

# Competing fiat currencies and debt enforcement\*

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## Abstract

International (low inflation) currencies are in general easily available everywhere. However, home currencies remain the means of payment mostly used in the vast majority of countries. This observation conflicts with the literature on currency competition which predicts that, in absence of transaction costs, agents will prefer to use the less inflationary currency. In this paper, we provide an example for more inflationary currencies being used, despite the availability of a less inflationary currency. We construct a model in which enforcement of debts is imperfect. Inflation affects incentives to default and, thereby, the level of interest rates, which determines the choice on which currency to accept in trade and the end-of-period decision on which currency to hold. We find that, if the level of banks' reserve requirements is not too high, the more inflationary currency is preferred in equilibrium even though no transaction costs to use the less inflationary currency exist.

*Keywords* : Money, Currency competition, Banking

*JEL Classification* : E41, E50

## 1 Introduction

International currencies, characterized by low inflation rates, are easily available in most economies. However, home currencies remain the means of payment predominantly used in the vast majority of countries. This observation is puzzling and conflicts with the literature on currency competition which predicts that, in absence of transaction costs, agents will prefer a less inflationary currency to hold and to accept in trade. In this paper, we intend to provide an example for more inflationary currencies being used despite the existence of less inflationary

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currencies and despite the absence of costs in using them. That is, we establish conditions under which the more inflationary currency may be effectively preferred to the less inflationary currency.

To provide a rationale for the use of home currencies, we propose an avenue different from the one that assumes a cost to the use of one or both of the competing currencies. We suggest that the use of domestic currencies is associated with the technology available for enforcing borrowers. We consider the issuance of credit backed by deposits, in an environment where full enforcement is not feasible. This entails that the rate of return on currencies is in part determined by incentives eliciting voluntary debt repayment. We show that, under certain conditions, the inflation rate of a currency in which debts are nominated may function as a commitment device.

The mechanism can be described as follows. Depending on the enforcement technology available to the institutions which provide credit, that for simplicity we call banks, the currency in which debts are denominated may affect the outside option for defaulters. In particular, if banks are only able to enforce agents who carry out transactions in the market and are only able to do so temporarily, then the punishment on defaulters would be stronger if the currency in which they took out loans is more inflationary. The reason is that defaulters would choose not to participate in the market for a while in order to avoid being enforced by banks. Doing this would entail a lower benefit, the more inflationary the currency borrowed were, since the currency would be less valuable at the moment to use it to purchase goods. Thus, borrowing a more inflationary currency would be a better commitment device than borrowing a less inflationary currency. In a rational-expectations framework, depositors anticipate that borrowers can commit better to paying interest on loans denominated in the more inflationary currency. Hence, they prefer to hold and deposit the more inflationary currency since this provides a higher compensation for their deposits.

We refer to the technology described in the previous paragraph as imperfect enforcement, since it enables banks to enforce some agents (those who take part in the market) but not all of them and, further, banks can enforce agents only for some time. To highlight the assumptions that give rise to the preference for the more inflationary currency when enforcement is imperfect, we also consider the case of a perfect enforcement technology and the case where no enforcement is feasible. In all cases, we assume no costs to use either of the currencies. We show that, even though agents may prefer the more inflationary currency when enforcement is imperfect, they never do so when enforcement is perfect or not feasible at all. The reason is that, with perfect enforcement as well as with no enforcement, agents do not get a punishment that depends on the currency that is borrowed. With perfect enforcement, they get no punishment since default is not an option. With no enforcement, they are free to use the funds of the defaulted loan in the same period of the loan; consequently, inflation does not affect its value, and borrowing in a more inflationary currency does not make agents less willing to default, which is contrary to the case with imperfect enforcement.

Our result may seem contradictory with what Kareken and Wallace (1981) defined as the *dominance result*. The *dominance result* may be stated as follows: if, in absence of transaction costs, there are two fiat currencies with positive price, their returns must be equal; if one of the currencies had a lower return, in equilibrium it would be driven out by the other currency so that its price would be zero. The explanation to this apparent contradiction is that, while in Kareken and Wallace the inflation rate entirely determines the rate of return of the currencies, in our model this is not generally the case. In particular, one currency may have a higher return than a second currency, despite its inflation being higher, as long as the interest rate in that currency is sufficiently higher than the interest rate in the second currency. As we will show, this may occur when inflation affects the incentives to default.

In order to put in place the mechanism described, we develop a Lagos and Wright (2005) model where agents can make deposits and take out loans, as in Berentsen, Camera and Waller (2007). The first difference with Berentsen *et al.* is that we consider an environment with two currencies, instead of one. Agents are allowed to borrow and deposit in either of the currencies. Thus, the only feature that potentially makes currencies differ in our model is their inflation rates. Another difference with respect to Berentsen *et al.* is that we add the imperfect enforcement set-up, which is key to let incentives to default depend on the currency denomination of loans.

As mentioned above, the literature which addresses the competition between currencies relies mainly on transaction costs to define the existence of national-currencies equilibria. Transaction costs allow low-return currencies to circulate in equilibrium despite the existence of a competing, higher return currency. For example, Martin (2006) studies a cash-in-advance economy where there is a cost for sellers to accept two currencies and a proportional cost of trading on the foreign exchange market. The national currency may circulate in equilibrium unless its growth rate is too high. Matsui (1998) studies different equilibria in a two-country model where taxes must be paid in local currency and government injects local currency through purchases of goods. Engineer (2000) considers a decentralized economy where the domestic currency has lower transaction costs but a higher growth rate than the foreign currency.

Several papers that belong to the so-called first and second generations of the search-theoretic approach have proved fruitful in studying national-currencies equilibria. A first group of papers study currency competition by assuming that there is no ex-ante difference in the fiat monies that compete, except for the physical properties that make them distinguishable (for instance, Matsuyama, Kiyotaki and Matsui (1993), Shi (1995) and Trejos and Wright (1996)). Hence, these papers focus on the role of expectations for a currency to be accepted in trade and emphasize the multiplicity of equilibria when it comes to studying the acceptability of money. Thus, this literature does not aim to provide an explanation for the use of domestic currencies but rather to explain why its use is one of the possible equilibrium outcomes.

A second group of search-theoretic models assume an exogenous difference in currencies' returns. Since in this class of models money is indivisible, the

difference in return is modeled through the behavior of a fraction of agents, taken as exogenous. In Li and Wright (1998), a proportion of agents are government agents who are assumed to use the home currency. In Curtis and Waller (2000), government agents impose a fine with some probability on traders who use the foreign (illegal) currency, whereas in Curtis and Waller (2003) and Camera, Craig and Waller (2004) they may confiscate money holdings in their meetings with private agents. That framework made it possible to have two currencies with different returns coexisting and avoid the *dominance result* because in a search-environment arbitrage opportunities are reduced and because of the indivisibility of money. Thus, this literature provided insight on the features that allow for either one or both of the currencies to be used in equilibrium and on the equilibrium properties in terms of purchasing power of the monies. However, once one attempts to analyze the effect of money growth on currencies' choice -which, for instance, the divisible money set-up by Lagos and Wright allows for-, obtaining one equilibrium in which the more inflationary currency circulates without assuming costs to the use of the competing currency is not straightforward. The purpose of this paper is to suggest the issue of default on debts as a key feature to give rise to such equilibrium.

Our work is also related to the literature on the optimal inflation rate. In particular, our work builds partly on the literature that states a benefit from inflation owing to its effect on incentives to default. Berentsen *et al.* develop a Lagos and Wright model with banks that provide loans and take deposits to analyze how inflation affects welfare. They show that when no enforcement on borrowers is feasible, inflation may be welfare improving because it makes the outside option for defaulters less attractive. As a result, inflation may allow for an increase in the borrowing interest rate without promoting default by borrowers, which in turn increases the value of money and so the goods that money can purchase. Since consumption is increasing in inflation, inflation has a positive impact on welfare. Previously, Aiyagari and Williamson (2000) have presented computational results on the benefits of inflation arising from the role of inflation in increasing the punishment for defaulters. This argument is also studied in Antinolfi, Azariadis and Bullard (2007) and Diaz and Perera-Tallo (2008).<sup>1</sup> We aim to contribute to this literature by studying how the results are affected when defaulters have a competing currency as an outside option. Indeed, we will see that conclusions obtained with one currency may not hold when a second currency is introduced. Furthermore, whereas the literature on the optimal inflation rate examines the conditions under which inflation turns out to be beneficial from a social planner's perspective, our purpose is to identify conditions under which agents may choose to use the more inflationary currency.

In the next section we present the model. In section 3, we characterize the symmetric equilibrium. In section 4, we focus on the home-currency equilibrium

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<sup>1</sup>Other features of the environment have been highlighted to play a role in the welfare benefits of inflation. For instance, it has been argued that inflation can improve welfare in environments where search effort is endogenous (see Li (1995) and Berentsen, Rocheteau and Shi (2007)) or that it can reduce the level of socially costly cash activities such as theft (see He, Huang and Wright (2008)).

and study the tree different set-ups concerning the enforcement technology mentioned above. Finally, section 5 concludes.

## 2 Environment

The original framework we build on is the divisible money model by Lagos and Wright (2005). The main advantage of this framework is that it facilitates the introduction of heterogeneity in production and consumption preferences as well as the divisibility of money, keeping the distribution of money holdings degenerate and, thus, analytically tractable. More precisely, we base our model on the model developed by Berentsen et al (2007).

Time is discrete and goes for ever. There is a continuum of infinitely lived agents of unit mass and one perfectly divisible and non-storable good that comes in many types. Agents can potentially produce only one type of the good and they can potentially consume all types of the good. Agents discount across periods with factor  $\beta \in (0, 1)$ .

In each period, two competitive markets open sequentially. Before the first market opens, agents get an idiosyncratic preference shock by which they either want to consume but cannot produce (with probability  $(1 - s)$ ) or can produce but do not want to consume (with probability  $s$ ). We call "consumers" the agents who get the first type of shock and "sellers" those who get the second type. After the first shock has realized, a second shock occurs which affects only consumers: each consumer learns that he has either a preference for the type of the good produced by himself (he only gets utility from his own type of the good and none from others' types of the good), with probability  $\frac{b}{1-s}$ , or a preference for others' types of the good (he only gets utility from others' types of the good and none from the type produced by himself), with probability  $\frac{1-b-s}{1-s}$ . We call "home-consumers" the consumers who prefer their own type and "buyers" the consumers who prefer others' types.

In the first market, buyers get utility  $u(q)$  when they consume a quantity  $q$  of some type of the good, with  $u'(q) > 0$ ,  $u''(q) < 0$ ,  $u'(0) = +\infty$  and  $u'(\infty) = 0$ . For simplicity, we assume that producing a quantity  $q$  represents a disutility equal to  $c(q) = q$ . We do not explicitly model the choice of home consumers; instead we assume that they get some fixed utility from consuming their own type of good.

In the second market all agents may consume and produce. Consuming  $x$  gives utility  $U(x) = \ln(x)$ .<sup>2</sup> Disutility from producing  $x$  is equal to  $y$ , where one unit of labor yields one unit of good. According to the preference revealed at the beginning of the period, buyers consume types of goods produced by others while home-consumers consume their own type.

In addition, two different monies exist. We call them home currency and

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<sup>2</sup>The assumption on a logarithmic utility function in the second subperiod is a sufficient although not necessary condition for the results that follow. What is actually necessary is that  $U(x^*) - x^*$ , where  $x^*$  is determined by  $U'(x^*) = 1$ , is sufficiently high, or that  $U(0)$  is sufficiently low.

foreign currency. Both are intrinsically useless. We assume that they are issued by a national central bank and a foreign central bank, respectively, even though we analyze only one country. Indeed, what is important for our purpose is that two different currencies with potentially different inflation rates are available. Central banks' decisions are exogenous. We call  $M_t$  and  $M_t^f$  the money stocks of home currency and foreign currency, respectively, in period  $t$ .

The growth rates of home and foreign currency are  $\gamma_h$  and  $\gamma_f$  where  $\gamma_h, \gamma_f > 0$ , so that  $M_{t,h} = \gamma_h M_{t-1,h}$  and  $M_{t,f} = \gamma_f M_{t-1,f}$ . Agents receive lump-sum transfers from each authority equal to  $\tau_h M_{t-1,h}$  and  $\tau_f M_{t-1,f}$  at the beginning of the second market in period  $t$ , where the subscript  $-1$  indicates the previous period ( $+1$  indicates the following period) and  $\tau_j = \gamma_j - 1$ . Agents may convert currencies at no cost during the second market, although they cannot do it before or during the first market.

Agents can deposit and borrow money (home as well as foreign currency) by resorting to banks. Banking activities take place after the first preference shock and before the second one. Banks are competitive and face an exogenous level  $r$  of reserve requirements; i.e., they can only issue a total amount of loans  $L_j$  by keeping a ratio  $r = D_j/L_j$  of total deposits to total loans, where  $j = h, f$  indicates home or foreign currency and  $0 \leq r \leq 1$ .

Loans are issued as bilateral contracts between an agent and a bank by which the bank gives an amount  $l_j$  of money ( $l_j$  is the amount of an individual loan) to the agent and the agent must pay it back during the second market (or subperiod) together with the interest on it. Deposits are taken by banks and paid back during the second subperiod with the corresponding interest. The timing of events is depicted in Figure 1.

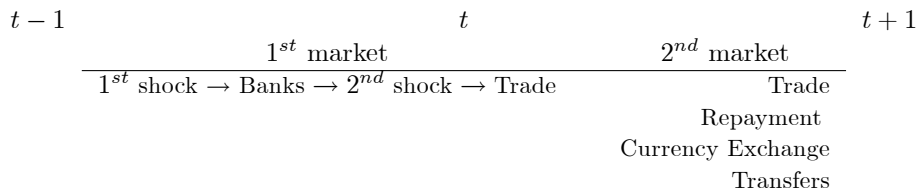


Figure 1: Timing of events

We will analyze three different set-ups regarding the technology for enforcing loans repayment: perfect enforcement, no enforcement and imperfect enforcement. Perfect enforcement means that banks are able to oblige agents to work to repay their loans and the interest. By contrast, no enforcement means that banks are unable to enforce borrowers and, therefore, must establish conditions so that repayment is voluntary. By imperfect enforcement, we mean that banks have an enforcement technology at their disposal by which they can force repayment by those agents who enter the second market, in which settlement of debts takes place. However, they cannot enforce agents who decide not to enter the second market. In addition, when enforcement is imperfect banks can enforce

agents who participate in the second market only in the period of the loan, but they are not able to do so afterwards.

In the cases of no enforcement and imperfect enforcement, we assume that banks possess a record keeping technology that allows to punish defaulters by excluding them from the banking system for the rest of their lifetime; i.e., after defaulting, agents are prevented from borrowing and depositing.<sup>3</sup> Moreover, we assume that defaulters are excluded from the monetary transfers as well.<sup>4</sup>

Banks operate at a cost  $\kappa$  per unit of money loaned. This cost is aimed to capture the resources devoted to the enforcement technology and the record keeping on borrowers.

In order to motivate a role for money, we assume anonymity of traders so that, for trade to take place, sellers require compensation at the same time as they produce. This assumption rules out bilateral credit; however, it does not conflict with the existence of lending in this model because this only requires that agents are identified by banks (which is not the same as being identified by partners in trade).

Dealing with two currencies requires additional precisions. First, we intend to focus our currency-choice analysis on the currencies' returns rather than on expectations (which has already been done in the literature, as mentioned in the introduction). Thus, we assume that a group of agents exist, namely government agents, who exogenously accept the home currency; analogously, some agents exogenously accept the foreign currency. However, they do not force the use of any of the currencies.<sup>5</sup> In sum, neither transaction costs nor legal restrictions are associated to the use of either of the currencies.

Markets are competitive so that agents take prices as given. Competitive pricing was first analyzed in a Lagos-Wright framework by Rocheteau and Wright (2004). As they, and previously Temzelides and Yu (2004), point out, the existence of competitive markets does not make money inessential as long as the double coincidence problem and anonymity are still features of the environment studied.

### 3 Symmetric equilibrium

We look at symmetric and stationary equilibria in which  $\gamma_h = M_{+1,h}/M_h = \phi/\phi_{+1}$ ,  $\gamma_f = M_{+1,f}/M_f = \phi_f/\phi_{+1,f} = \phi/\phi_{+1} * e/e_{+1}$  and  $e_{+1}/e = \gamma_h/\gamma_f$ , where  $\phi$  and  $\phi_f$  are the price of home currency and the price of foreign currency in real terms.  $e$  is the exchange rate that converts one unit of foreign currency

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<sup>3</sup>We follow Berentsen *et al.* who make this assumption in order to study the effect of inflation on welfare when enforcement is not feasible. We depart from their set-up in that we introduce the case of imperfect enforcement.

<sup>4</sup>This assumption could be relaxed without affecting the main results.

<sup>5</sup>This assumption is made to rule out the coordination problem that typically arises in monetary models. It allows for a situation in which, in a particular equilibrium in which one currency circulates but not the other, an agent starts using the latter without dealing with expectation issues about the acceptability that the currency will have.

in home currency. Imposing stationarity implies that end-of-period real money holdings are constant:

$$\begin{aligned}\phi_{-1}M_{-1,h} &= \phi M_h \\ \phi_{-1,f}M_{-1,f} &= \phi_f M_f\end{aligned}$$

A representative agent starts each period with amounts  $m_h$  of home currency and  $m_f$  of foreign currency. His expected utility when each period starts is  $V(m_h, m_f)$ . When an agent enters the second market, his expected utility is  $W(m_h, m_f, d_h, d_f, l_h, l_f)$ , according to his money holdings, deposits and loans in each currency. To simplify the notation, we write the function  $V$  as  $V(m_j)$  and the function  $W$  as  $W(m_j, d_j, l_j)$  where  $m_j$ ,  $d_j$  and  $l_j$  denote  $(m_h, m_f)$ ,  $(d_h, d_f)$  and  $(l_h, l_f)$ , respectively.

### 3.1 The second market

Agents decide whether to enter the second market or not. Buyers always decide to enter the second market since they can only consume types of the good produced by other agents and their consuming zero goods entails infinite negative utility. Instead, home-consumers may decide not to enter the second market, since they consume the type of good produced by themselves.

For agents who enter the second market, their program is to solve:

$$\begin{aligned}W(m_j, d_j, l_j) &= \max_{x, h, m_{+1, j}} U(x) - y + \beta V_{+1}(m_{+1, j}) \\ \text{s.t. } y &= x + \sum_j \phi_j [m_{+1, j} - m_j - (1 + i_{d, j}) d_j + (1 + i_{l, j}) l_j - \tau_j M_{-1, j}]\end{aligned}\tag{1}$$

where  $j = h, f$ . Agents can hold deposits  $d$ , loans  $l$  and money in home currency and/or foreign currency.  $i_d$  is the deposit interest rate and  $i_l$  is the borrowing interest rate. We rewrite (1) using the budget constraint:

$$\begin{aligned}W(m_j, d_j, l_j) &= \max_{x, m_{+1, j}} U(x) - x + \beta V_{+1}(m_{+1, h}, m_{+1, f}) \\ &\quad + \sum_j \phi_j [m_j + (1 + i_{d, j}) d_j - (1 + i_{l, j}) l_j - m_{+1, j} + \tau_j M_{-1, j}]\end{aligned}$$

The first-order conditions on  $x$  and  $m_{+1, j}$  are:

$$\begin{aligned}U'(x) &= 1 \\ \beta V'_{+1}(m_{+1, j}) &= \phi_j\end{aligned}\tag{2}$$

The envelope conditions on  $m_j$ ,  $d_j$  and  $l_j$  are:

$$\begin{aligned}W_{m, j} &= \phi_j \\ W_{d, j} &= \phi_j (1 + i_{d, j}) \\ W_{l, j} &= -\phi_j (1 + i_{l, j})\end{aligned}\tag{3}$$



Home-consumers who do not enter the second market simply maximize the difference between their utility from consumption  $U(x)$  and their disutility from working  $x$ . However, they cannot adjust their money holdings since they do not trade.

## 3.2 The first market

### 3.2.1 Sellers

In the first market, sellers decide how much to produce in exchange for each currency,  $q_{s,j} = \{q_{s,h}, q_{s,f}\}$ , and how much to deposit in each currency,  $d_j$ . Their program is:

$$\begin{aligned} \max_{q_{s,j}, d_j} & -c(q_s) + W(m_{-1,j} - d_j + p_j q_{s,j}, d_j, 0) \\ \text{s.t.} & d_j \leq m_{-1,j} \end{aligned}$$

where  $q_s = q_{s,h} + q_{s,f}$ .

The first-order condition on  $q_{s,j}$  is:

$$-1 + \phi_j p_j = 0 \quad (4)$$

The first-order condition on  $d_j$  is:

$$-W_{m,j} + W_{d,j} - \lambda_{d,j} = 0$$

Using (3), it becomes

$$\phi_j i_{d,j} - \lambda_{d,j} = 0$$

### 3.2.2 Consumers

Consumers must choose the consumption quantities  $q_{b,j} = \{q_{b,h}, q_{b,f}\}$ , to be purchased with each currency, and the amount of loans in each currency  $l_j$  before the second shock; i.e., before learning if they have a preference for other agents' types of the good or for their own type of the good. Their program is:

$$\begin{aligned} \max_{q_{b,j}, l_j} & \frac{b}{1-s} [u(q_b) + W(m_{-1,j} + l_j - p_j q_{b,j}, 0, l_j)] + \frac{1-b-s}{1-s} W(m_{-1,j} + l_j, 0, l_j) \\ \text{s.t.} & p_j q_{b,j} \leq m_{-1,j} + l_j \\ & l_j \leq \bar{l}_j \end{aligned}$$

where  $q_b = q_{b,h} + q_{b,f}$  and  $\bar{l}_j$  is the borrowing limit.

The first-order condition on  $q_{b,j}$  is:

$$u'(q_b) - W_{m,j} p_j - \lambda_j p_j = 0$$

where  $\lambda_j$  is the multiplier associated to the cash constraint. Using (3) and (4), this condition becomes:

$$u'(q_b) = \frac{\lambda_j}{\phi_j} + 1$$

The first-order condition on  $l_j$  is:

$$\begin{aligned} \frac{b}{1-s} \left[ u'(q_b) \frac{dq_{bj}}{dl_j} + W_{m,j} \left( 1 - p_j \frac{dq_{bj}}{dl_j} \right) + W_{l,j} \right] + \frac{1-b-s}{1-s} (W_{m,j} + W_{l,j}) \\ + \frac{b}{1-s} \lambda_j \left( 1 - p_j \frac{dq_{bj}}{dl_j} \right) - \frac{\lambda_{l,j}}{1-s} = 0 \end{aligned}$$

where  $\lambda_{l,j}$  is the multiplier associated to the borrowing constraint. We may rewrite the first-order condition on  $l_j$  as follows:

$$u'(q_b) = 1 + \frac{\lambda_{l,j}}{b\phi_j} + \frac{(1-s)i_{l,j}}{b} \quad (5)$$

### 3.3 Banks' problem

Banks must hold a proportion  $r$  in the form of deposits for each unit of money loaned. Therefore,  $d_j = \frac{r(1-s)l_j}{s}$  where  $d_j$  and  $l_j$  are the amounts of an individual deposit and loan, respectively. Banks solve the following problem per borrower:

$$\begin{aligned} \max_{l_j} \sum \phi_j l_j (i_{l,j} - r i_{d,j} - \kappa) \\ \text{s.t. } l_j \leq \bar{l}_j \\ \text{s.t. } b [u(q_b) - \phi_j (1 + i_{l,j}) l_j] - (1-b-s) \phi_j i_{l,j} l_j \geq \Gamma_j \end{aligned}$$

where  $\kappa$  is the bank's operating cost per unit of money loaned and  $\bar{l}_j$  is the borrowing limit in each currency which we take as given by now and will endogenize later.<sup>6</sup> The first constraint is the borrowing constraint. The second constraint is the participation constraint of the borrower: each bank has to offer a pay-off to the borrower that is at least the same as the pay-off he may get while resorting to another bank,  $\Gamma_j$ .

The first-order condition on  $l_j$  is:

$$i_{l,j} - r i_{d,j} - \kappa - \lambda_{l,j} - \lambda_{\Gamma,j} \phi_j [b + (1-s) i_{l,j}] = 0$$

where  $\lambda_{l,j}$  and  $\lambda_{\Gamma,j}$  are the multipliers associated to the borrowing constraint and the participation constraint, respectively. Since in a competitive equilibrium profits are zero, we know that:

$$i_{l,j} - r i_{d,j} - \kappa = 0 \quad (6)$$

Therefore, the first-order condition for banks may be written as follows:

$$-\lambda_{l,j} - \lambda_{\Gamma,j} \phi_j [b + (1-s) i_{l,j}] = 0$$

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<sup>6</sup>We assume that banks do not subsidize their activities in one of the currency with the profits made in the other currency. This is equivalent to assume that some banks operate in the home currency and some banks operate in the foreign currency.

### 3.4 Marginal value of money

The expected utility for an agent who starts a period with an amount  $m_j$  of money is:

$$\begin{aligned} V(m_j) &= b[u(q_b) + W(m_{-1,j} + l_j - p_j q_{b,j}, 0, l_j)] \\ &\quad - s[c(q_s) + W(m_{-1,j} - d_j + p_j q_{s,j}, d_j, 0)] \\ &\quad + (1 - b - s)W(m_{-1,j} + l_j, 0, l_j) \end{aligned}$$

Therefore, the marginal value of currency  $j$  is:

$$\begin{aligned} V'(m_j) &= b \left[ u'(q_b) \frac{dq_b}{dm_j} + W_{m,j} \left( 1 - p_j \frac{dq_b}{dm_j} \right) \right] \\ &\quad + s \left[ W_{m,j} \left( 1 - \frac{dd_j}{dm_j} \right) + W_{d,j} \frac{dd_j}{dm_j} \right] + (1 - b - s) W_{m,j} \end{aligned} \quad (7)$$

Using (3), (7) becomes:

$$V'(m) = b \frac{u'(q_b)}{p_j} + s \phi_j (1 + i_{d,j}) + (1 - b - s) \phi_j$$

Using (2) and (4), we have:

$$\frac{\gamma_j - \beta}{\beta} = b[u'(q_b) - 1] + i_{d,j}s \quad (8)$$

The right-hand side of this equation represents the marginal cost of acquiring an additional unit of the currency  $j$  while the left-hand side represents its marginal benefit given by the increase in consumption  $q_b$  with probability  $b$  and the deposit interest rate with probability  $s$ .

## 4 Home-currency equilibrium

We will focus on equilibria in which agents hold home currency and do not hold foreign currency. As mentioned above, we assume that there are "home government agents" and "foreign government agents" who always accept (but not force to use) home currency and foreign currency, respectively. This implies that, if an agent wants to use a currency different from that one used by all other agents, he is able to do it. Moreover, this is a feature shared by both currencies; they do not differ in this respect.

### 4.1 Unconstrained home-currency equilibrium: perfect enforcement

In this subsection, we analyze an unconstrained home currency equilibrium; i.e., an equilibrium in which home currency circulates and agents may borrow as much as they desire since banks have the power to fully enforce agents. For

simplicity, we will in general ignore the subscripts to indicate home currency. In this equilibrium, (8) becomes:

$$\frac{\gamma_h - \beta}{\beta} = b[u'(q_b) - 1] + i_d s \quad (9)$$

We know that in an unconstrained equilibrium  $\lambda_l = 0$ . Hence, combining (5) and (8) we have:

$$\frac{\gamma_h - \beta}{\beta} = (1 - s) i_l + i_d s \quad (10)$$

Besides, banks are competitive so banks' profits are zero; this implies:

$$i_d = \frac{i_l - \kappa}{r} \quad (11)$$

We can now define an unconstrained home currency equilibrium:

**Definition 1** *An unconstrained home currency equilibrium is  $\{q_b, i_l, i_d\}$  that satisfy (9), (10) and (11).*

**Proposition 1** *When  $\gamma_h \leq \gamma_f$ , an unconstrained home-currency equilibrium exists if:*

$$\frac{\gamma_h - \beta}{\beta} \geq (1 - s) \kappa \quad (12)$$

$$\frac{\gamma_h - \beta}{\beta} \leq \frac{\kappa}{1 - r} \quad (13)$$

*If  $\gamma_h < \gamma_f$ , the unconstrained home-currency equilibrium is unique.*

For an unconstrained equilibrium to exist, it must be that  $i_d \geq 0$  (or  $i_l \geq \kappa$ ). Furthermore, the existence of this equilibrium requires  $i_l \geq i_d$ ; otherwise, agents would demand infinite loans to deposit and earn interest. Using (10) and (11) these conditions reduce to (12) and (13) in Proposition 1.

The higher  $r$  and  $s$  the more likely that (12) and (13) are verified. When  $r$  is high it is more likely that  $i_d < i_l$  and that the zero-profit condition is satisfied. On the other hand, when  $s$  is high, agents are less likely to become borrowers and pay the cost  $\kappa$ : credit is more useful when  $s$  is high because agents are less willing to hold money across periods if they are producers more frequently.

Consider that only the home currency is available. Then, from Definition 1, we verify that  $di_l/dr \geq 0$  and  $di_d/dr \leq 0$ . If  $r$  increases, the borrower has to pay more for each real unit of money loaned, whereas  $i_d$  decreases because banks must compensate more units of money per loan. The consumption quantity, the individual amount borrowed and the real price of money are decreasing in  $r$  since  $dq_b/dr < 0$ ,  $dl/dr < 0$  and  $d\phi/dr < 0$ . The effect of  $\gamma_h$  on the endogenous variables is summarized by the following derivatives:  $di_l/d\gamma_h > 0$ ,  $di_d/d\gamma_h > 0$ ,  $dq_b/d\gamma_h < 0$  and  $d\phi/d\gamma_h < 0$ .

Having defined the unconstrained equilibrium, we intend to determine if its existence is conditioned on the absence of a less inflationary currency. We say that the home-currency equilibrium is "robust" if it is not eliminated after a marginal increase in  $\gamma_h$  when a competing currency is available. We state the following proposition:

**Proposition 2** *The unconstrained home-currency equilibrium is not robust to an increase in  $\gamma_h$  when a foreign currency is available and  $\gamma_h = \gamma_f$ .*

To see why the unconstrained home currency equilibrium is not robust to the availability of a less inflationary foreign currency, consider the case with two currencies, home and foreign, and  $\gamma_h > \gamma_f$ . If both currencies were held across periods, from (8), two first-order conditions on money should be satisfied:

$$\begin{aligned}\frac{\gamma_h - \beta}{\beta} &= b[u'(q_b) - 1] + i_{d,h}s \\ \frac{\gamma_f - \beta}{\beta} &= b[u'(q_b) - 1] + i_{d,f}s\end{aligned}$$

Using (6) to replace  $i_{d,h}$  and  $i_{d,f}$ , they become:

$$\begin{aligned}\frac{\gamma_h - \beta}{\beta} &= b \left[ 1 + \frac{s}{r(1-s)} \right] [u'(q_b) - 1] - \frac{s\kappa}{r} \\ \frac{\gamma_f - \beta}{\beta} &= b \left[ 1 + \frac{s}{r(1-s)} \right] [u'(q_b) - 1] - \frac{s\kappa}{r}\end{aligned}$$

The LHS (left-hand side) of each of these equations represents the marginal cost of getting an additional unit of home (foreign) currency whereas the RHS (right-hand side) represents the marginal benefit of getting an additional unit of home (foreign) currency. If  $\gamma_h \neq \gamma_f$ , one of these two conditions cannot be satisfied since RHS is the same in both of them. The agent chooses the currency with the lowest marginal cost, in this case the foreign currency since  $\gamma_h > \gamma_f$ . If  $\gamma_h = \gamma_f$ , increasing  $\gamma_h$  would destroy the unconstrained home currency equilibrium: agents can get a higher utility (from a higher quantity  $q_b$ ) by using foreign currency.

## 4.2 Constrained home-currency equilibrium with no enforcement

In this subsection, we assume that no enforcement is feasible; i.e., banks are not able to enforce agents, regardless of agents entering the centralized market or not. Hence, banks have to set a borrowing constraint so that all agents (agents who prefer to consume their own type of good and agents who prefer to consume others' types of the good) choose to repay loans instead of defaulting on them.

The first-order condition on money (8) is:

$$\frac{\gamma_h - \beta}{\beta} = b[u'(q_b) - 1] + i_{d,h}s \quad (14)$$

with

$$i_d = \frac{i_l - \kappa}{r} \quad (15)$$

After defaulting, it is easy to verify that agents will use foreign currency because they cannot use the banking system and home currency inflation is assumed to be equal or higher than foreign currency inflation. The marginal value of money for a defaulter satisfies:

$$\frac{\gamma_f - \beta}{\beta} = b [u'(\hat{q}_b) - 1] \quad (16)$$

where  $\hat{q}_b$  is the quantity consumed by a defaulter. We denote  $\hat{m}_{-1}$  the holdings of money by a defaulter at the end of the period. Hence, we have:

$$\begin{aligned} \hat{m}_{-1} &= p_f \hat{q}_b \\ \hat{m}_{+1} &= \gamma_f \hat{m}_{-1} \end{aligned} \quad (17)$$

In a symmetric equilibrium, money holdings by a non-defaulter are  $m_{-1} = M_{-1}$ . We also know that the deposit constraint and cash constraint bind, so that money holdings by a non-defaulter satisfy:

$$\begin{aligned} m_{-1} &= d = \frac{r(1-s)l}{s} \\ pq_b &= m_{-1} + l \end{aligned} \quad (18)$$

Using (4) and (18) we can write the amount of an individual loan in real terms as:

$$\phi l = \frac{sq_b}{s + r(1-s)} \quad (19)$$

Let us analyze next the decision by a buyer on whether to repay his loan or not before entering the second market in the period in which he borrowed. We call  $W$  the expected lifetime utility if he does not default and  $\bar{W}$  his expected lifetime utility if he defaults:

$$\begin{aligned} W &= U(x) - y_b + \beta V_{+1}(m_{+1}) \\ \bar{W} &= U(x) - \bar{y}_b + \beta \bar{V}_{+1}(m_{+1}) \end{aligned}$$

where  $y_b$  and  $\bar{y}_b$  are the amounts of hours worked by the buyer who does not default and the buyer who defaults in the current period, respectively.  $\bar{V}_{+1}$  corresponds to the expected lifetime value for a buyer who defaults in the current period.

Banks set a borrowing constraint in order to prevent default. They choose  $\bar{l}$ , the maximum amount loaned, such that the expected lifetime utility for a non-defaulter equals the expected lifetime utility for a defaulter; i.e.,  $W = \bar{W}$ . In the case of the buyer, the borrowing constraint may be written as follows:

$$\begin{aligned} & -y_b + \frac{\beta}{1-\beta} [bu(q_b) - sq_s + U(x) - by_b - sy_s - (1-b-s)y_c] \\ &= -\bar{y}_b + \frac{\beta}{1-\beta} [bu(\hat{q}_b) - sq_s + U(x) - b\hat{y}_b - s\hat{y}_s - (1-b-s)\hat{y}_c] \end{aligned} \quad (20)$$

where  $y_b$ ,  $y_s$  and  $y_c$  are the amount of hours worked in the second subperiod of each period by the buyer, the seller or the home-consumer who are not defaulters and did not default in the past, respectively.  $\hat{y}_b$ ,  $\hat{y}_s$  and  $\hat{y}_c$  are the hours worked by a defaulter the period after defaulting and from then on. If an agent defaults, he saves working hours since he does not repay the loan nor the interest (i.e.;  $\bar{y}_b < y_b$ ), but he is punished by being permanently excluded from the banking system, so that his consumption and working hours as a defaulter may be different from those as non-defaulter.

Since the consumer's decision on defaulting may depend on the consumer's type, we should also consider the decision by the home-consumer. We state the following Lemma:

**Lemma 1** *The pay-off to a defaulter is independent of his type (buyer or home-consumer). The borrowing constraint set by banks when no enforcement is feasible is*

$$\phi \bar{l} = \frac{\beta b [u(\hat{q}_b) - u(q_b)] - [\gamma_f - \beta(1-b)] \hat{q}_b + [\beta b + (1-\beta) \gamma_h] q_b}{(\gamma_h - 1 - i_l)(1-\beta) - \beta(1-s) \kappa} \quad (21)$$

The borrowing constraint (21) results from replacing  $y_b$ ,  $y_s$ ,  $y_c$ ,  $\bar{y}_b$ ,  $\hat{y}_b$ ,  $\hat{y}_s$  and  $\hat{y}_c$  and setting  $l = \bar{l}$  in (20). (17) and (18) are also used, as well as (4) to set  $\phi p = 1$  and  $e \phi p_f = 1$  and (15) to replace  $i_d$ .<sup>7</sup>

According to Lemma 1, the pay-off to a defaulter is the same regardless of his being a buyer or a home-consumer. Indeed, the expected lifetime utility in the case of defaulting or repaying is the same for both the buyer and the home-consumer. The gain from defaulting is also the same because they both save  $\phi l(1+i_l)$  working hours in the period of default: at the moment of defaulting, the buyer has already spent the money borrowed to buy goods in the first market so he avoids working to buy  $l(1+i_l)$  units of money, while the home-consumer needs not work to buy  $i_l l$  units of money and uses the money borrowed  $l$  to get his optimal money holdings working less than otherwise. To rewrite (21), we make use of (19):

$$\frac{s q_b}{(1-s)r+s} = \frac{\beta b [u(\hat{q}_b) - u(q_b)] - [\gamma_f - \beta(1-b)] \hat{q}_b + [\beta b + (1-\beta) \gamma_h] q_b}{(\gamma_h - 1 - i_l)(1-\beta) - \beta(1-s) \kappa} \quad (22)$$

We can now state the following definition:

**Definition 2** *A home currency constrained credit equilibrium with no enforcement is  $\{q_b, \hat{q}_b, i_l, i_d\}$  that satisfy (14), (15), (16) and (22).*

We call  $\bar{\gamma}$  the value of  $\gamma_h$  such that  $\gamma_h = \gamma_f$  and  $i_d = 0$ . We write (22) setting  $i_d = 0$  (or  $i_l = \kappa$ ) and  $\gamma_h = \gamma_f = \bar{\gamma}$ ,

$$\frac{s q_b}{(1-s)r+s} = \frac{\beta b [u(\hat{q}_b) - u(q_b)] + (\beta b + \bar{\gamma}_h)(q_b - \hat{q}_b) + \beta(\hat{q}_b - \bar{\gamma}_h q_b)}{(\bar{\gamma}_h - 1)(1-\beta) - \kappa(1-\beta s)}$$

<sup>7</sup>It is straightforward to verify that the same borrowing constraint results from equating the expected pay-offs from defaulting and not defaulting that correspond to a home-consumer. The calculation of hours worked for this and next subsection is presented in the appendix.

We know that  $q_b = \hat{q}_b$  when  $\gamma_h = \gamma_f = \bar{\gamma}$ ; thus,

$$\bar{\gamma} = 1 + \frac{s(1-\beta s)\kappa}{s+(1-s)r\beta} \quad (23)$$

Hence, as long as  $\kappa > 0$ ,  $\bar{\gamma} > 1$  (if  $\kappa = 0$ ,  $\bar{\gamma} = 1$ ). To see why, consider the case in which  $\kappa = 0$ . The gain for an agent who defaults consists of the working hours saved in the current period,  $(1+i_l)\phi l = \phi l$ . The cost of defaulting is given by the working time in each period necessary to acquire extra money holdings, which is equal to  $\phi l_{+1} - \beta\phi_{+1}l_{+1} = (\gamma_h - \beta)\phi l$ . The discounted lifetime sum of this cost is  $\sum_{t=0}^{\infty} \beta^t (\gamma_h - \beta)\phi l = \frac{(\gamma_h - \beta)\phi l}{1-\beta}$ . Hence,  $\gamma_h = 1$  corresponds to the level of  $\gamma_h$  for which the agent is just indifferent between defaulting or not when  $\kappa = 0$ .<sup>8</sup> As a consequence, when  $\kappa > 0$ ,  $\gamma_h$  must be higher than one since otherwise the pay-off to a defaulter would be higher than in the case with  $\kappa = 0$  but the cost would be the same, which cannot occur in equilibrium. Hence, the inflation rate must be positive for an equilibrium with credit to exist.

At  $\gamma_h = \bar{\gamma}$ , from (14)  $u'(q_b)$  is:

$$u'(q_b) = \frac{\gamma_h - \beta}{b\beta} + 1 \quad (24)$$

We now differentiate (22) with respect to  $\gamma_h$  (having previously replaced  $i_l$  using (14) and (15)):

$$\frac{dq_b}{d\gamma_h} = \frac{\left(1 - s + \frac{1}{\beta}\right)(1-\beta)r q_b}{(\gamma_h - 1)[s + \beta(1-s)r] - s(1-\beta s)\kappa + (1-\beta)rbq_b u''(q_b)}$$

Notice that we have set  $d\hat{q}_b/d\gamma_h = 0$  according to (16) and use (24) to replace  $u'(q_b)$ . Replacing  $\gamma_h = \bar{\gamma}$  with (23), we verify that  $dq_b/d\gamma_h$  evaluated at  $\bar{\gamma}$  is negative:

$$\left. \frac{dq_b}{d\gamma_h} \right|_{\bar{\gamma}} = \frac{1 - s + \frac{1}{\beta}}{bu''(q_b)} \quad (25)$$

Therefore, when no enforcement is feasible, increasing the home money growth  $\gamma_h$  makes the quantity consumed by a buyer who does not default decrease.

**Proposition 3** *The constrained home-currency equilibrium with no enforcement is not robust to an increase in  $\gamma_h$  when a foreign currency is available and  $\gamma_h = \gamma_f$ .*

To explain Proposition 3, we need to determine the choice of an agent on which currency (or currencies) to bring from one period to the following. Our procedure consists of stating the currency choice when the home currency is used in equilibrium and  $\gamma_h = \gamma_f$  in order to assess whether this choice is affected

<sup>8</sup>When the money growth rate is equal to 1 there are no lump-sum monetary transfers, so we do not need to consider them in the cost of defaulting in this case.



when  $\gamma_h$  increases while  $\gamma_f$  is kept unchanged. For this, we calculate the net gain  $B$  from bringing an amount  $m_{+1}$  of home currency to the period  $t + 1$ :

$$B = \beta [bu(q_b(m_{+1})) + (si_d + 1 - b)\phi_{+1}m_{+1}] - m_{+1}\phi$$

The first term is the discounted utility from bringing an amount  $m_{+1}$  of home currency: it allows to consume  $q_b$  acquired with  $m_{+1}$  with probability  $b$  and to save disutility from working equal to  $\phi_{+1}m_{+1}$  with probability  $(1 - b)$ . Besides, with probability  $s$  an extra amount  $i_d\phi_{+1}m_{+1}$  of disutility is saved.

Rewriting it:

$$B = \beta bu(q_b) + [\beta (si_d + 1 - b) - \gamma_h] \frac{r(1-s)q_b}{s+r(1-s)} \quad (26)$$

Differentiating (26) with respect to  $\gamma_h$  gives:

$$\begin{aligned} \frac{dB}{d\gamma_h} &= \beta bu'(q_b) \frac{dq_b}{d\gamma_h} + [\beta (si_d + 1 - b) - \gamma_h] \frac{r(1-s)}{s+r(1-s)} \frac{dq_b}{d\gamma_h} \\ &\quad + \frac{r(1-s)q_b}{s+r(1-s)} \beta s \frac{di_d}{d\gamma_h} \end{aligned}$$

We derive  $di_d/d\gamma_h$  from (14) and insert it in the previous expression. We also replace  $i_d s$  with (14) to get:

$$\frac{dB}{d\gamma_h} = \frac{\beta b [su'(q_b) - r(1-s)u''(q_b)q_b]}{s+r(1-s)} \frac{dq_b}{d\gamma_h} + \frac{r(1-s)q_b}{s+r(1-s)} \quad (27)$$

Therefore, the manner in which an increase in  $\gamma_h$  affects the currency choice depends on the derivative  $dq_b/d\gamma_h$ . When banks do not possess enforcement power,  $dB/d\gamma_h < 0$  as we can verify by inserting (25) in (27):

$$\frac{dB}{d\gamma_h} = \left\{ \frac{su'(q_b) [\beta(1-s) + 1]}{u''(q_b)} - r\beta q_b(1-s)^2 \right\} \frac{1}{s+r(1-s)} < 0$$

As a result, in the case of no enforcement the home-currency equilibrium cannot be supported for  $\gamma_h > \gamma_f$  as it is claimed in Proposition 3. The explanation for this relies on the usual effect caused by an increase in the inflation rate: since the cost of holding money increases, agents choose to hold a lower amount of money and consume less so that the marginal value of money increases as well. The deposit interest increases owing to the increase in  $\gamma_h$  but does not compensate for the higher cost of holding home currency, so demand for home currency diminishes. Increasing  $\gamma_h$ , on the other hand, does not modify the outside option for defaulters, which depends only on  $\gamma_f$ . Therefore, the borrowing constraint for loans in home currency is not relaxed after an increase in  $\gamma_h$ , which precludes an increase in the deposit interest rate sufficient to make agents hold the home currency at the end of the period. Evaluating the equilibrium at  $\gamma_h = \gamma_f = \bar{\gamma}$  in which agents use the home currency, a marginal increase in  $\gamma_h$  makes agents switch to the foreign currency, since  $\gamma_f$  is kept unchanged and the benefit from holding one currency is decreasing in its inflation rate.

### 4.3 Constrained home-currency equilibrium with imperfect enforcement

In this subsection, we consider the case of imperfect enforcement; i.e., banks are able to enforce agents who voluntarily enter the second market and trade. However, they cannot enforce agents who do not take part in the second market. Agents who prefer the types of the good produced by other agents will always enter the market, since they would not be able to consume otherwise and doing so would entail infinite negative utility ( $\ln(0) = \infty$ ). Therefore, banks are concerned only with home-consumers who do not need to enter the market to consume. Indeed, home-consumers could decide not to enter the market in order to avoid being enforced and save working effort. Banks set a borrowing constraint to prevent default by these agents. They choose  $\check{l}$  such that the expected lifetime utility for a home-consumer who does not default equals the expected lifetime utility for a home-consumer who defaults; i.e.  $W_c = \check{W}_c$ , with:

$$\begin{aligned} W_c &= U(x) - y_c + \beta V_{+1}(m_{+1}) \\ \check{W}_c &= U(x) - y_c^* + \beta \check{V}_{+1}(m_{+1}) \end{aligned}$$

where  $y_c^*$  is the amount of hours worked by the defaulter in the period he defaults and  $\check{V}_{+1}$  corresponds to the expected lifetime value for a defaulter in that period. The borrowing constraint  $W_c = \check{W}_c$  is then:

$$\begin{aligned} & -y_c + \frac{\beta}{1-\beta} [bu(q_b) - sq_s + U(x) - by_b - sy_s - (1-b-s)y_c] \\ &= -y_c^* + \beta [bu(\check{q}_b) - sq_s + U(x) - b\check{y}_b - s\check{y}_s - (1-b-s)\check{y}_c] \\ & \quad + \frac{\beta^2}{1-\beta} [bu(\hat{q}_b) - sq_s + U(x) - b\hat{y}_b - s\hat{y}_s - (1-b-s)\hat{y}_c] \end{aligned} \quad (28)$$

where  $\check{y}_b$ ,  $\check{y}_s$  and  $\check{y}_c$  are the amounts of hours worked by the defaulter the period after he defaults if he turns out to be buyer, seller or home-consumer, respectively, and  $\hat{y}_b$ ,  $\hat{y}_s$  and  $\hat{y}_c$  are the hours worked by a defaulter the second period after defaulting and from then on.  $\check{q}_b$  is the quantity consumed by the defaulter the period after defaulting if he happens to be buyer.

In the period after defaulting the agent holds the money borrowed and not reimbursed. If the agent is home-consumer or seller in that period, he uses this money to save working hours in the second market. If he is buyer in this period, he uses this money to consume in the first market so that working hours in the period after defaulting are the same as in all other periods. So we verify that  $\check{y}_b = \hat{y}_b$  (see appendix).

The quantity consumed by the defaulter if he is buyer the period after defaulting is:

$$\check{q}_b = \frac{m_{-1} + l_{-1}}{p} \quad (29)$$

since his money holdings when entering the first market are the sum of his

previous money holdings and the loan defaulted from the previous period.<sup>9</sup> Since  $l$  is a multiple of  $m_{-1}$  ( $l_{-1}$  is a multiple of  $m_{-2}$ ) and  $m$  grows at a rate  $\gamma_h$ , it follows that  $l_{-1} = l/\gamma_h$ . Using (4), (18) and (19) we can rewrite (29) as:

$$\check{q}_b = q_b \frac{\frac{s}{\gamma_h} + r(1-s)}{s+r(1-s)} \quad (30)$$

Thus,  $\check{q}_b = q_b$  if  $\gamma_h = 1$  and  $\check{q}_b < q_b$  if  $\gamma_h > 1$ .

**Lemma 2** *The borrowing constraint set by banks when imperfect enforcement is feasible is*

$$\phi \check{l} = \frac{\beta}{1-\beta} \frac{b[u(q_b) - (1-\beta)u(\check{q}_b) - \beta u(\hat{q}_b)] + \beta(1-b)(q_b - \hat{q}_b) + \gamma_f \hat{q}_b - q_b}{i_l + \beta(1-b)\left(\frac{1}{\gamma_h} - 1\right) + \frac{\beta\kappa(1-s)}{1-\beta}} \quad (31)$$

(31) comes from (28), in which  $y_b, y_s, y_c, y_c^*, \check{y}_b, \check{y}_s, \check{y}_c, \hat{y}_b, \hat{y}_s$  and  $\hat{y}_c$  have been replaced,  $l$  has been set equal to  $\check{l}$  and  $l_{-1}$  to  $\frac{\check{l}}{\gamma_h}$ . Replacing  $i_l$  and  $\phi \check{l} = \phi l$  with (19) in (31) yields

$$\begin{aligned} & \frac{1-\beta}{\beta} \frac{q_b}{s+r(1-s)} \\ &= \frac{b[u(q_b) - (1-\beta)u(\check{q}_b) - \beta u(\hat{q}_b)] + \beta(1-b)(q_b - \hat{q}_b) + \gamma_f \hat{q}_b - q_b}{\frac{\gamma_h - \beta}{\beta} r - br[u'(q_b) - 1] + s\beta(1-b)\left(\frac{1}{\gamma_h} - 1\right) + \frac{1-\beta s}{1-\beta} \kappa s} \end{aligned} \quad (32)$$

We can now state the following definition:

**Definition 3** *A home-currency constrained equilibrium with imperfect enforcement is  $\{q_b, \hat{q}_b, \check{q}_b, i_l, i_d\}$  that satisfy (14), (15), (16), (30) and (32).*

We call  $\check{\gamma}$  the value of  $\gamma_h$  such that  $\gamma_h = \gamma_f$  and  $i_d = 0$  when enforcement is imperfect.

**Lemma 3** *For  $\kappa > 0$ , the money growth rate consistent with zero nominal interest rate when imperfect enforcement is feasible,  $\check{\gamma}$ , is higher than 1.*

**Proof.** See appendix. ■

Lemma 3 implies that the inflation rate must be positive to support an equilibrium with credit, as in the previous subsection. If  $\check{\gamma} = \gamma_h = \gamma_f$  and  $i_d = 0$ , we know from (14) and (16) that  $q_b = \hat{q}_b$ ; thus,

$$\frac{\frac{1-\beta s}{\beta(1-\beta)} \kappa s}{s+r(1-s)} = \frac{b[u(q_b) - u(\check{q}_b)]}{q_b} + \frac{\check{\gamma} - 1}{1-\beta} - \frac{s(1-b)\left(\frac{1}{\check{\gamma}} - 1\right)}{s+r(1-s)} \quad (33)$$

<sup>9</sup>Note that the defaulter receives monetary transfers the same period of default, even though he will not receive them from then on, since the default is noticed only at the end of the period.

Hence, the solution for  $\check{\gamma}$  is implicitly determined by (33). Before evaluating how the borrowing constraint is affected by an increase in  $\gamma_h$ , we calculate  $d\check{q}_b/d\gamma_h$  from (30):

$$\frac{d\check{q}_b}{d\gamma_h} = \frac{dq_b}{d\gamma_h} \frac{\frac{s}{\check{\gamma}} + r(1-s)}{s+r(1-s)} - \frac{s}{s+r(1-s)} \frac{q_b}{(\check{\gamma})^2} \quad (34)$$

The second term in the right-hand side of (34) corresponds to the depreciation of the loan owing to an increase in  $\gamma_h$  and it is always negative. The first term corresponds to the variation in the price of money  $\phi$  when  $\gamma_h$  increases, which is reflected in the change in  $q_b$ . If  $q_b$  increases after an increase in  $\gamma_h$ , this means that  $\phi$  increases which allows for a higher  $\check{q}_b$ .

Thus, differentiating (32) with respect to  $\gamma_h$  yields

$$\frac{q_b}{s+r(1-s)} \left[ \frac{r}{\beta^2} - s \frac{1-b+bu'(\check{q}_b)}{(\gamma_h)^2} \right] = \frac{dq_b}{d\gamma_h} \left\{ \begin{array}{l} \frac{bu'(q_b)}{1-\beta} - \frac{b[u(q_b)-\beta u(\check{q}_b)] + [\gamma_f - \beta(1-b)]\check{q}_b}{(1-\beta)q_b} \\ + b \frac{\frac{r}{\beta} u''(q_b)q_b - [r(1-s) + \frac{s}{\gamma_h}] u'(\check{q}_b)}{s+r(1-s)} + \frac{bu(\check{q}_b)}{q_b} \end{array} \right\}$$

where we have used (34) and  $d\hat{q}_b/d\gamma_h = 0$ .

Evaluating at  $\gamma_h = \gamma_f = \check{\gamma}$  (so that  $i_d = 0$ ) yields

$$\left. \frac{dq_b}{d\gamma_h} \right|_{\check{\gamma}} = \frac{\frac{q_b}{s+r(1-s)} \left( \frac{r}{\beta^2} - s \frac{1-b+u'(\check{q}_b)b}{\check{\gamma}^2} \right)}{\frac{bu'(q_b)-\check{\gamma}+\beta(1-b)}{1-\beta} - b \frac{[r(1-s) + \frac{s}{\check{\gamma}}] u'(\check{q}_b) - \frac{r}{\beta} u''(q_b)q_b}{s+r(1-s)} - b \frac{u(q_b)-u(\check{q}_b)}{q_b}} \quad (35)$$

**Proposition 4** *If the following condition holds,*

$$r \leq \frac{s\beta}{\check{\gamma}} \quad (36)$$

*then the constrained home-currency equilibrium with imperfect enforcement is robust to an increase in  $\gamma_h$  when a foreign currency is available and  $\gamma_h = \gamma_f = \check{\gamma}$ .*

According to Proposition 4, (36) is a sufficient condition for the home-currency equilibrium to be robust to an increase in  $\gamma_h$  when a foreign currency is available,  $\gamma_h = \gamma_f = \check{\gamma}$  and  $\gamma_f$  is held fixed. If (36) holds, the numerator in (35) is negative, while the denominator in (35) is always negative. Hence, if (36) is verified,  $dq_b/d\gamma_h$  evaluated at  $\gamma_h = \gamma_f = \check{\gamma}$  is positive. To get to (36), note that for the numerator in (35) to be negative it must be that:

$$\frac{r}{\beta^2} - s \frac{1-b+u'(\check{q}_b)b}{\check{\gamma}^2} < 0$$

We can rewrite this condition taking into account that  $\check{q}_b < q_b$  and therefore  $u'(\check{q}_b) = u'(q_b) + \epsilon$ , where  $\epsilon$  is some positive number:

$$\frac{r}{\beta^2} - s \frac{1 - b + [u'(q_b) + \epsilon] b}{\check{\gamma}^2} < 0$$

We use (16) to replace  $u'(q_b)$  (which is equal to  $u'(\hat{q}_b)$  for the parameters values that we are examining) and simplify:

$$\frac{1}{\beta} \left( \frac{r}{\beta} - \frac{s}{\check{\gamma}} \right) < \frac{sb}{\check{\gamma}^2} \epsilon$$

We write it as a sufficient condition by ignoring the RHS and rearrange to get (36).

To show that the denominator in (35) is always negative, we use the mean value theorem to rewrite it as follows:

$$\frac{bu'(q_b) - \check{\gamma} + \beta(1 - b)}{1 - \beta} - b \frac{\left[ r(1 - s) + \frac{s}{\check{\gamma}} \right] u'(\check{q}_b) - \frac{r}{\beta} u''(q_b) q_b}{s + r(1 - s)} - bu'(q_m) \left( 1 - \frac{\check{q}_b}{q_b} \right) < 0$$

where  $q_m \in (\check{q}_b, q_b)$  is such that  $u'(q_m)(q_b - \check{q}_b) = u(q_b) - u(\check{q}_b)$ . We set  $u'(\check{q}_b) = u'(q_b) + \epsilon$  and  $u'(q_m) = u'(q_b) + \varepsilon$ , with  $\epsilon, \varepsilon > 0$ . We also use (30) and rearrange to get:

$$\frac{\beta bu'(q_b) - \check{\gamma} + \beta(1 - b)}{1 - \beta} + b \frac{\frac{r}{\beta} u''(q_b) q_b - \epsilon \left[ r(1 - s) + \frac{s}{\check{\gamma}} \right] - \varepsilon \left( s - \frac{s}{\check{\gamma}} \right)}{s + r(1 - s)}$$

Finally, we replace  $u'(q_b)$  using (16) and simplify so that all remaining terms are negative:

$$\frac{r}{\beta} u''(q_b) q_b - \epsilon \left[ r(1 - s) + \frac{s}{\check{\gamma}} \right] - \varepsilon \left( s - \frac{s}{\check{\gamma}} \right) < 0$$

We know that the solution for  $\check{\gamma}$  is determined by the implicit function in (33), where  $q_b$  and  $\check{q}_b$  are functions of  $\check{\gamma}$  and are determined by (14) and (30) when  $\gamma_h = \check{\gamma}$  and  $i_d = 0$ . If the solution for  $\check{\gamma}$  from (33) satisfies (36), we know that  $dq_b/d\check{\gamma} > 0$ . Finally, from (27),  $dB/d\check{\gamma} > 0$  when  $dq_b/d\check{\gamma} > 0$  and, therefore, the home-currency equilibrium evaluated at  $\gamma_h = \gamma_f = \check{\gamma}$  is robust to a marginal increase in  $\gamma_h$ .

Thus, we have showed that in the case of imperfect enforcement, under certain conditions, increasing  $\gamma_h$  makes more profitable to hold the home currency. This implies that a constrained home-currency equilibrium is robust to increasing  $\gamma_h$  even if a less inflationary currency is available and no transaction costs specific to the use of foreign currency are assumed.

The explanation for this result resides in the link between the inflation rate of a currency and the borrowing constraint associated to that currency. Since

defaulters are obliged to skip one period to use the loan defaulted, the inflation of the currency in which the loan is nominated matters. The more inflationary a currency is, the less valuable is a loan defaulted in that currency and the smaller are the incentives to default. Consequently, the borrowing constraint on the loans nominated in a particular currency may be relaxed when the inflation rate in that currency increases. This, in turn, allows for an increase in the borrowing interest rates: since the gain from defaulting is reduced with inflation, the cost of repayment may be increased without inciting agents to default. The increase in the borrowing interest rate is reflected in the deposit interest rate to satisfy the zero-profit condition for banks. As a result, agents may prefer to take the more inflationary currency across periods: should they deposit their money holdings, they would be better compensated for deposits in the more inflationary currency than for deposits in the less inflationary currency. The higher demand for the former currency makes its price increase as well, which allows buyers to get a higher quantity of goods in the market, as illustrated by the derivative in (35) when (36) holds.

The key feature of a constrained equilibrium is that the increase in the deposit interest rate after an increase in the inflation rate may exceed the one that occurs in an unconstrained equilibrium. In general, when inflation increases, the deposit interest rates should increase to compensate agents for the highest cost of holding money. However, in a constrained equilibrium interest rates are set below their market-clearing level to prevent default. Hence, the relaxation of the borrowing constraint is an additional channel through which the interest rate increases after an increase in inflation. This additional channel is what explains the preference for a currency which loses purchasing power more rapidly than another.

The preference for the more inflationary currency is not possible when there is no enforcement, however, because the link between the inflation of a currency and the incentives to default loans nominated in it does not exist, since agents are not obliged to wait to use the money borrowed and so they spend it in the period of default. Inflation in general does not affect their outside option, which is entirely determined by the currency that they will use after defaulting. If the outside option consists in using the foreign currency, increasing home inflation has no effect on it. Hence, the borrowing constraint is not affected either and the mechanism through the interest rates described in the previous paragraph cannot take place. Therefore, an increase in the home inflation makes agents switch to the foreign currency in this case.

Let us now explain condition (36). This condition states that the level of reserve requirements must be sufficiently low for inflation to increase the quantity consumed by the non-defaulter. The term  $r/\beta^2$  in the numerator in (35) comes from the increase in the borrowing interest rate when  $\tilde{\gamma}$  increases and is lower the lower is  $r$ . The term  $s(1-b)/\tilde{\gamma}^2$  comes from the decrease in the real value of the money borrowed when  $\tilde{\gamma}$  increases. Asking  $r$  to be low enough is equivalent, then, to ask that the depreciation of the money borrowed is more important than the increase in the interest rate when  $\gamma_h$  increases, so that the borrower is less willing to default. Note that, if  $r = 1$  (i.e., if the level of reserve

requirements is 100%), this condition never holds.

### 4.3.1 Numerical example

To get an example where the more inflationary currency is preferred, we assume  $u(q_b) = q_b^\eta$ , with  $\eta < 1$ . We give arbitrary values to all parameters except  $\gamma_h$  and  $\gamma_f$ . We set  $r = 0.2$ ,  $b = 0.4$ ,  $s = 0.48$ ,  $\eta = 0.8$ ,  $\kappa = 0.02$  and  $\beta = 0.9$ . Then we set  $\gamma_h = \gamma_f = \check{\gamma}$  and so we can compute  $\check{\gamma}$  using (33), where the solution for  $\check{q}_b$  comes from (30) and the solution for  $q_b$  comes from (14) when  $\gamma_h = \check{\gamma}$  and  $i_d = 0$ . We get  $\check{\gamma} = 1.00951$ . We verify that condition (36) holds since  $s\beta/\check{\gamma} = 0.428 > r = 0.2$ . We also verify that (12) holds, since  $(\check{\gamma} - \beta)/\beta = 0.1217 > \kappa(1 - s) = 0.01$ , which ensures existence of credit. This means that  $q_b$  is increasing in  $\gamma_h$  at  $\gamma_h = \gamma_f = \check{\gamma}$ . Then, we can compute  $q_b$  from (32), where  $\hat{q}_b$  and  $\check{q}_b$  are defined by (16) and (30), respectively.

In Figure 2 we have plotted  $q_b$  as a function of  $\gamma_h$ , from  $\gamma_h = \gamma_f = \check{\gamma}$  to the value of  $\gamma_h$  for which the wedge between the borrowing interest rate and the deposit interest rate is zero (for higher values of  $\gamma_h$ , the wedge becomes negative which precludes the existence of a home currency equilibrium with credit).

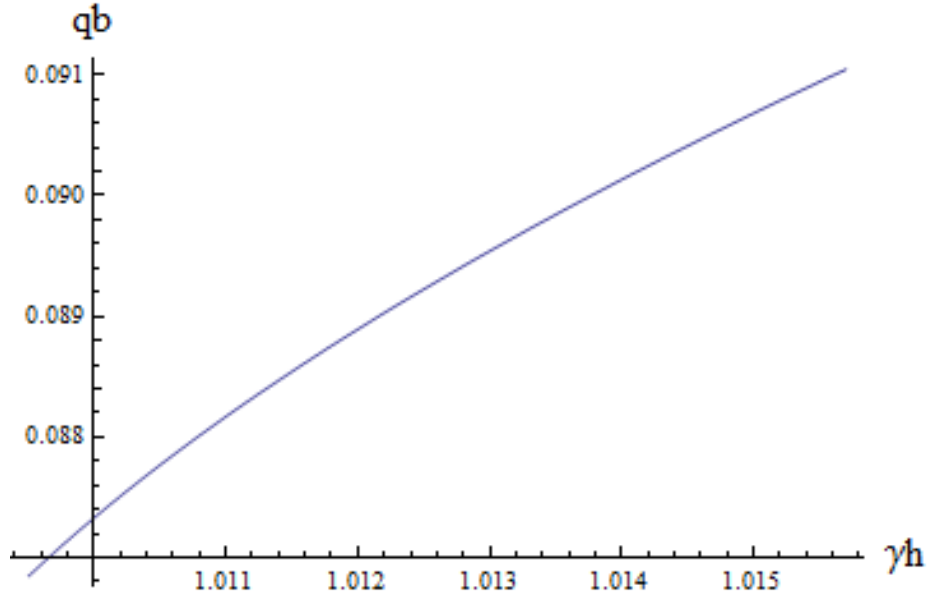


Figure 2: Consumption by the non-defaulter as a function of  $\gamma_h$

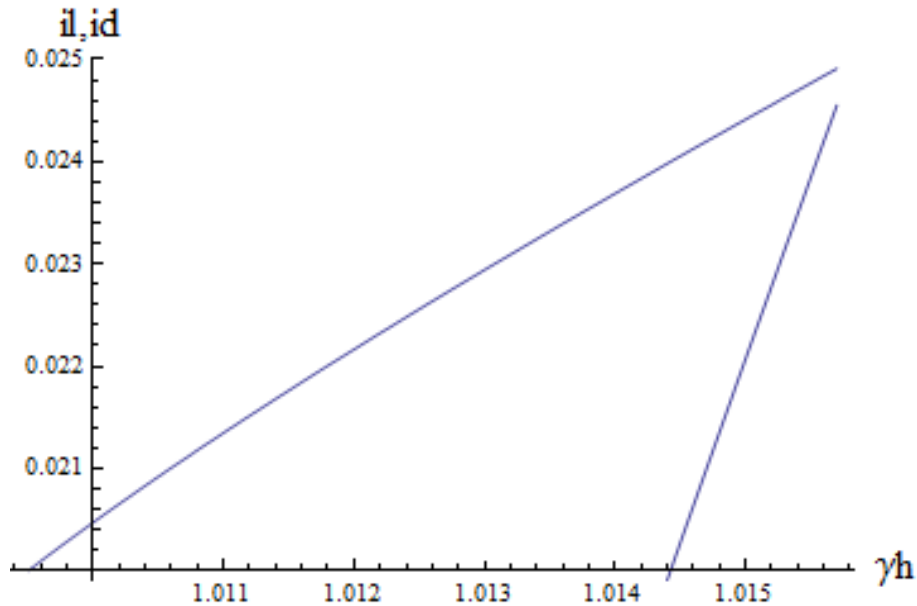


Figure 3: Interest rates as a function of  $\gamma_h$ .

Notice that, in this example, condition (12) holds but condition (13) does not. To ensure the existence of an unconstrained credit equilibrium, both conditions were required. The first one states that home inflation must not be too low compared to the bank cost for agents to be willing to use credit instead of relying only on their money holdings. This condition applies for the definition of both an unconstrained equilibrium and a constrained equilibrium, since in both cases we require that agents choose to use the banking system instead of only using money.

The second condition states that home inflation must not be too high for the banks' zero profit condition to be verified. If inflation is very high, the deposit interest rate is also high to compensate agents for the cost of carrying money. In addition, when inflation increases, the deposit interest rate increases faster than the borrowing interest rate owing to the zero profit condition and the existence of reserve requirements lower than 100%. Eventually, for  $\gamma_h$  sufficiently high as defined in (13), the deposit interest rate would have to be higher than the borrowing interest rate to satisfy the zero profit condition, which cannot occur in equilibrium. However, this condition needs not hold in the case of a constrained equilibrium. In this case, the deposit interest rate is not at its market-clearing level (since enforcement is not perfect, the borrowing interest rate is lower than its market-clearing level to prevent default by borrowers and so depositors cannot be fully remunerated). Hence,  $\gamma_h$  may violate (13) in a constrained equilibrium without implying that  $i_d > i_l$ , as we can verify from Figure 3.



## 4.4 Welfare

To calculate welfare, we first calculate the total output  $Y$  in the second market:

$$Y = by_b + sy_s + (1 - b - s)y_c$$

Replacing  $y_b$ ,  $y_s$  and  $y_c$  and simplifying,

$$Y = x + \frac{s(1-s)\kappa q_b}{s+r(1-s)}$$

Then, welfare  $\mathcal{W}$  is

$$\mathcal{W} = \frac{1}{1-\beta} \left[ U(x) - x - \frac{s(1-s)\kappa q_b}{s+r(1-s)} + bu(q_b) - sq_s \right]$$

Differentiating  $\mathcal{W}$  with respect to  $\gamma_h$  yields

$$\frac{d\mathcal{W}}{d\gamma_h} = \frac{1}{1-\beta} \left[ -\frac{s(1-s)\kappa}{s+r(1-s)} + b[u'(q_b) - 1] \right] \frac{dq_b}{d\gamma_h}$$

Evaluating  $d\mathcal{W}/d\gamma_h$  in  $\gamma_h = \gamma_f$  such that  $i_d = 0$ ,

$$\left. \frac{d\mathcal{W}}{d\gamma_h} \right|_{i_d=0} = \frac{1}{1-\beta} \left[ \frac{\gamma_h - \beta}{\beta} - \frac{s(1-s)\kappa}{s+r(1-s)} \right] \frac{dq_b}{d\gamma_h} \quad (37)$$

The term in brackets in (37) is positive since a condition for credit to exist is  $(\gamma_h - \beta)/\beta \geq (1-s)\kappa$ , which is required for  $i_d \geq 0$ . Therefore, if  $dq_b/d\gamma_h > 0$ , welfare is increasing in  $\gamma_h$ . By contrast, if  $dq_b/d\gamma_h < 0$  and foreign money is available with a growth rate  $\gamma_f = \gamma_h$ , increasing  $\gamma_h$  would make agents switch to the foreign currency and welfare would not be affected.<sup>10</sup>

## 5 Conclusion

We have presented a model where agents can choose between two currencies with potentially different inflation rates in order to identify conditions under which they prefer to hold and to accept in trade the more inflationary currency. In particular, we have showed that the borrowers' enforcement technology and the weight of loans relative to the money stock (i.e., the level of banks' reserve requirements) are important to get the home-currency equilibrium when a foreign less inflationary currency is available.

Our set-up should be improved in a number of ways. On the one hand, different pricing mechanisms, such as forms of bilateral bargaining, should be explored since this will certainly provide novel insight on the conditions for the home-currency equilibrium to exist. On the other hand, the enforcement

<sup>10</sup>Note that we have not included utility by home-consumers in the calculation of welfare since this would not alter the conclusions on the effect of inflation on it.

technology could be modeled as a policy variable. This would allow us to study explicitly combinations of the money growth rate and the policy variable for the choice of the level of enforcement. In addition, an important step would be to introduce a second country. This would make possible to ask about the conditions for different currencies being used in different countries, among other questions, and to compare with a large body of literature that studies two-country set-ups. All this is part of future research.

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## Appendix

### Hours worked in the case of no enforcement

The amount of hours worked by the agent who does not default is:

$$\begin{aligned} y_b &= x + \phi(1 + i_l)l + \phi m_{+1} - \phi \tau m_{-1} = x + i_l \phi l + q_b \\ y_s &= x - \phi(1 + i_d)d - \phi p q_s + \phi m_{+1} - \phi \tau m_{-1} = x - i_d \frac{r(1-s)}{s} \phi l - \frac{b}{s} q_b \\ y_c &= x + \phi(1 + i_l)l - \phi l + \phi m_{+1} - \phi m_{-1} - \phi \tau m_{-1} = x + \phi i_l l \end{aligned}$$

The amounts of hours worked by the buyer and the home-consumer who default in the period of default and thereafter are:

$$\begin{aligned} \bar{y}_b &= x + \phi e \hat{m}_{f+1} - \phi \tau m_{-1} = x + \gamma_f \hat{q}_b - (\gamma_h - 1)(q_b - \phi l) \\ \bar{y}_c &= x - \phi l + \phi e \hat{m}_{+1} - \phi m_{-1} - \phi \tau m_{-1} = x + (\gamma_h - 1) \phi l + \gamma_f \hat{q}_b - \gamma_h q_b \\ \hat{y}_b &= x + \phi e \hat{m}_{f+1} = x + \gamma_f \hat{q}_b \\ \hat{y}_s &= x - \phi p q_s - \phi e \hat{m}_{f-1} + \phi e \hat{m}_{f+1} = x - q_s + (\gamma_f - 1) \hat{q}_b \\ \hat{y}_c &= x - \phi e \hat{m}_{f-1} + \phi e \hat{m}_{f+1} = x + (\gamma_f - 1) \hat{q}_b \end{aligned}$$

### Hours worked in the case of imperfect enforcement

The amounts of hours worked by the non-defaulter are as in the case with no enforcement. The hours worked by the home-consumer when enforcement is imperfect in the period of default and the period after default are:

$$\begin{aligned} y_c^* &= x \\ \check{y}_b &= x + e \phi \hat{m}_{f+1} = x + \gamma_f \hat{q}_b \\ \check{y}_s &= x + e \phi \hat{m}_{f+1} - \phi m_{-1} - \phi l_{-1} - \phi p q_s \\ &= x + \gamma_f \hat{q}_b - \left(1 + \frac{b}{s}\right) q_b + \left(1 - \frac{1}{\gamma_h}\right) \phi l \\ \check{y}_c &= x + e \phi \hat{m}_{f+1} - \phi m_{-1} - \phi l_{-1} = x + \gamma_f \hat{q}_b - q_b + \left(1 - \frac{1}{\gamma_h}\right) \phi l \end{aligned}$$

The hours worked by the defaulter in subsequent periods are the same as in the case with no enforcement.

### Proof of Lemma 3

To verify that  $\check{\gamma} > 1$ , we write (32) setting  $i_d = 0$  (or  $i_l = \kappa$ ) and  $\gamma_h = \gamma_f = \check{\gamma}$ ,

$$\begin{aligned} &\frac{1-\beta}{\beta} \frac{q_b}{s+r(1-s)} \\ &= \frac{b[u(q_b) - (1-\beta)u(\check{q}_b) - \beta u(\hat{q}_b)] + \beta(1-b)(q_b - \hat{q}_b) + \check{\gamma}\hat{q}_b - q_b}{s\beta(1-b)\left(\frac{1}{\check{\gamma}} - 1\right) + \frac{1-\beta s}{1-\beta}\kappa s} \end{aligned}$$

If  $\check{\gamma} = \gamma_h = \gamma_f$  and  $i_d = 0$ , we know from (14) and (16) that  $q_b = \hat{q}_b$ ; thus,

$$\frac{\frac{1-\beta s}{\beta(1-\beta)}\kappa s}{s+r(1-s)} = \frac{b[u(q_b) - u(\check{q}_b)]}{q_b} + \frac{\check{\gamma} - 1}{1-\beta} - \frac{s(1-b)\left(\frac{1}{\check{\gamma}} - 1\right)}{s+r(1-s)} \quad (38)$$

If  $\check{\gamma}$  were 1, LHS would be higher than RHS ( $s\kappa\frac{1-\beta s}{1-\beta} > 0$ ) in (38). We can verify that RHS increases with  $\gamma_h$  when setting  $\gamma_h$  to 1. The derivative of RHS with respect to  $\gamma_h$  is:

$$\begin{aligned} \frac{dRHS}{d\gamma_h} = & \beta b \frac{\left[ u'(q_b) \frac{dq_b}{d\gamma_h} - u'(\check{q}_b) \frac{d\check{q}_b}{d\gamma_h} \right] q_b - \frac{dq_b}{d\gamma_h} [u(q_b) - u(\check{q}_b)]}{(q_b)^2} \quad (39) \\ & + \frac{s\beta(1-b)}{(s+r(1-s))(\gamma_h)^2} + \frac{\beta}{1-\beta} \end{aligned}$$

Using the expression for  $d\check{q}_b/d\gamma_h$  in (34) to rewrite (39) and evaluating it at  $\check{\gamma} = \gamma_h = \gamma_f = 1$  yields:

$$\left. \frac{dRHS}{d\gamma_h} \right| = \frac{s\beta[1+b(u'(q_b)-1)]}{s+r(1-s)} + \frac{\beta}{1-\beta} > 0$$

Therefore,  $\check{\gamma}$  should be higher than one to satisfy this constraint.