Temporary disequilibrium and money in a Classical approach

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Introduction

The Classical approach conceives equilibrium as a state in which the rates of profit are uniform across all industries. Sraffa himself assumed that uniformity without any justification. The dominant tradition in Classical theory is centred on the analysis of equilibrium seen as a 'normal' or 'long-term' position defined by the technique and the state of distribution. According to Ricardo, that equilibrium results from persistent causes whereas 'accidental' causes move the market magnitudes away from their natural positions. In that approach, market prices are non theoretical magnitudes, only their motion can be modelled. Gravitation models of this type intend to build convergent dynamics in order to justify the long-run uniformity of the rates of profit.

Our approach is different: disequilibrium is the expression of the same systematic causes as those which govern equilibrium. Our starting point is Marx's analysis, which stresses that the social division of labour is not reduced to the general interdependency between sectors and goes with a divergence between the individual estimates and the social evaluation (market prices) of produced commodities.

We return to Marx's problem and propose an answer to the open question of the determination of exchange values. Following Torrens and Marx (in his study of reproduction schemes), it is assumed that profits are reinvested in their own sector. In their analysis the real magnitudes are determined by a system of equations which express the capitalists' accumulation targets and the

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budget constraints. We go farther in the analysis of disequilibrium by introducing, on the one hand, a market mechanism for the determination of monetary prices and the allocation of inputs and, on the other hand, an institutional device for the settlement of monetary imbalances, which both lead to the final effective allocations and productions. The forces behind the dynamics are the capitalists' accumulation desire, the disequilibrium resulting from the contradiction between these desires and the physical constraints as represented by the wage basket, the methods of production and the existing quantities. We determine the prices, the rates of profit and the produced quantities within a temporary disequilibrium framework. The study of those dynamics shows that the economy may converge towards the natural quantities and the Classical equilibrium prices or towards a limit cycle.

The model is an attempt to take into account some features which are often neglected in modern Classical theory, namely: the transactions in disequilibrium and in equilibrium as well, the formation of prices, individual real and monetary disequilibria, the role of price expectations, money and institutional rules for monetary settlements, and the real effects of the integration of money in the theory of value.

I. Disequilibrium and Classical tradition: a broken promise

I.1. Marx and the social division of labour

The social division of labour is at the basis of market economy. Smith distinguished between the social division of labour and its technical division inside the manufacture. Only the first (which we now call division of labour, to simplify) generalises the exchange relations between individuals: "Every man thus lives by exchanging, or becomes in some measure a merchant, and the society itself grows to be what is properly a commercial society" (Smith (1776), I.4.1), by creating interdependency relationships between producers: "In civilized society he stands at all times in need of the cooperation and assistance of great multitudes" (Smith (1776), I.2.2). An individual's wealth then depends of his production but also of prices (buying prices for inputs and selling prices for outputs) he does not control.

A few exceptions apart, the Classical theory of value is centred on the determination of equilibrium prices as the solution of a system of equations in which the division of labour is reduced to the differences between products and the interdependencies between sectors. Marx's analysis moved aside from that tradition. In the main two works on political economy published in his lifetime (*A Contribution…*, 1859, *Capital*, book I, 1867), Marx started his analysis of capitalism by constructing a theory of value in disequilibrium, disequilibrium being conceived as a necessary consequence of the division of labour and not as a gap to an equilibrium position due to 'accidental causes' *à la* Ricardo. His remarkable analysis of the division of labour is unfortunately made obscure by his project to elaborate a labour theory of value that he intended to oppose to Ricardo's. When one gets rid of that ambition, Marx's analysis becomes the main historical reference for the analysis of market (or exchanges) (see Benetti and Cartelier, 1999).

Following Marx: "As a general rule, articles of utility become commodities, only because they are products of the labour of private individuals or groups of individuals who carry on their work independently of each other" (*Capital*, Book I, Chapter 1, p. 47, see also p. 30). In such a society every capitalist decides his production independently of the others, and without knowing what they are doing, when however the result of everybody's action depends on the others. Individual freedom and interdependencies are the two sides of the same coin: "the seeming mutual independence of the individuals is supplemented by a system of general and mutual dependence through or by means of the products" (*ibid*. p. 72-3). The consequence is what, in his colourful language, Marx designated as "the salto mortale of the commodity" when it is sold (*ibid.* p. 71, and *A Contribution*, p. 47), resulting from the ignorance of exchange values, itself a consequence of the division of labour. Hence the agents' disequilibria, which have nothing to do with price rigidities. The market mechanism implies that, at every moment, the actual prices differ from those on which the agents made their calculations. In disequilibrium, the amounts of goods received by the producers, as well as the prices, are not those that were foreseen.

It is that economy that Marx analysed by means of the labour theory of value. He then arrived directly to the central notion concerning the "twofold character of the labour embodied in commodities" (*Capital* p. 29), of which Marx claimed authorship and which he considered as "the pivot on which a clear comprehension of political economy turns" (*ibid*.). Labour is

simultaneously private and social. What is 'private' is the concrete and heterogeneous labour provided by individuals engaged in the production of use-values, the quantities of which result from individual decisions taken independently from each other. Social labour, which is homogeneous and abstract, is the substance of value, and its quantity determines the exchange values. Marx explained the relationship between the two notions of labour in the following terms: "The point of departure is not the labour of individuals considered as social labour, but on the contrary the particular kinds of labour of private individuals, i.e., labour which proves that it is universal social labour only by the supersession of its original character in the exchange process. Universal social labour is consequently not a ready-made prerequisite but an emerging result." (*A Contribution…*, p. 16). The question is then to determine values inside the unity of production and circulation, viz. the amounts of social labour which are formed through exchanges by starting from given quantities of private labours. Here appears the basic weakness of the labour theory of value, which was noticed by Marx himself: "Thus a new difficulty arises: on the one hand, commodities must enter the exchange process as materialized universal labour-time, on the other hand, the labour-time of individuals becomes materialized universal labour-time only as the result of the exchange process" (*ibid*.). Hence a contradiction: the 'universal social labour' is simultaneously the condition and the result of exchange.

To the best of our knowledge, neither Marx nor the Marxists have solved that problem. That is why Marx's theory of value eventually admits the primacy of production. The canonical model of labour values determines values as the solution $v = (I - A)^{-1}l$ of a system of linear equations. But that loophole is not acceptable as it contradicts the whole of Marx's analysis of the division of labour: vector l cannot represent heterogeneous private labours when vector v is a vector of values and therefore of social homogeneous labour, and l cannot be a vector of social labour either since, as Marx clearly showed, that vector is not a datum analogous to the matrix A of technical coefficients.

Therefore, Marx's labour theory of value does not hold its promise to determine exchange values in an economy with division of labour in which, except by fluke, the agents are in disequilibrium. That failure is very harmful to Marx's theory inasmuch it deprives his analysis of the commodity (or of the division of labour) of its indispensable theoretical ground that neither philosophy nor

history can provide. Section II proposes a study of an economy in disequilibrium in which the reproduction of capital deals with a process of exchange inspired by Marx's analysis and which may help to solve the problem he left open. In section III a Classical model of a monetary bisector economy in temporary disequilibrium illustrates our approach. The dynamics of that economy are sketched in section IV. We must explain before why and how our approach differs from the post-Sraffian tradition.

I.2. Equilibrium and disequilibrium in the Classical tradition

Today, the dominant theory is an equilibrium theory on all markets, those of commodities, labour and money. Only equilibrium is considered as amenable to a rigourous analysis. That methodological requirement (the 'discipline of equilibrium', in Lucas's words) discards a field of research and also expresses implicitly a faith in the automatic equilibrium of markets. It might seem surprising that a parallel can be drawn with the modern version of the Classical theory due to Sraffa. The reference to prices of production, or long-term prices according to the usual interpretation, with a uniform rate of profit, presumes that production meets the social demand. That condition is probably inherent to any theory of value which aims at defining permanent prices, be it the labour theory of value or that of production prices.

What about a return to the Classical theory that Sraffa expected? That theory has many facets. The dynamic nature of the Ricardian theory has often been underlined. Whatever dramatic those dynamics are, with the long-run fall of the rate of profit, they do not refer to disequilibrium but rather to a succession of equilibria, as shown by the maintained hypothesis of a uniform rate of profit at each date. As for the theory of gravitation, its objective is not so much to analyse economies out of equilibrium than to provide some justification to the hypothesis of uniform rates of profit. Ricardo's position has more to do with personal convictions than with economic analysis: only long-term equilibria are a theoretical object, because they reveal the consequences of permanent causes, viz. the technique and distribution (Sraffa, 1960 § 4, gives an extreme form to that assertion when he states that the rates of profit *must* be uniform), while market prices are but non theoretical magnitudes. (The idea to consider the good working of gravitation as an axiom is also taken up by other contemporary authors, see for instance Schefold (1998)).

Obviously enough, an analysis restricted to equilibria states cannot address the issue of the determination of prices in an economy with division of labour. Instead of answering a question, it simply deletes it: any contradiction between private and social evaluations disappears at equilibrium and the economy behaves as if it was centralised.

One may wonder if the lack of significant influence of the Sraffian critique on the evolution of economic theory is not due to its reference to the notion of equilibrium. But the Classical tradition is far from being limited to that approach and, be it only for the understanding of economic crises, some Classical authors have attempted to improve the analysis by integrating the dynamics of prices and quantities in a more convincing way. In that line of thought, the attention can be drawn on Torrens's (1821) work. Torrens was indeed the very first to point at the physical constraints imposed to the reproduction of capital when the sectoral profits are totally reinvested in the same branch (see however section IV on the limits of his analysis).

II. Towards a theory of market disequilibrium

Starting from Marx's analysis of the division of labour, we propose a solution to the Marxian indetermination of social values on the ground of private labours, independently of any reference to the notion of exploitation (which is not relevant for the problem at stake). The solution we propose discards the labour theory of value and answers to Marx's question by introducing two devices inseparable from the division of labour itself: one is money, the other is a market mechanism which determines prices and allocations. Each producer is assumed to take, before the opening of the market, irrevocable decisions which lead him to bring to the market some amount of product (supply side) and to ask for some amount of money (demand side). The market mechanism described below then determines prices and allocations which usually differ from those the agents expected.

II.1. Prices and division of labour

Our analysis of the exchange process relies on the notion of the 'twofold character of price', which is inspired by the 'twofold character of the labour embodied in commodities' we have seen in Marx. We distinguish two notions:

- the 'expected prices' are those used by the producer to determine his production plan given his expected budget constraint, from which he deduces his demand for inputs.

- the 'market prices' are those at which transactions occur on the market.

Expected prices

Each producer knows his technique of production and the amount of product obtained at the end of the previous period. In an economy with division of labour, he must foresee the quantity he will produce but he knows neither the selling price of his product nor the buying prices of his inputs. By their private character, these plans are only based on the fragile ground of personal estimates (generally incorrect) of the market prices. The expected price is a personal interpretation of the state of the market, i.e. a personal opinion on the evaluation by the other agents (the society) of his own product. At these expected prices, capitalists determine the quantities they intend to produce and the inputs required for their productions. In our model each capitalist aims at maximizing accumulation in his own sector (Torrens, 1821, chap. VI, sect. VI). A different behavioural hypothesis could have been retained but, in any case, individual decisions must satisfy the expected budget constraint: the expected value of the purchases must not exceed the expected value of the sales (credit is discarded by hypothesis).

It must be stressed that our expected price has no relationship with Hicks's. In Hicks's theory, expected prices are a link between the present and the future and their aim is to overcome the limits of what Hicks calls the 'static theory': "supplies (and ultimatley demand too) are governed by expected prices as much as by current prices" (Hicks, 1938, p. 116). Here, the supplies and demands at date t depend on prices at t and of the expectations at date t of prices at date $t+1$. In our approach, by contrast, the expected price does not coexist with the market price: expected prices are but the basis for the formation of market prices at the beginning of the current period.

Market prices

Individual productions receive a social evaluation on the market. Which are the prices resulting from the whole set of individual actions governed by expected prices?

'Market' is conceived as the set of places of exchanges, or trading posts, one for each commodity. According to Cantillon (1755), the market price of good i is determined by the ratio between the aggregated monetary expense on the good i market and the quantity of that good brought to the market[1]. The exchange value of good i then results from the whole set of private estimates and defines the market prices (or 'social prices') and the allocations. Except by fluke, the market value differs from the expected price.

II.2. Money and temporary disequilibrium

Monetary economy

Money is the institutional expression of the division of labour. It allows each agent to act independently of the others (to buy without depending on the sale of his product). Market prices are endogenous and the existence of a universal means of exchange on all markets is the condition for the consistency of market prices. We give privilege to that function (Wicksell (1906) p. 7: "Of the three main functions, only the last [medium of exchange] is in a true sense characteristic of money"). The quantity of money is endogenous, and it is issued by a bank at the producers' requests. The amount of money demanded by each agent is equal to the expected value of his supply of goods, which itself is equal to the expected value of his purchases of goods. Each agent commits himself to reimburse the money he received from the bank (this is why money does not appear in the *ex ante* budget constraint), and money is destroyed after it has accomplished its circular flow. As we assume that the creation of money, its circulation and its destruction take place within a very short time interval, the rate of interest is ignored. There is neither financing of present purchases by future resources (no credit) nor financing of future purchases by today's money (money is not considered here as a store of value). In short, money is not a specific good in the initial endowments; it is not either a credit or a store of value and its amount is an endogenous magnitude. A similar conception of money, though not dominant in economic theory, can be traced back at least to Wicksell (1906), is found in post-Keynesian approaches and is also adopted by Drèze and Polemarchakis (2000) in a general equilibrium framework. Unlike these authors, however, our analysis introduces a market mechanism

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[1] The notion of "trading post" was introduced in the modern theory of strategic market games by Shapley and Shubik (1977). These authors make use of a similar Cantillon rule to determine the market price.

determining monetary prices both in disequilibrium and equilibrium.

Since the availability of a means of exchange allows each agent to buy goods independently of his sales, monetary imbalances appear and must be settled. The agent's disequilibrium is real (he does not fulfil his own plan) and monetary (positive or negative balances appear). Monetary disequilibrium is objective, observable and measurable. It defines and determines quantitatively the disequilibrium of the economy.

Let us have a closer look at monetary disequilibrium. The amount of money that an agent spends at a given time is the total expected value of his product, whereas the amount he actually receives is the value of that product at market prices. These two amounts result from distinct evaluations (the first is private, the second social) of the same product and their difference gives a balance, which may be either positive or negative. These balances follow from the monetary organization of the society and from the fact that the means of exchange is not an initial endowment (by contrast, monetary imbalances cannot be conceived in the so-called 'monetary economy' in disequilibrium as described by Arrow and Hahn (1971), p. 337 *et sq*.). The cause of an agent's negative balance is that his expected price was higher than the market price: the total value of purchases exceeds sales. The constraint budget, which was met *ex ante* when calculations were made at expected prices, is violated *ex post* at market prices. An agent who has not financed the whole of his purchases is not the ultimate owner of the goods he got on the market: in our monetary economy, the property of good is fully transferred only after balance settlements. Different institutional rules can be considered for these settlements: the one we retain will be explained in more details in Section III.

Let us conclude. That analysis of market in disequilibrium provides a way to solve the problem Marx left open. In that scheme money takes the place that labour had in Marx's theory. Individual evaluations of production are made in monetary terms (at expected prices), not in terms of concrete labour. Similarly for the social evaluation of products, which is made in monetary terms, not in that of social labour. The mysterious passage from private labour to social labour, which is deemed to occur through the exchange process in Marx's theory, disappears. The question of the incommensurability between heterogeneous private labours and homogeneous social labour, which is set by Marx's analysis, becomes irrelevant as the monetary magnitudes

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associated with private evaluations are of the same nature as the monetary magnitudes socially determined on markets. Our market model is therefore a potential candidate as a theoretical ground for Marx's analysis of an economy characterized by division of labour.

Temporary disequilibrium

After the balance settlements –still to be described-, all magnitudes defining the temporary disequilibrium are determined. The available goods which were initially in their producer's hands are redistributed among the sectors and used as inputs (and, for some good, partly excluded from accumulation). The effective production of each sector is then determined. Since the market prices are known, so are the rates of profit, which are not uniform across sectors.

In that theory, disequilibrium is the 'normal' state of an economy with division of labour and not, as in Ricardo's approach, an accidental departure from the natural position. This was intended as a justification of the notion of price of production. But when disequilibrium is seen as a necessary consequence of the division of labour, the problem is no longer that of stability, but that of the study of a particular state defined by the technique, distribution, the produced quantities, and also the divergence between the private and the social estimates.[2] The aim of the analysis is then the determination of prices and of produced quantities in that case. The study of transitory states of the economy, which is but a minor aspect of the stability analysis, becomes the central point in our approach. It is only when that step is made that the dynamical analysis can start. To that end, a hypothesis on the formation of price expectations is required, as it allows us to build a sequence of temporary disequilibria which describes the evolution of the economy. A first objective is to check that the dynamics are not explosive. The similarities in the mathematical tools used in that dynamical analysis and in that of the stability of equilibrium cannot not hide the differences between our approach to disequilibrium and that of the Ricardian tradition.

II.3. Equilibria

A full equilibrium (or long-period position) of the economy is defined by the implementation of

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[2] That is why we reject the interpretation proposed by Garegnani (1976) according to which the expected price would be an 'accidental cause' of the gap between the market price and the natural price. That gap is a direct consequence of the division of labour, not an accidental mistake.

two conditions: (i) *market temporary equilibrium*, that is the equality between expected and market prices; and (ii) *reproduction equilibrium*, that is the equality between the two rates of accumulation.

Market temporary equilibrium implies that all production plans are met and that the monetary balances are zero. Note that this equilibrium holds only by a sheer fluke (and at the next period the economy will be again in disequilibrium). It is not the result of an underlying stable adjustment process (as the Hicksian temporary equilibrium (Hicks, 1938)) and it is not either the temporary equilibrium of Classical models (Roemer, 1980; Franke, 1987). That is why the dynamics of our model are those of a sequence of temporary disequilibria.

Reproduction equilibrium implies that the structure of production is constant. Full equilibrium is market equilibrium combined with the full success of reproduction. At full equilibrium, the economy is on a von Neumann maximum growth path. In that case, the rate of profit is uniform and equal to the maximum growth rate of the economy, and the market prices coincide with the Classical prices of production. The structure of relative prices and the proportions in production are given by the right and left dominant eigenvectors of the input-output matrix.

In our approach, equilibrium is a specific configuration of the economy when there is no gap between private and social evaluation of production, a rather exceptional state.

III. A Classical bisector model

The model[3] we study is based on the following hypotheses: (i) Labour does not appear explicitly, every worker being replaced by the corresponding wage basket, which is incorporated in the means of production; (ii) there are two sectors and one method of production per sector, constant returns prevail, production takes one period, all capital is circulating and all goods are perishable; (iii) goods can be disposed of freely; (iv) each capitalist aims at maximising accumulation in his own sector.[4]

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[3] For a complete study of the model, without or with money, see Benetti *et al.* (2012 and 2014).

[4] Maximising accumulation is a quite usual hypothesis in the Classical tradition, connected with the idea that accumulation is the best way to get future profits. In this tradition, the hypothesis that capitalists invest in their own

We moreover assume that all producers have the same monetary price expectations. That hypothesis, which is quite natural as soon as one admits static expectations (as we shall do), will be commented below.

III.1. Prices and market disequilibrium

As each producer maximises his production plan under the budget constraint he foresees, the expected value of his product is equal to the expected value of the inputs it intends to get. The production plans only depend on the relative expected price of the two goods. In our model, the whole production goes to the market: each agent i ($i = 1,2$) sells on the market the whole of his product, then buys again the amount of input i he needs (in that 'wash sales' version, producer i intervenes on both sides of market i, first as a seller, then as a buyer). Each agent asks to and gets from the bank a quantity of money equal to the expected value of his product. He shares that quantity among the two markets by considering his production plan, which takes into account the technical coefficients and the expected values of the inputs.The application of the Cantillon rule then determines the monetary market price of good i as the ratio between the aggregate amount of money on market i (stemming from both producers) to the quantity of good i supplied on that market (and provided by agent i). These physical quantities and the money are swapped for each other on both markets.

The corresponding formalisation is simple: let a_{ij} be the quantity of input j per unit of product i, q_i^- the quantity produced at the end of the previous period and available at the beginning of the new period before the opening of the market and, finally, let p_i^e be the expected monetary price of good i. On those bases, capitalists determine their production plans, denoted q_i^n (index n is for 'notional'), which meet the budget constraint:

$$q_i^n = \frac{q_i^- p_i^e}{a_{ii} p_i^e + a_{ij} p_j^e} \tag{1}$$

Formula (1) shows that plans only depend on the relative price $p^e = p_1^e / p_2^e$, not on monetary prices.

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sector is found in Torrens (1821) and in Marx's (1885) enlarged reproduction schemes. That hypothesis excludes the existence of a capital market and allows us to isolate the role of money as a pure means of exchange.

The producers get from the bank the quantity $p_i^e q_i^-$ of money and spend it on both markets. The application of the Cantillon rule relative to the price formation then determines the market prices and the market allocations of commodities

$$p_i = \frac{p_i^e d_i}{s_i} = \frac{p_i^e (a_{ii} q_i^n + a_{ji} q_j^n)}{q_i^-} \tag{2}$$

where d_i is the total demand of good i. The product $p_i^e d_i$ is the monetary expense on market i, and s_i is the the quantity of good i brought to the market.

Flukes apart, prices expectations are not met. The production plans are not fulfilled and the capitalists are in real disequilibrium. That disequilibrium is, in Marx's terms, 'private' since the agents' plans reflect their personal point of view. If the agents' price expectation for good i is greater than its market price, they overvalue that good and undervalue the other good j. Formula (2) shows that $p_i \gtrless p_i^e \Leftrightarrow d_i \gtrless s_i$: the undervalued commodity is in excess demand and is called 'scarce', while the overvalued commodity is in excess supply and is called 'superabundant'. The allocations differ from demands: the quantity of some good the producers receive on a market is either smaller or greater than that he had scheduled.

These individual disequilibria have a social counterpart, which takes the form of monetary disequilibria: if each agent spends a monetary sum equal to the expected value of his sales, the money he receives differs from his expectations as soon as his price expectation is wrong. Those of the agents who underestimated their good receive more money than they have spent and have therefore a positive monetary balance; and conversely for too optimistic agents. As money is endogenous and is a pure means of exchange, the following equality holds by construction: quantity of money issued = total expenses = total receipts, or $M = p_1^e q_1^- + p_2^e q_2^- = p_1 q_1^- + p_2 q_2^-$, hence $q_1^-(p_1^e - p_1) + q_2^-(p_2^e - p_2) = 0$. The sum of monetary balances is zero and $p_1^e \lessgtr p_1 \Leftrightarrow$ $p_2^e \gtrless p_2$: if agent 1 underestimates his product, agent 2 overevaluates his. The individual monetary disequilibrium is the adequate expression of the 'salto mortale' of the commodity when it is sold, i.e. transmuted into money. It is worth noting that, even in the extreme case here retained of uniform price expectations, exchanges in disequilibrium result in monetary imbalances. Those common expectations are not self-fulfilling.

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III.2. Balances settlement and effective production

The monetary disequilibrium let the two agents in an assymetric position: the capitalist with a negative balance is unable to reimburse the bank. He is not the ultimate owner of a part of the goods he bought on the market: a liquidator can seize them. The institutional rule we admit is the following: the producer with a negative money balance obtains the money he needs to pay back the bank by transferring some of his real assets to the agent with positive money balance. We assume that the transferred goods are evaluated at market prices and that the physical composition of the transferred basket is determined by the capitalist with a positive balance, who is in a position to impose it. He chooses it in order to maximise his next effective production. Then the effective productions are fully determined. The capitalist with a positive monetary balance (i.e., the one who underestimated the value of his production) will eventually get the inputs allowing him to accumulate the whole of his profits, beyond his initial plan based on lower expected profits. Except in the pathological case referred to in note 7, the whole of his product is fully accumulated (see Benetti *et al*., 2014). As for the agent in the red, he will ultimately get a basket of inputs that he will not be able to use in its totality for production because the proportions differ from his (given) technical coefficients. For these two reasons (reimbursement of the debt and inadequate proportions), the corresponding product will be lower than expected.

Let capitalist 1 be the one with a positive monetary balance after the market. He accumulates all goods he possesses, those bought on the market and those obtained from agent 2 during the settlement of monetary balances. His effective production amounts to

$$q_1 = \frac{q_1^- p_1}{a_{11}p_1 + a_{12}p_2} \qquad (3)$$

and is greater than the expected production.

Producer 2, by contrast, must deliver some inputs to capitalist 1 for the settlement of his negative monetary balance. The basket he delivers being in the proportion desired by producer 1, the proportions of the input basket remaining to producer 2 fit no longer the technical coefficients of industry 2, therefore an input is not totally used and the effective production of good 2 is expressed by means of the *min* function:

$$q_2 = \min(\frac{\bar{q_1} - a_{11}q_1}{a_{21}}, \frac{\bar{q_2} - a_{12}q_1}{a_{22}})$$

The production of good 2 is lower than its producer expected and, in the normal case (see note 7), the initial quantity $\bar{q_2}$ is not fully accumulated.

Does the mechanism we consider violate the principle of voluntary exchange, which expresses that no agent is obliged to sell or buy more than he had planned at market prices?[5] The question is sensible in Bénassy's (1986) model in which plans and exchanges are made at the same fix price. But it is irrelevant in the present model because the agents' plans are based on expected prices while exchanges are ruled by the initially unknown market prices. A comparison between expected and effective positions is meaningless when these prices differ. Furthermore, in our model, the exchange remains incomplete as long as monetary imbalances have not been settled by means of an institutional procedure. After the market and before that settlement, no agent can know if he bought too much or too little of a good. The very notion of voluntary exchange does not fit with that framework.

III.3. A noteworthy relationship

In our model, there appears a relationship stating that the value of the quantity of input i used in industry j is the same as the value of inputs j used in industry i:

$$q_i a_{ij} p_j = q_j a_{ji} p_i \tag{4}$$

That relationship follows from the equalities between uses and resources. Let us consider good i. The balance of its uses and resources determines the quantity e_i which is not accumulated in any sector and is excluded (hence notation 'e') from accumulation:

$$e_i = \bar{q_i} - q_i a_{ii} - q_j a_{ji} \tag{5}$$

Consider now producer i. The equality of his sales and his expenses determines the value he will not invest, which we denote v_i:

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[5] In general strategic market games, it may also be the case that an agent receives more than he had asked for. Shapley and Shubik (1977, p. 947) pointed at that phenomenon but only noticed that "it is a matter of letting one's stomach rather than one's purse absorb the fluctuations".

$$v_i = p_i \bar{q_i} - p_i q_i a_{ii} - p_j q_i a_{ij} \qquad (6)$$

By combining these relationships and those corresponding to good j and sector j, one shows that:

$$(p_i e_i - v_i) + (p_j e_j - v_j) = 0 \qquad (7)$$

Therefore[6] relation (4) holds if receipts that producer i does not devote to accumulation is equal to the value of the amount of good i excluded from accumulation. When that condition holds for i, it automatically holds for j. In our model, the producer who gets a positive monetary balance accumulates all of his effective profits ($v_i = 0$ for that agent) and the commodity he produces in the current period is entirely accumulated[7] ($e_i = 0$).[8] The equivalence relation (4) between the values of external productive resources is then met.[9]

IV. Dynamics

That section summarizes the results obtained by Benetti *et alii* (2014). The effective productions being determined within a given period, an expectation hypothesis allows us to link the periods to each other and therefore to study the dynamics of the model. One may then wonder whether the dynamics converges or not towards a full equilibrium state. For simplicity, we assume static expectations. Even under such a hypothesis, an analytical study of the dynamics seems out of

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[6] Equality (4) is not a Walras law: in particular, e_i is not a magnitude relative to the behaviour of any capitalist, since it results from the accumulation behaviour in both sectors. *A priori*, inequalities of the type $p_i e_i \neq v_i$ do not imply any type of disequilibrium by themselves.

[7] At least in the 'normal' case. One can identify a 'pathological' case in which relation (4) does not hold because the totally accumulated commodity is not the one produced in the sector in monetary excess.

[8] For the other good j, e_j and $v_j = p_j e_j$ are positive magnitudes.

[9] The same equality (4) also holds in: (*i*) the 'Torrens model' in which the whole production of both sectors is totally accumulated; (*ii*) a model which generalises it, in which the rates of accumulation are positive and exogenous (see Benetti *et alii*, 2013, model 1) ; (*iii*) a bisector temporary disequilibrium model without money, in which the effective quantities which intervene in relation (4) are evaluated at expected prices (see Benetti *et alii*, 2012).

reach because the dynamics depend on the nature and the amounts of goods excluded from accumulation as described by the functions *min*, and which may vary from a period to the next. In our model, however, that study remains possible because of the existence of a simple linear relation between the relative price and the relative quantity, which is derived from (4):

$$\frac{q_i}{q_j} = \frac{a_{ji}}{a_{ij}} \frac{p_i}{p_j} \tag{8}$$

That relation allows us to express the real dynamics by an induction equation relative to either the proportion of productions or the relative price.

Real dynamics

When the proportion is that of the Perron-Frobenius row-eigenvector of the technical matrix and the price expectations are the corresponding column-eigenvector, all goods are accumulated, the price and quantity expectations are met and the economy follows a regular growth path at maximum rate (von Neumann growth rate). Otherwise, the proportion and relative price vary from a period to the next, there are balance settlements and some goods are excluded from accumulation. Even if it can be shown that these dynamics are never explosive, several evolutions remain possible. The dynamics of the relative quantities depend on the sign of the determinant of the technical matrix: if the determinant is positive, the system converges towards a von Neumann growth path; if it is negative, either that convergence is local (and may be global), or there exists a limit cycle of order two, depending on the ratio between the second and the first (or dominant) eigenvalue of the matrix.

Thanks to relation (8), the dynamics of the relative price are basically the same as those of the relative quantities. In particular, in case of convergence, the relative price tends towards the right eigenvector of the technical matrix *A*, i.e. towards the price of production as considered in the Classical theory.

Torrens

We may compare these results with Torrens's (1821), a classical reference for the physical reproduction of capital and the accumulation process. In section VI of Chapter VI of his book, Torrens made use of numerical examples to study the reproduction of an economy where capitalists accumulate all profits in their own sector. The corresponding equations determine the

sectoral accumulation rates, which by definition are equal to the rates of profit. Then disequilibrium prices are also determined. The equilibrium configuration requires "good proportions" (in Torrens's terms) between sectors, which are those corresponding to the left dominant eigenvector of the input matrix **A** and coincide also with the von Neumann proportions. Torrens shows that this equilibrium is unstable. Such a state defines what Torrens called a "general glut". This explains Torrens's original position in the debate on Say's law: since supply creates its own demand only if the system is in the "good proportions", Say's law is neither always wrong, as claimed by Malthus, nor always right, as claimed by Ricardo.

Interpreted in the terms of our model, the total accumulation of profits in Torrens's model presumes that price expectations are met at each date, which means that all capitalists are in equilibrium in an economy which is itself in disequilibrium: that configuration, which is exceptional in our model, is the rule in Torrens's. Our model is more stable than Torrens's because the market mechanism we consider partly discards some good from accumulation.

Nominal dynamics

Rather unexpectedly, there exists an asymmetry between the dynamics of quantities and those of nominal prices. The rule adopted for quantities makes reference in equation (5) to function *min* and is not amenable to a simple analytical study.

A doubling of nominal prices at date 0 leads to the same proportional change at any date. In the case of convergence of relative prices and quantities, the nominal prices at equilibrium are also doubled. Consider now the effects of an exogenous non proportional shock at some date on expected monetary prices when the economy is assumed to be at equilibrium at that date, all goods being entirely accumulated. As the relative expected price is changed, the agents' demands are modified: some good becomes superabundant and is not totally accumulated, monetary imbalances requiring monetary settlements appear and effective productions are modified. The economy is thus submitted to real and monetary disequilibria. These phenomena last all along the transitional dynamics until a new equilibrium is reached in the long run. The economy then recovers its initial von Neumann rate of growth (no long-run rate effect), but the real effects of the short-term shock in terms of levels of production are permanent. There is a long-run level effect, with no catch up.

Conclusion

The main novelty of our model is to introduce money and a market mechanism in a Classical framework taking disequilibrium into account.

Money is considered as a pure means of exchange and is issued by a bank at the agents' requests (endogenous money). Why does money matter? The behaviour of a monetary economy differs from that of a barter economy obeying otherwise the same rules (Benetti *et al.* 2012): even if equilibrium is the same in both models, relative prices differ in disequilibrium, as well as the amounts of goods and their allocations between agents. Money also matters because monetary imbalances must be settled at each period. The presence of a means of exchange requires the definition of institutional rules related to its issue and to balance settlements, which affect the allocation of inputs between agents and the dynamics. The model calls for variants concerning legal rules and for extensions, beyond the peculiar rules we have adopted: for instance financing monetary balances by transferring securities instead of real capital or, alternatively, introducing credit and interest rates.

Our bisector model may serve as a basis for a macroeconomic study inspired by a Classical approach, which puts disequilibrium at the core of economic analysis. The grounds of that approach are Marx's analysis, which contrasts private and social valuations of commodities. However, when Marx refers to a distinction between private and social labour, we refer only to monetary quantities, social valuations being determined by a market mechanism. We propose to reconsider the problem of exchange in a framework where the stress is put on money rather than labour.

Our approach also differs from the post-Sraffian long-period analysis, of which it is a complement. The fruitful Sraffian and post-Sraffian analysis of production prices only goes on half of the way. It is useful to complement it with a theory of prices and allocations in temporary disequilibrium, which is the normal state of an economy characterized by the division of labour. The uniformity of the rates of profit may be the result of the dynamics.

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