320). More specifically, Schefold divides time in successive 'long periods' – 'decades' in his terminology – during which prices of produced commodities remain at their normal levels. Normal prices change spasmodically at the instant of time which separates one decade from the other. Schefold does not explain, however, why such changes occur only between two decades. In the theory of rent, a change of normal prices follows an increase in demand which requires a change in the productive methods, for instance a switch to a less productive type of land. Nothing of the sort happens in Schefold's model; it is a complete mystery why prices are frozen for long periods of time, and then change suddenly. In his formalisation, the precise length of a decade is an essential characteristic, whereas in the usual classical theory of prices the unit period of time (usually referred to as a 'year') has no incidence.

Schefold's construction becomes even stranger when one realises that a different rule applies to the prices of commodities (including extracted guano) than to the price of the *in situ* resource. Commodity prices remain at their normal levels during each period, but "an essential change in the price of the resource takes place within each period" (*ibid.*). We do not understand how such an asymmetrical treatment can be justified. *All* prices should be allowed to change within a period, not only the prices of the resources in the ground. It seems to us that Schefold's distinction is inspired more by a nostalgic desire to remain within the confines of the familiar classical formalisation with its constant long-term prices than by economic reasoning.

To sum up, we reject Schefold's critiques and his model for two reasons:

- From a methodological point of view, we maintain that our original corn-guano model is the core of what might become the 'Sraffian' theory of exhaustible resources. This model can be adapted, alongside the paths explored in our second model, to take into account distinctions and refinements which we dropped on purpose from the original model. Schefold's distinction between *in situ* guano and produced guano falls within this type of complication.

- From an analytical point of view, Schefold's alternative model relies on weird assumptions on price changes and competition, and can be criticised on several points. It is worth mentioning that if the prices of produced commodities (including the produced guano) are stable for a decade while that of *in situ* guano changes, numerous opportunities for arbitrage are open. For instance, it is profitable to buy a produced commodity at the end of a decade and sell it at the beginning of the next, after the price increase. Schefold's implicit thesis is that a competitive economy cannot adapt itself smoothly to the presence of exhaustible resources and suffers a dramatic crisis at the end of every decade. The definition of a decade, which is essential for the determination of the ensuing chaotic dynamics, remains unclear.

8 The Misunderstandings of Kurz and Salvadori

For many years Heinz Kurz and Neri Salvadori have worked on a theory of exhaustible resources of Sraffian inspiration (Salvadori, 1987; Kurz and Salvadori, 1995, 1997, 2000, 2001). Although formally their model has remained basically the same, their interpretation of it has changed substantially over the years. What has remained constant, however, is their fundamental misunderstanding of the notions of nominal and real rates of interest. In fact they have consistently evaded any reflection on the concept of profit, which as we have shown at the beginning of the paper should start from the simple observation that profit results from the comparison of two values, those of investments at date t and of receipts at date $t+1$. The real rate of profit is based upon a comparison of both values expressed in terms of a standard of value. In the standard Sraffian theory, where prices are identical from period to period, the choice of the standard of value does not matter. If relative prices change over time, however, the measured rate of profit depends upon the standard, and the question is to select the relevant standard. But Kurz and Salvadori fail to see this, as can be illustrated by their critique of the corn-guano model.

Kurz and Salvadori (2001) begin by an analysis of the corn-guano model on the assumption that the real wage is given. This assumption differs from the one we have made above - that of a given (real) rate of profit -, but the two cases can be been examined just as easily, as we indicated in our study (Bidard and Erreygers, 2001a: 251-2) and briefly repeat here. In the case of a given real rate of profit, we need to specify the numeraire/standard of value in order to know what this rate stands for. Unless there is evidence to the contrary, we assume that the standard of value remains constant over time. Given a stable numeraire, the model then determines the sequence of corn prices $\{p(t)\}$, of royalties $\{z(t)\}$ and of wages $\{w(t)\}$. In the case of a given real wage, we need to specify the numeraire/standard of value in order to interpret the rates of profit that will be determined by the model. Again we assume that the standard remains constant over time. Given the numeraire, the model now determines the sequence of corn prices $\{p(t)\}$, of royalties $\{z(t)\}$ and of profit rates $\{r(t)\}$. Let corn be the numeraire, i.e. let us take d units of corn ($d > 0$) as our unit of prices:

$$dp(0) = 1, \quad dp(1) = 1, \quad dp(2) = 1, \quad ... \tag{19}$$

For this numeraire a *unique* sequence of real profit rates will be determined by the price equations, whatever the specific value of d.

Kurz and Salvadori write down correctly the price equations, but then they lose contact with the ground and start to drift. The trouble begins when they state: "The sequence of nominal rates of profit $\{r_t\}$ is assumed to be given." (Kurz and Salvadori, 2001: 284). It has escaped their attention that 'nominal' rates of profit make no sense without reference to a numeraire. If they had first specified the numeraire, they would have noticed that no sequence of nominal rates of profit can be given, since there is simply no room for it. Believing that a sequence of 'nominal' rates of profit must be specified to solve the model, they discover that any sequence of numbers can be fitted in, and that there remains one degree of freedom. In their opinion: "This means that there is room for a further equation fixing the numeraire." (Kurz and Salvadori, 2001: 285). Such a statement is seriously misleading. In fact, their procedure boils down to the use of a sequence of implicit *changing* numeraires, so defined that they yield the desired rates of profit. This is how it goes. Suppose that $\{r(0), r(1), r(2), ...\}$ is the sequence of real profit rates obtained by taking d units of corn as the numeraire, and let $\{r'(0), r'(1), r'(2), ...\}$ be an arbitrary sequence of nonnegative numbers. Suppose that at time t the numeraire consists of $\alpha(t)$ units of corn, i.e. suppose we have:

$$d(0)p(0) = 1, \quad d(1)p(1) = 1, \quad d(2)p(2) = 1, \quad ... \tag{20}$$

If we choose the coefficients $d(0), d(1), d(2), ...$ in such a way that:

$$d(t+1) = \frac{1 + r(t)}{1 + r'(t)} d(t), \quad t = 0, 1, 2, ... \tag{21}$$

then we obtain the sequence $\{r'(0), r'(1), r'(2), ...\}$ as profit rates of the model. Observe that the $d(t)$ coefficients are defined up to a scalar only - this is the degree of freedom to which Kurz and Salvadori refer in the quotation above.

So Kurz and Salvadori's main message is that it is possible, by means of a judicious choice of numeraires, to obtain an arbitrary series of profit rates as a solution to the corn-guano model.[4] These are of course nothing but examples of the fake rates of return which we defined at the beginning of the paper. Already in Bidard and Erreygers (2001a: 246) we explicitly rejected the manipulation of prices caused by changes in the numeraire when time goes by, and advocated the use of the same numeraire at all times. We think that somebody who wants to determine whether a tree's diameter increases over time should use a constant measure every year; Kurz and Salvadori suggest that this can be done just as well by a measure which changes erratically from year to year. We disagree: putting the cart before the oxen – specifying the rates of profit before defining the numeraire – is not the way to make progress.

9 Conclusion

Building a theory of exhaustible resources appears to be quite a challenge for those working within the confines of the classical theory. In a competitive economy, the current price of an exhaustible resource rises at a rate equal to the rate of interest and, therefore, the structure of relative prices is changing. We have examined several attempts to deal with this problem. Parrinello fails to define the current royalty in a consistent way. Schefold assumes that the

[4] It is not difficult to see that we can easily obtain a sequence of only *negative* profit rates.

prices of the produced commodities, including the extracted resource, change every period, whereas the price of the in situ resource is stable for long periods, then is adjusted suddenly between two 'decades'. This evolution leaves open the opportunity of arbitrage and depends crucially on the missing definition of the decade. Kurz & Salvadori's reading of their equations has been significantly modified after 1995, but their recent interpretation is ultimately based on a confusion regarding the definition of the (real) rate of profit and the role of the numeraire.

Our own approach starts from a simple corn-guano model, with one commodity (corn) and one exhaustible resource (guano). In a first version, corn is chosen as numeraire; in a second version of the model, the use of a mixed numeraire (corn and labour) illustrates the difficulties linked to the fact that the measure of the rate of profit depends on the numeraire when the relative prices evolve with time. The same model also serves as a proxy for multisector models. Under the hypotheses explicitly retained (mainly, competition and perfect foresight), the critiques formulated against this approach have been shown to be unconvincing: the level of royalty is determined by the competitiveness hypothesis; the distinction between the in situ resource and the extracted resource is inessential, and the basic model is easily adapted to take it into account, if necessary; the fact that the rate of profit changes with time and depends on the (fixed) numeraire does not mean that the sequence of rates is arbitrary. At this stage of the debate, our approach appears to be the only solid construction to the competitive theory of exhaustible resources inspired by the classical ways of thought.

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