Renewable Resources in a Long-Term Perspective: The Corn-Tuna Model

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1 Introduction

In models of classical, Sraffian or long-term inspiration, natural resources are usually confined to 'Ricardian' lands, i.e. resources of which the quality and quantity remain unchanged over time, no matter how they are used. When they become scarce, a positive price must be paid for their use. The theory of rent examines all the intricacies to which this may give rise; a few of the main contributions to this literature are: Sraffa (1960, ch. 11), Quadrio Curzio (1966), Schefold (1971), Montani (1975), Kurz (1978), Abraham-Frois and Berrebi (1980), Steedman (1982), D'Agata (1983), Salvadori (1983), Woods (1987), Bidard and Woods (1989), Erreygers (1990), and Bidard (2010). Considerably less work has been done on either exhaustible or renewable resources. Exhaustible resources have been treated by Parrinello (1983, 2004), Kurz and Salvadori (1997, 2000), and Bidard and Erreygers (2001a, 2001b); this literature is reviewed by Ravagnani (2008). The corn-guano model presented by Bidard and Erreygers has been subject to a debate (see, among others, Kurz and Salvadori, 2001; Parrinello, 2001; and Schefold, 2001), and applied to the problem of waste treatment (Hosoda, 2001).

As far as renewable resources are concerned, there is very little besides the salmon model of Kurz and Salvadori (1995: 351-357). In their model they conecived the renewable resource - salmon - as a nonbasic commodity. They examined whether long-period equilibria exist for a given demand of salmon, and arrived at the conclusion that in some circumstances both stable and unstable equilibria may exist. They also showed that in some cases the population of wild salmon is doomed.

My purpose here is to construct a simple model for the study of renewable resources in a long-term framework. My model is similar, yet distinct from the model presented by Kurz and Salvadori. Since I draw my inspiration from existing simple long-term models, I begin with a brief recapitulation of two of these models, the corn model (Section 2) and the corn-guano model (Section 3). The core of the paper consists of the presentation, analysis and discussion of the corn-tuna model. The characteristics of the corn-tuna model are meant to reflect salient features of the use of renewable resources (Section 4). I start the analysis of the model with a comparison of two 'pure' systems of production (Section 5) before considering the 'mixed' system (Section 6). This leads to the examination of two different government policies (Section 7). I then explore what can be learned from this model, and point out some of its limitations and weaknesses (Section 8).

2 The Corn Model

I find it appropriate to start with a brief reminder of a simple Ricardian model which is very familiar to those who have studied the classical theory: the corn model (see Chapter 1 of Bidard, 2004 for a more extensive presentation). Let us assume that there exists only one produced commodity, corn, which is produced by means of itself, land and labour. Land and labour are assumed to be of uniform quality. The following table summarizes the two processes which are available in this economy.

Table 1: Processes in the Corn Model

Inputs				Outputs	
Corn	Land	Labour		Corn	Land
a	1	l	$\rightarrow$	b	1
0	1	0	$\rightarrow$	0	1

The first process, the single production process of corn, is used year in, year out. The occurrence of land on the side of both inputs and outputs means that the quantity and the quality of land are not affected by its use in production. The same holds if land is left fallow, i.e., if it is not used in production. This is represented by the second process.

The two price relations which correspond to these processes are:

$$bp_c + p_l \leq (1+r)(ap_c + p_l + lw) \qquad [y_{1,t}] \tag{1}$$

$$p_l \leq (1+r)p_l \qquad [y_{2,t}] \tag{2}$$

with r the rate of profits, p_c the price of corn, p_l the price of land, w the wage, and $y_{1,t}$ and $y_{2,t}$ the activity levels of the two processes in period t. The above notation means that if the activity level of a process is positive ($y_{i,t} > 0$), then the corresponding price relation must hold as an equality; by contrast, for an inactive process ($y_{i,t} = 0$) the inequality may be strict (i.e. the process cannot sustain the ruling rate of profit).

Let us assume that the demand for corn is such that there is plenty of land. This means that part of the land is not used; in other words, the fallow process is always active ($y_{2,t} > 0$). In that case, (2) implies $p_l = 0$. Taking corn as numéraire ($p_c = 1$), equation (1) determines the relation between the rate of profits and the wage expressed in units of corn (i.e. w/p_l). If the rate of profit is treated as the independent variable, it should not exceed the limit value of $(b-a)/a$, otherwise the wage will be negative. Alternatively, if the wage is treated as the independent variable, it should not exceed the limit value of $(b-a)/l$, otherwise the rate of profit will be negative.

Instead of taking corn as numéraire, we could also take labour ($w=1$). We then obtain a relation between the rate of profits and the price of corn expressed in labour (i.e. p_c/w, the inverse of the wage expressed in corn). It is easy to show that the relation is equivalent to the one obtained with corn as numéraire.[1] In other words, whatever the numéraire - a mixed corn/labour numéraire would yield exactly the same result - we always find the same relation between the "real rate of profits" and the "real wage".

If the demand for corn continues to increase, eventually all the land will be used for corn production, and no land will be left fallow ($y_2=0$). There is then room for the activation of a second corn production process, which allows to obtain a greater net output of corn. The second process is more expensive than the first; their joint operation leads to a positive price (and hence rent) for land, with the rent equal to the difference in cost between the two corn production methods. When the rate of profit is positive, however, paradoxical results can occur; these are examined in the classical theory of (intensive) rent.

3 Exhaustible Resources

3.1 The Hotelling Rule

I now make the model a bit more complex by introducing the existence of an exhaustible resource, say resource g. Before we specify how this changes the corn-model, let us consider how a rational resource owner will behave in a competitive environment. As long as he owns a positive amount of the resource, an owner faces the choice between selling his resource immediately, or letting it lie idle and sell it in the future. Nothing is lost if we reduce his problem to the choice of selling one unit at time t or waiting for one year and selling it at time $t+1$. In the first case the sale gives him an immediate revenue of $p_{g,t}$, i.e. the price of one unit of the resource at time t, which he can then invest. In the second case he simply waits and obtains a revenue of $p_{g,t+1}$ at time $t+1$. If one of the two options were more profitable than the other, all resource owners would follow the most profitable course of action, which would mean that either the *whole* supply of the resource would be exploited at time t, or that *none* of it would. In general this is impossible: in any period of time, except the one in which the resource is depleted, part

[1] In the first case we have $\frac{w}{p_c}=\frac{b-(1+r)a}{(1+r)l}$, and in the second $\frac{p_c}{w}=\frac{(1+r)l}{b-(1+r)a}$.

of the supply is exploited and part of it is conserved. This means that in equilibrium the two options must be equally profitable. If the first option allows you to obtain a rate of return r when you invest the proceeds of selling the resource, then so must the second. This leads to the following equilibrium condition:

$$r = \frac{p_{g,t+1}}{p_{g,t}} - 1 \tag{3}$$

from which it easily follows that:

$$p_{g,t+1} = (1 + r)p_{g,t} \tag{4}$$

Equation (4) implies that the price of the exhaustible resource, i.e. its royalty, rises at a rate equal to the ruling rate of profit. This is the result known as the *Hotelling rule*, after the seminal contribution of Harold Hotelling (1931). Since exhaustible resources are typically goods which are used for the production of other goods, the Hotelling rule implies that one can expect that the prices of *all* goods will change over time.

3.2 The Corn-Guano Model

The classical theory of prices is often identified with the theory of long-term or normal prices, which are assumed to be constant as long as there are no changes in the methods of production. On this interpretation, the classical theory seems to be at odds with the Hotelling rule. A representative of this view is Bertram Schefold, who starts from the adage that "the classical approach relies on the conception of normal prices and is inseparable from it" (Schefold, 2001: 320), and therefore accepts only with the greatest reluctance the possibility of changing relative prices. I do not share Schefold's view, and reject his way of dealing with the issue. I maintain that the classical theory cannot ignore changing relative prices when dealing with exhaustible resources, and must therefore find a way of coping with changing relative prices. In two previous contributions Bidard and Erreygers (2001a, 2001b) have shown, by means of the corn-guano model, that this is indeed possible.

The corn-guano model has been conceived as a methodological tool: its analytical simplicity allows us to shed light on the original economic features linked to the introduction of exhaustible resources. However, its structure is rich enough to initiate the reader to the study of the dynamics of models with exhaustible resources. There is only one produced commodity, called corn, and one exhaustible resource, called guano. The following table describes the available processes.

Table 2: Processes in the Corn-Guano Model

Inputs				Outputs	
Corn	Guano	Labour		Corn	Guano
a_1	1	l_1	$\to$	b_1	0
a_2	0	l_2	$\to$	b_2	0
0	1	0	$\to$	0	1

The first, or 'guano process', specifies how corn can be produced by means of guano. The second, or 'backstop process', is an alternative corn production method; the name is suggested by the property that it will be used after the exhaustion of the stock of guano.[2] The third process expresses the fact that unused quantities of guano are transferred without loss of quality to the next period. All processes admit constant returns.

3.3 Corn as Numéraire

During any period, the operated methods must yield the same rate of profit whereas the non-operated method(s) do not yield more. As is usual in Sraffian models, I treat the rate of profit as exogenously given. What this means in real terms depends upon the standard of value adopted by investors. Let us assume that the numéraire acts as a standard of value. In the first version of the model (Bidard and Erreygers, 2001a), corn is chosen as numéraire. The rate of profit cannot be too high; I assume it is such that:

$$0 \leq r < \frac{b_1 - a_1}{a_1}, \quad 0 \leq r < \frac{b_2 - a_2}{a_2} \tag{5}$$

(these inequalities are required in order that the two corn production processes can sustain that rate of profit). To make things interesting, I assume that the production coefficients are such that if guano were free, the guano process would be cheaper than the backstop process.[3]

[2]Both processes obviously also require land. Assuming land to be of uniform quality and always abundantly available, its price will be zero. Hence land plays no role in this model, and therefore I have dropped it from the inputs and outputs.

[3]If this were not the case, guano would never be used and we would be back in the simple corn model. The condition holds if we have:

$$\frac{b_2 - (1+r)a_2}{l_2} < \frac{b_1 - (1+r)a_1}{l_1}.$$

For simplicity, I suppose that the date T when the stock of guano becomes exhausted is known. The underlying hypotheses may be that the initial stock and the demand for corn are exogenously given and that the guano method is continuously used until exhaustion.[4]

As long as guano is not exhausted, the guano preservation process is operated ($y_{3,t} > 0$), therefore the price of guano, or royalty, rises at a rate equal to the rate of profit (the Hotelling rule). Since the speed of evolution of the royalty is determined, it suffices to know its level at some date. A simple economic argument is:

> At the moment of exhaustion (...) we expect the backstop method to be used alongside the guano-method. Only by fluke would the then remaining supply of guano be sufficient to satisfy the whole demand for corn by means of the guano process: normally the remaining quantity will be too low, and the backstop process must be operated to fill the gap. (Bidard and Erreygers, 2001a: 249)

The coexistence of the two processes in the period of exhaustion requires that they are equally costly at that time. This condition determines the royalty at the date T of exhaustion, which coincides with the differential rent between the two processes to produce corn. Thanks to the Hotelling rule, the royalties in the preceding periods can be calculated by backward induction. The last unknown, viz. the real wage, is then obtained. As a consequence of the increasing royalties, the real wage decreases as time passes, a phenomenon at variance with the behaviour of Ricardian models without exhaustible resources. One may alternatively assume that the real wage is given and show that the current rate of profit declines up to the exhaustion period.

3.4 A Corn-Labour Numéraire

In a second version of the model (Bidard and Erreygers, 2001b), the numéraire is a given combination of d units of corn and k units of labour, with both d and k positive (by contrast, in our first version we assumed $d = 1$ and $k = 0$). For period t, which begins at date t and ends at date $t + 1$, the price system

[4] Ex post, one must check that the last assumption is consistent with the analysis of prices, i.e. one must check that, up to date T, the guano method is cheaper that than the backstop method.

is such that:

$$\begin{aligned} p_{c,t+1} &\leq (1+r)(a_1 p_{c,t} + p_{g,t} + l_1 w_t) && [y_{1,t}] && (6) \\ p_{c,t+1} &\leq (1+r)(a_2 p_{c,t} + l_2 w_t) && [y_{2,t}] && (7) \\ p_{g,t+1} &= (1+r) p_{g,t} && [y_{3,t}] && (8) \\ 1 &= d p_{c,t} + k w_t && && (9) \end{aligned}$$

where $p_{c,t}$ is the price of corn, $p_{g,t}$ the price (or royalty) of guano, w_t the wage, and $y_{1,t}$ and $y_{2,t}$ the activity levels of the two corn production processes, all at date t. During the exhaustion period T, these two processes are operated simultaneously. After the exhaustion of guano, only the second process can be operated, and expressions (6) and (8) become irrelevant.

Since the relative price of corn and labour changes with time, the profitability of a given process now depends on the composition of the numéraire, in the same way as the profitability of an international firm depends on whether it is calculated in dollars or euros. The dynamics depend on the numéraire and become complex. The study of system (6) to (9) shows that the price equations admit one degree of freedom, say the level of $p_{c,T}$. Once $p_{c,T}$ is known, the system can be solved by means of backward induction. This procedure, however, leads to a negative price $p_{c,T-\tau}$ for τ great enough, except for one specific choice of $p_{c,T}$. That choice defines what we have called the 'natural path'. In fact, there exists an infinite number of paths with positive prices for T periods, but for large T all these paths are close to the natural path. Comparable complications occur in multisector models (several produced commodities) even when the numéraire is made of a unique commodity, but their resolution is similar when the formulae obtained for the corn-guano model are conveniently re-interpreted.

4 Renewable Resources

4.1 The Character of Renewable Resources

Typical for exhaustible resources is that nature has stopped producing them, or produces them at such an extremely slow rate that humans do not notice it. Renewable resources, by contrast, are goods which are produced by natural processes, although sometimes only if a number of critical conditions are satisfied. Think of pelagic fish: as long as the stock of fish swimming in the sea is higher than the critical mass necessary for reproduction and lower than the maximum carrying capacity of the environment, the stock of fish normally increases, which means that fish is produced. One can then harvest fish without endangering the survival of the species, but only if one does not catch too much of it. Sadly enough, there are plenty of examples of overfishing culminating in the extinction or near-extinction of fish species.

In some cases it may be possible to produce renewable resources also by man-made processes. Once again, fish is a good example: various forms of fish farming have become increasingly important in the last decades. This contrasts sharply with the essentially static level of capture fisheries over the last two decades. In its most recent two-yearly report on the state of world fisheries and aquaculture, the FAO wrote:

> Global fish production continues to outpace world population growth, and aquaculture remains one of the fastest-growing food producing sectors. In 2012, aquaculture set another all-time production high and now provides almost half of all fish for human food. This share is projected to rise to 62 percent by 2030 as catches from wild capture fisheries level off and demand from an emerging global middle class substantially increases. If responsibly developed and practised, aquaculture can generate lasting benefits for global food security and economic growth. (FAO, 2014: iii-iv)

In the following model I try to incorporate some of these features. I assume there exists a renewable resource - tuna - which can be produced by nature and by man. Making some strong assumptions, I will explore the competition between the natural method and the man-made method and examine whether and when these methods can co-exist. Let me recall that the exhaustible resource we considered in the previous section, guano, was a means of production, not a consumption good. In the model we are about to present the renewable resource is a consumption good. So we move from a model with one consumption good to a model with two consumption goods.

4.2 Catching Fish in the Wild vs. Fish Farming

First of all we have to clarify the interplay of biological and economic factors involved in the production of fish (see, e.g., Shaefer 1954, 1957). In the absence of fishing, the population P tends to grow according to its natural increase n, which can be seen as a function of the population level, i.e. $n = f(P)$. Let us assume there exists a maximum population level, L, at which the population of fish stops growing, i.e. $f(L) = 0$. If there is fishing, the population changes with the difference between the natural increase and the amount of fish caught in the sea. The catch c depends basically upon two factors, the population of fish P and the effort level e, so that we can assume that we have a functional relationship $c = g(P, e)$. For a given effort level, a larger population normally leads to a higher catch ($\partial c/\partial P > 0$): the more fish are swimming in the sea, the easier it is to catch them. Likewise, for a given population, a higher effort level leads to a higher catch ($\partial c/\partial e > 0$): the more boats are out on the seas, the more fish is caught. To simplify the analysis, I take the following natural increase and catch functions:

$$f(P) = k_1 P(L - P) \tag{10}$$

$$g(P, e) = k_2 P e \tag{11}$$

where k_1 and k_2 are positive constants (Schaefer, 1954: 29-31). The natural increase function (10) determines a relationship between the population and its natural increase which has an inverted U-shape (see Figure 1). The catch function (11) implies that the catch per effort is directly proportional to the population.

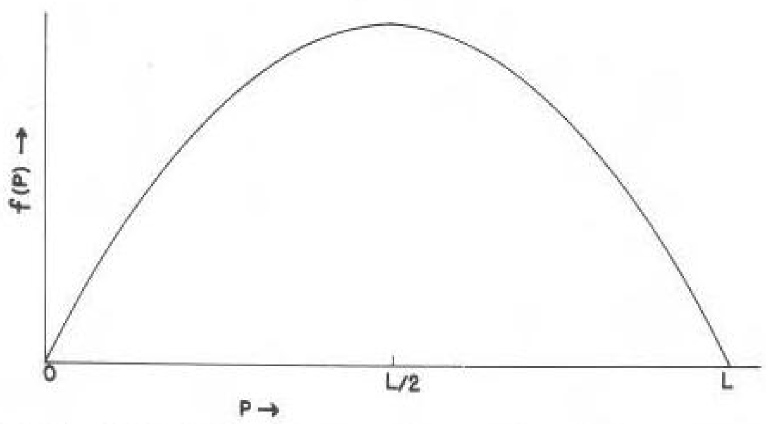

Figure 1. Natural rate of increase of a population which grows according to the Verhulst-Pearl logistic.

Source: Schaefer (1954: 30).

In a long-term perspective, we are especially interested in *sustainable* catch levels. A catch is sustainable if the effort level ensures that the population of fish remains constant. This means that, given the population of fish, the effort level is chosen in such a way that the catch c coincides with the natural increase n. The peak of the natural increase curve represents the maximum sustainable yield, which for (10) is equal to $k_1(L/2)^2$. It is easy to see that to any population $0 \leq P \leq L$ we can associate a sustainable catch $c^*(P) = f(P)$. Given the effort function we can also determine a sustainable effort level $e^*(P)$, where $c^*(P) = g(P, e^*(P))$. For the natural increase and effort functions (10) and (11) we have $c^*(P) = k_1 P(L - P)$ and $e^*(P) = [k_1(L - P)] / k_2$. The relationship between $c^*(P)$ and $e^*(P)$ has typically also an inverted U-shape. For instance, for our specific catch and effort functions, we have $c^*(P) = \{k_2 e^*(P) [k_1 L - k_2 e^*(P)]\} / k_1$. This reflects the fact that any catch below the maximum sustainable yield can be sustained either by a large population (and hence a low effort level) or by a small population (and hence a high effort level). For society it seems best that a large-population equilibrium is obtained, i.e. an equilibrium on the right-hand side of Figure 1. The first reason is that a large-population equilibrium is stable: if there is a small but temporary deviation of the effort level from its sustainable level, the population will tend to return to its equilibrium value. The opposite holds for a small-population equilibrium, which means that there is a substantial risk of overfishing and dwindling populations. The second reason is purely economic: in a small-population equilibrium the cost of producing the same amount of fish as in a comparable large-population equilibrium is much higher.

Fish farming, by contrast, resembles any other industrial or agricultural activity. Fish farms are very much the same as pig farms or cattle farms.

4.3 The Corn-Tuna Model

While fish farming can be integrated without any problem into a Sraffian model, catching fish in the wild poses a real challenge. One problem is that for a given population of fish, the natural method of producing fish is not necessarily characterized by constant returns to scale: doubling the effort level may not double the catch of fish. Moreover, as we have just seen, in a sustainable (and hence long-term) equilibrium the effort level is determined by the population of fish. This means we are obliged to introduce some biological elements into the model.

The first consumption good in our model, corn, is produced by means of itself and land. The second consumption good, tuna, which is the model's renewable resource, can either be caught in the sea by means of boats (this is the natural process) or be produced artificially in aquaculture ponds (the man-made process). In the first case boats are needed, and to make the model a bit more realistic I assume that every year a fraction of the existing fleet must be replaced. With regard to the second case, I assume that ponds also need to be renewed from time to time, so that each year new ponds have to be constructed. Ignoring land and sea, the model therefore consists of 4 goods (corn, tuna, boats and ponds) and 5 processes. I stress that I make the probably unrealistic assumption that wild and farmed tuna are indistinguishable commodities. Consumers are indifferent between the two: they have a demand for tuna, not for wild or farmed tuna.

The available processes are described in the following table.

Table 3: Processes in the Corn-Tuna Model

Inputs						Outputs			
Corn	Tuna	Boat	Pond	Labour		Corn	Tuna	Boat	Pond
a_{11}	0	0	0	l_1	$\rightarrow$	b_{11}	0	0	0
a_{21}	0	1	0	l_2	$\rightarrow$	0	$b_{22,t}$	b_{23}	0
a_{31}	0	0	0	l_3	$\rightarrow$	0	0	1	0
a_{41}	a_{42}	0	1	l_4	$\rightarrow$	0	b_{42}	0	b_{44}
a_{51}	0	0	0	l_5	$\rightarrow$	0	0	0	1

The first process is the single corn production process of the economy. The second and fourth processes are the two available methods to produce tuna: the former using boats to catch fish in the wild, the latter using ponds in which fish are farmed. The third process is the construction process for boats, and the fifth that of ponds. Since boats are used only to catch wild tuna, processes 2 and 3 will always be operated jointly; hence $\{2, 3\}$ is the wild tuna technique. The same holds with respect to processes 4 and 5, and so $\{4, 5\}$ constitutes the farmed tuna technique.

As mentioned, some caution is needed with regard to the second process. The amount of tuna caught per boat, $b_{22,t}$, is context-dependent and hence it has a time index. Recall that the total catch of tuna in a given period is a function of the existing population of tuna and of the effort level. While the population at time t will be denoted by P_t, the effort level in the period between time t and time $t+1$ will be measured by the size of the fishing fleet active in this period, i.e. $y_{2,t}$.[5] The catch function can therefore be written as $g(P_t, y_{2,t})$. Since the total catch is equal to the catch per boat $b_{22,t}$ multiplied by the number of boats $y_{2,t}$, we have $b_{22,t} = g(P_t, y_{2,t})/y_{2,t}$. Given the effort function (11) the average tuna catch per boat $b_{22,t}$ is a simple linear function of the population P_t:

$$b_{22,t} = k_2 P_t \tag{12}$$

As far as the b_{23} coefficient is concerned, I assume it is smaller than 1. This expresses the fact each year some boats break down or perish, so that

[5] To be precise, the effort consists of a combination of boats, corn and labour, but since these are always used in the same proportion, there is no harm in treating boats as a proxy.

new boats have to be constructed in order to keep the fleet constant. The number $1-b_{23}$ measures the fraction of the fleet which must be replaced each year in order to keep the fleet constant. A similar reasoning underpins the assumption that the b_{44} coefficient is smaller than 1: ponds also have to be replaced, and the replacement rate is equal to $1 - b_{44}$.

Please note that in order to simplify the analysis, I assume that tuna is not used as a means of production, except in aquaculture. In this respect the model presented here is slightly different from the salmon model of Kurz and Salvadori (1995: 352). Given that overfishing can lead to the exhaustion of fish stocks, the wild tuna technique is to a certain extent comparable to the guano process, whereas the farmed tuna technique resembles the backstop process in the corn-guano model.

Finally, when I talk about the sea, I always understand that this is not a privately owned good. Whereas the amount of fish in the sea may be large or small, the sea itself is assumed to be vastly large. The sea therefore does not have a price, and no rent will ever be paid for its use. Nevertheless, it may occur that boat owners are in a situation to capture the advantage of using the sea as a productive agent. The 'sea rent' must therefore be understood as a kind of extra-profit earned by boat owners. Although land is (usually) privately owned, I assume here that it is so plentiful that its owners never earn rent. This means that we can drop the land from both the quantity and the price equations.

4.4 The Quantity Side

The quantity equations specify that the activity levels are such that the system is capable of satisfying the net demands for corn and tuna of the current period, and of producing everything which is needed to continue production in the next period. This may require an expansion of the fishing fleet and of the number of ponds.

Let the net demands for corn and tuna in period t be equal to $d_{1,t}$ and $d_{2,t}$. The activity level of process i in period t is represented by $y_{i,t}$. My choice of units implies that $y_{1,t}$ stands for the area of land used for corn production, $y_{2,t}$ for the effective number of boats used to catch tuna, $y_{3,t}$ for the number of new boats constructed, $y_{4,t}$ for the effective number of ponds used to produce tuna, and $y_{5,t}$ for the number of new ponds constructed.

The quantity equations can be written as follows:

$$y_{1,t}b_{11} = y_{1,t+1}a_{11} + y_{2,t+1}a_{21} + y_{3,t+1}a_{31} + y_{4,t+1}a_{41} + y_{5,t+1}a_{51} + d_{1,t} \quad (13)$$

$$y_{2,t}b_{22,t} + y_{4,t}b_{42} = y_{4,t+1}a_{42} + d_{2,t} \quad (14)$$

$$y_{2,t}b_{23} + y_{3,t} = y_{2,t+1} \quad (15)$$

$$y_{4,t}b_{44} + y_{5,t} = y_{4,t+1} \quad (16)$$

The first equation specifies that the output of corn must be equal to the sum of the aggregate input of corn in the next period and the net demand for corn in the current period. The second equation stipulates the same with respect to the output of tuna. The third equation expresses that the size of the fleet of period $t+1$ is equal to the sum of what remains of the fleet used in period t and the number of new vessels produced in period t. The fourth equation states the same with regard to the number of ponds. With an obvious notation, the quantity equations can be written more compactly in the following matrix form:

$$y_t B_t = y_{t+1} A + d_t \quad (17)$$

Given that the tuna catch per boat depends on the population of tuna (cf. (12)), the activity levels can be determined only if we know how the demands for corn and tuna as well as the population of tuna change over time. One could try to derive the dynamics of the system by means of these equations and inequalities making assumptions about the level of demand in each period, the evolution of the population of tuna, and the initially available quantities of the different goods. But since a choice has to be made among different processes, we also need to consider the price side.

4.5 The Price Side

As usual in classical models, I assume that none of the available methods makes extra-profits, and that all methods which are actually operated break even. I take corn to be the numéraire good, and the rate of profit r to be given, constant over time, and nonnegative.

Let the prices of the four goods in period t be denoted as $p_{i,t}$, $i = 1, 2, 3, 4$, the wage as w_t and the 'sea rent per boat' as z_t. Since corn is the numéraire, we have $p_{1,t} = 1$ in every period. The prices, wage and rent must be such that the following conditions hold:

$$\begin{array}{rcll} b_{11} & \leq & (1+r)(a_{11}+l_1 w_t) & [y_{1,t}] \quad (18) \\ b_{22,t}p_{2,t+1}+b_{23}p_{3,t+1} & \leq & (1+r)(a_{21}+p_{3,t}+l_2 w_t)+z_t & [y_{2,t}] \quad (19) \\ p_{3,t+1} & \leq & (1+r)(a_{31}+l_3 w_t) & [y_{3,t}] \quad (20) \\ b_{42}p_{2,t+1}+b_{44}p_{4,t+1} & \leq & (1+r)(a_{41}+a_{42}p_{2,t}+p_{4,t}+l_4 w_t) & [y_{4,t}] \quad (21) \\ p_{4,t+1} & \leq & (1+r)(a_{51}+l_5 w_t) & [y_{5,t}] \quad (22) \end{array}$$

Using an obvious matrix notation the price side can be written more compactly as:

$$B_t p_{t+1} \leqq (1+r)(Ap_t + lw_t) + sz_t \quad (23)$$

$$y_t B_t p_{t+1} = (1+r)y_t(Ap_t + lw_t) + y_t s z_t \quad (24)$$

Boat owners are in a position to capture rent only if they can produce tuna more cheaply than tuna farmers, and if they are unable to satisfy the demand for tuna. We will be more specific about that in what follows.

4.6 Stationary Conditions

In the following sections I focus on situations which correspond to the long-term positions usually studied in classical theory. Let us make the strong assumption that the demand for corn and tuna remain constant over time. I will explore whether there is room for long-term equilibria characterized by constant prices and activity levels, and where the catch of wild tuna is sustainable. This implies that I will consider only situations where the population of tuna remains constant. I will try to find out whether it is possible that the same set of processes is used for an extended period of time, with the prices and activity levels equal to their long-term equilibrium levels.[6]

[6] Of course, in reality changes occur all the time. The perspective adopted here is that it makes sense to focus on long-term equilibrium positions if the changes are slow and gradual, and if the price and quantity dynamics occasioned by these changes are stable, so that the prices and activity levels will never deviate very much from their long-term equilibrium levels. However, as we will see below, there may be circumstances in which stability is not guaranteed.

5 Wild vs. Farmed Tuna

5.1 Two Pure Systems

It seems useful to begin the analysis with a comparison of two 'pure' systems of production. We will look at the possible combination of the two at a later stage.

The first pure system is the one in which tuna is caught in the wild only. Two conditions are vital here: it must be cheaper the catch tuna in the wild than to farm it, so that there is no incentive to switch to aquaculture, and the catch must be both sufficient to satisfy demand and sustainable, so that the existing population of tuna remains the same. This will be called the *Wild Tuna System*. In formal terms, only processes $\{1, 2, 3\}$ are operated, and the activity levels and prices are determined by the following systems of equations (since the activity levels and prices are assumed to be constant, we can drop the time index from the quantity and price equations):

Wild Tuna System

$$\left.\begin{array}{c} y_1 b_{11} = y_1 a_{11} + y_2 a_{21} + y_3 a_{31} + d_1 \\ y_2 b_{22} = d_2 \\ y_2 b_{23} + y_3 = y_2 \end{array}\right\} \qquad \text{(W.Q)}$$

$$\left.\begin{array}{c} b_{11} = (1 + r)(a_{11} + l_1 w) \\ b_{22} p_2 + b_{23} p_3 = (1 + r)(a_{21} + p_3 + l_2 w) \\ p_3 = (1 + r)(a_{31} + l_3 w) \end{array}\right\} \qquad \text{(W.P)}$$

We already know that the catch per boat is $b_{22} = k_2 P$. The existing population P must be such that the supply and demand of tuna coincide with the sustainable catch ($y_2 b_{22} = d_2 = c^*(P)$), and the size of the fleet with the corresponding effort level ($y_2 = e^*(P)$). For the natural increase and catch functions (10) and (11) this means that the fleet size is equal to $y_2 = [k_1(L - P)]/k_2$.

The second pure system is the one in which tuna is produced in aquaculture ponds only. The crucial condition here is that it must be cheaper to farm tuna than to catch it in the wild. But there is another aspect which must be taken into account: if no tuna is caught in the wild and the population of tuna is below its maximum (but not zero), the population will tend to increase and it might be that at some future date catching tuna in the wild becomes more profitable than farming tuna. It must therefore be cheaper to farm tuna than to catch it in the wild for *all* possible population levels. In this system, which I will call the *Farmed Tuna System*, only processes

$\{1, 4, 5\}$ are operated, and the activity levels and prices are determined by the following systems of equations:

Farmed Tuna System

$$\left.\begin{array}{c} y_1 b_{11} = y_1 a_{11} + y_4 a_{41} + y_5 a_{51} + d_1 \\ y_4 b_{42} = y_4 a_{42} + d_2 \\ y_4 b_{44} + y_5 = y_4 \end{array}\right\} \quad \text{(F.Q)}$$

$$\left.\begin{array}{c} b_{11} = (1+r)(a_{11} + l_1 w) \\ b_{42} p_2 + b_{44} p_4 = (1+r)(a_{41} + a_{42} p_2 + p_4 + l_4 w) \\ p_4 = (1+r)(a_{51} + l_5 w) \end{array}\right\} \quad \text{(F.P)}$$

5.2 The Activity Levels

Let us begin by exploring how the activity levels are determined if either the Wild Tuna System of the Farmed Tuna System is operated. If only wild tuna is to be produced, the demand for tuna cannot be higher than the maximum sustainable yield. Hence we must have:

$$d_2 \leq k_1 (L/2)^2 \tag{25}$$

We already know that the catch per boat is $b_{22} = k_2 P$ and that a sustainble catch requires a fishing fleet equal to $y_2 = [k_1(L-P)]/k_2$. Since the total catch is then $y_2 b_{22} = k_1(L-P)P$, it follows that the population P must be such that:

$$k_1(L-P)P = d_2 \tag{26}$$

As can be seen from Figure 1, when the demand for tuna is lower than the maximum sustainable yield, there exist two population levels which are able to sustain the demand for tuna: the small-population equilibrium $P^S = \frac{L}{2} - \sqrt{(\frac{L}{2})^2 - \frac{d_2}{k_1}}$ and the large-population equilibrium $P^L = \frac{L}{2} + \sqrt{(\frac{L}{2})^2 - \frac{d_2}{k_1}}$. Once the population P and the fleet size y_2 are known, the third equation of (W.Q) determines the required number of new boats y_3, and the first equation determines the activity level of corn production y_1. More precisely, we have:

$$y_1 = \frac{d_1}{b_{11} - a_{11}} + \frac{[a_{21} + (1-b_{23})a_{31}]k_1(L-P)}{(b_{11}-a_{11})k_2} \tag{27}$$

$$y_2 = \frac{k_1(L-P)}{k_2} \tag{28}$$

$$y_3 = \frac{(1-b_{23})k_1(L-P)}{k_2} \tag{29}$$

If the supply of tuna comes only from farmed tuna, the second equation of (F.Q) determines the required number of ponds y_4, the third the required number of new ponds y_5, and the first the activity level of corn production y_1. In formal terms:

$$y_1 = \frac{d_1}{b_{11}-a_{11}} + \frac{[a_{41}+(1-b_{44})a_{51}]d_2}{(b_{11}-a_{11})(b_{42}-a_{42})} \tag{30}$$

$$y_4 = \frac{d_2}{b_{42}-a_{42}} \tag{31}$$

$$y_5 = \frac{(1-b_{44})d_2}{b_{42}-a_{42}} \tag{32}$$

5.3 The Costs of Producing Wild and Farmed Tuna

The choice between the Wild and Farmed Tuna Systems depends crucially upon which of the two systems produces tuna in the cheapest way. At first sight the comparison of the two systems seems impossible because they do not produce the same set of goods: in the Farmed Tuna System ponds do not occur, and in the Farmed Tuna System boats are inexistent. But the problem is only apparent. In fact, in both systems the first process determines the wage rate, and this is all that is needed to calculate the shadow prices of boats and ponds, even if they are not actually produced. Since the wage is the same in both systems, the (shadow) prices of boats and ponds are also the same in both systems. Hence the choice is determined exclusively by the price of production of tuna; the system with the lowest tuna price is the most efficient.

Let us formalize this reasoning. The wage w is determined exclusively by the first process and is equal to:

$$w = \frac{b_{11} - (1+r)a_{11}}{(1+r)l_1} \tag{33}$$

Given the wage, the (shadow) price of boats is determined by the third process:

$$p_3 = \frac{b_{11}l_3 - (1+r)(a_{11}l_3 - a_{31}l_1)}{l_1} \tag{34}$$

Likewise, the (shadow) price of ponds is determined by the fifth process:

$$p_4 = \frac{b_{11}l_5 - (1+r)(a_{11}l_5 - a_{51}l_1)}{l_1} \tag{35}$$

Given the wage and the price of boats, the price of tuna in the Wild Tuna System is determined by the second process. This gives us the price of production of wild tuna p_2^W:

$$p_2^W = \frac{b_{11}[l_2+(1+r-b_{23})l_3]-(1+r)[(a_{11}l_2-a_{21}l_1)+(1+r-b_{23})(a_{11}l_3-a_{31}l_1)]}{b_{22}l_1} = \frac{C^W(r)}{b_{22}} \tag{36}$$

In the Farmed Tuna System, the price of tuna is determined by the fourth process. This gives us the price of production of farmed tuna p_2^F:

$$p_2^F = \frac{b_{11}[l_4+(1+r-b_{44})l_5]-(1+r)[(a_{11}l_4-a_{41}l_1)+(1+r-b_{44})(a_{11}l_5-a_{51}l_1)]}{[b_{42}-(1+r)a_{42}]l_1} = \frac{C^F(r)}{b_{42}-(1+r)a_{42}} \tag{37}$$

For a given rate of profit, what can we say about the prices of tuna in the two systems? From (36) it follows that the catch per boat b_{22} has a crucial influence on the wild tuna price. When the catch per boat is relatively high, the wild tuna price is low and probably aquaculture will not be able to produce tuna more cheaply. But when the catch per boat is relatively low, the wild tuna price is high and may very well be above the farmed tuna price. In that case, traditional fishery loses the competition with aquaculture, which turns out to be the cheapest tuna producer. The threshold level of the catch per boat can be calculated with precision. The transition occurs for the value of b_{22} for which we have $p_2^W = p_2^F$; in other terms, aquaculture is more efficient than traditional fishery when the catch per boat sinks below the critical value:

$$\tilde{b}_{22} = \frac{C^W(r)}{C^F(r)}\left[b_{42} - (1+r)a_{42}\right] \tag{38}$$

In view of (12) it is possible to define a corresponding critical population level:

$$\tilde{P} = \tilde{b}_{22}/k_2 \tag{39}$$

5.4 Wild vs. Farmed Tuna

We can now be more specific in our conclusions about the comparison between the two systems. The Wild Tuna System will dominate the Farmed Tuna System if the demand for tuna does not exceed the maximum sustainable yield, and if in addition wild tuna can be produced more cheaply than farmed tuna. Nevertheless, there seem to be some Wild Tuna System equilibria which are very unlikely to occur. This is due to their unstable character.

When the first condition holds, we have seen there are two possible outcomes: a small-population equilibrium P^S and a large-population equilibrium P^L. The small-population equilibrium is clearly an unstable equilibrium: any deviation of the catch from its sustainable level $c^*(P^S) = k_1(L - P^S)P^S$ will entail that the population moves away from P^S. Although it is in principle possible that the economy remains in the small-population equilibrium, it is hard to see how the economy could ever reach it. The large-population equilibrium, on the other hand, is stable. It seems therefore that we must discard the small-population equilibria and concentrate on the large-population equilibria.

When we look at the second condition, it turns out that the probability that it is satisfied is always higher in a large-population equilibrium than in a small-population equilibrium. Since more boats are required in the small-population equilibrium than in the large-population equilibrium, the catch per boat is lower in the small-population equilibrium than in the small-population equilibrium ($b_{22}^S < b_{22}^L$). In the small-population equilibrium the price of production of wild tuna is therefore always higher than in the large-population equilibrium; the ratio of the two prices is equal to $b_{22}^L/b_{22}^S = P^L/P^S$.

To check whether the second condition is verified, we have to look at the critical value of the catch per boat $\tilde{b}_{22}$ (cf. (38)) or, equivalently, at the critical population level $\tilde{P}$ (cf. (39)). Three cases can be distinguished. If $\tilde{P} < P^L$, catching tuna in the wild is more efficient than farming tuna for a tuna population equal (or close) to P^L. Aquaculture will not come off the ground: it is simply too expensive. If $P^L < \tilde{P}$, catching tuna in the wild is less efficient than farming tuna for a tuna population equal (or close) to P^L. Hence, the fishing fleet will remain idle and the demand for tuna will be satisfied by tuna farmers. But if no wild tuna is caught, the population of tuna will increase, and after some time the point may be reached where it is once again more efficient to catch tuna at sea. We therefore have to distinguish two subcases: either a population level exists for which this point is reached, or no such population level exists. More precisely, the former arises if $\tilde{P} < L$, and the latter if $L \leq \tilde{P}$. In the first case, the likely outcome is a mixture of both systems of production - this will be examined in the next section. In the second case, no wild tuna will ever be caught: it is much too expensive to do so.

It is also interesting to note that it may depend upon the rate of profit which of the two systems is the most profitable one. It can occur that the Wild Tuna System is more profitable than the Farmed Tuna System for low rates of profit, but less profitable for high rates of profit. We may even see reswitching for very high rates of profit, as shown in the following example.

5.5 A Numerical Example (I)

Consider an economy characterized by the following data (for the moment we leave the value of the catch per boat undetermined):

Inputs						Outputs			
Corn	Tuna	Boat	Pond	Labour		Corn	Tuna	Boat	Pond
2	0	0	0	1	$\to$	10	0	0	0
1	0	1	0	1	$\to$	0	b_{22}	$\frac{9}{10}$	0
1	0	0	0	5	$\to$	0	0	1	0
1	1	0	1	1	$\to$	0	10	0	$\frac{9}{10}$
3	0	0	0	3	$\to$	0	0	0	1

Under the Wild Tuna System the prices and wage will be:

$$\begin{aligned} p_2^W &= \frac{-90r^2 + 391r + 131}{10b_{22}} \\ p_3 &= 41 - 9r \\ w &= 8 - 2r \end{aligned}$$

and the shadow price of ponds

$$p_4 = 27 - 3r$$

Under the Farmed Tuna System we obtain:

$$\begin{aligned} p_2^F &= \frac{-30r^2 + 257r + 117}{10(9 - r)} \\ p_4 &= 27 - 3r \\ w &= 8 - 2r \end{aligned}$$

and the shadow price of boats

$$p_3 = 41 - 9r$$

The catch per boat for which the wild and farmed tuna prices coincide is equal to:

$$\tilde{b}_{22} = \frac{-90r^2 + 391r + 131}{-30r^2 + 257r + 117}(9 - r)$$

For instance, when $r = 1/15 \simeq 6.67\%$, we have $p_2^W = 47/(3b_{22})$, $p_2^F = 1.5$, $p_3 = 40.4$, $p_4 = 26.8$ and $w = 118/15 \simeq 7.87$. It follows that the critical value $\tilde{b}_{22}$ is equal to $94/9 \simeq 10.44$.

To find out which of the two systems is the most efficient, we have to determine the value of the average catch per boat and check whether it is smaller or larger than the critical value $\tilde{b}_{22}$. Let us assume that the parameters of the natural increase and catch functions are $k_1 = 1/126900$ and

$k_2 = 1/1000$, and the maximum population $L = 20772$. The average catch per boat is therefore $b_{22} = P/1000$, which means that every boat is able to catch 0.1% of the existing population. The maximum sustainable yield of wild tuna is slightly higher than 850.

Suppose that the demand for tuna is equal to $d_2 = 832.23$, which is below the maximum sustainable yield. The two population levels which are able to sustain this demand are $P^S = 8883$ and $P^L = 11889$. We can safely discard the low-population equilibrium. For the high-population equilibrium we have $b_{22} = 11.889 > 10.44 \simeq \tilde{b}_{22}$, which implies that the Wild Tuna System dominates the Farmed Tuna System when $r = 1/15$. The required size of the fishing fleet is $y_2 = 70$, which means that in every period 7 new ships have to be built. The price of production of wild tuna is $p_2^W = 47000/35667 \simeq 1.32$.

In order to show the influence of the rate of profit on the comparison between the two systems, let us suppose that the demand for tuna is equal to $d_2 = 846$. This can be sustained by a tuna population $P^L = 10575$, with a catch per boat equal to $b_{22} = 10.575$ and a fleet size of $y_2 = 80$ boats. The wild tuna price is therefore equal to $p_2^W = (-360r^2 + 1564r + 524)/423$. From this we can infer that the Wild Tuna System dominates the Farmed Tuna System for $r < 1/10 = 10\%$, the Farmed Tuna System dominates the Wild Tuna System for $1/10 < r < 259/360$, and the Wild Tuna System again dominates the Farmed Tuna System for $r > 259/360 \simeq 71.94\%$.

6 The Mixed Tuna System

6.1 The Mixed Tuna System

At the end of our comparison of the Wild and Farmed Tuna Systems it transpired that there may be circumstances in which the demand for tuna will be satisfied by both wild and farmed tuna. For this to happen, the Wild Tuna System must dominate the Farmed Tuna System for large populations; in formal terms, the critical population $\tilde{P}$ must be such that:

$$L/2 \leq \tilde{P} < L \tag{40}$$

If the price of production of wild tuna were lower than that of farmed tuna, boat owners would be in a position to earn rents. These boat rents constitute extra profits, and act as incentives to increase the number of boats. Hence, the fleet size would increase, more will tuna would be caught, and the tuna population would decrease. Obviously, the process comes to an end only if the difference between the two costs of production disappears. An equilibrium is reached when the fishing fleet is expanded to its economic limit, i.e., up to the point where the price of production of wild tuna becomes equal to that of farmed tuna.

In the *Mixed Tuna System* all processes $\{1, 2, 3, 4, 5\}$ are operated and the following systems of equations hold:

Mixed Tuna System

$$\left.\begin{array}{c} y_1 b_{11} = y_1 a_{11} + y_2 a_{21} + y_3 a_{31} + y_4 a_{41} + y_5 a_{51} + d_1 \\ y_2 b_{22} + y_4 b_{42} = y_4 a_{42} + d_2 \\ y_2 b_{22} + y_3 = y_2 \\ y_4 b_{44} + y_5 = y_4 \end{array}\right\} \quad \text{(M.Q)}$$

$$\left.\begin{array}{c} b_{11} = (1+r)(a_{11} + l_1 w) \\ b_{22} p_2 + b_{23} p_3 = (1+r)(a_{21} + p_3 + l_2 w) \\ p_3 = (1+r)(a_{31} + l_3 w) \\ b_{42} p_2 + b_{44} p_4 = (1+r)(a_{41} + a_{42} p_2 + p_4 + l_4 w) \\ p_4 = (1+r)(a_{51} + l_5 w) \end{array}\right\} \quad \text{(M.P)}$$

Moreover, as we have just seen, the population of tuna must be equal to $\tilde{P}$.

6.2 The Activity Levels and Prices

Once we know the equilibrium population level, it is straightforward to calculate the activity levels. Since the catch per boat is $b_{22} = k_2 \tilde{P}$ and the size of the fleet $y_2 = \left[k_1(L - \tilde{P})\right] / k_2$, the supply of wild tuna is equal to the sustainable catch $k_1(L - \tilde{P})\tilde{P}$. It follows that the net amount of tuna which needs to be supplied by tuna farmers is equal to $d_2 - k_1(L - \tilde{P})\tilde{P}$. The activity levels are therefore:

$$y_1 = \frac{d_1}{b_{11}-a_{11}} + \frac{[a_{21}+(1-b_{23})a_{31}]k_1(L-\tilde{P})}{(b_{11}-a_{11})k_2} + \frac{[a_{41}+(1-b_{44})a_{51}]\left[d_2-k_1(L-\tilde{P})\tilde{P}\right]}{(b_{11}-a_{11})(b_{42}-a_{42})} \quad (41)$$

$$y_2 = \frac{k_1(L-\tilde{P})}{k_2} \quad (42)$$

$$y_3 = \frac{(1-b_{23})k_1(L-\tilde{P})}{k_2} \quad (43)$$

$$y_4 = \frac{d_2-k_1(L-\tilde{P})\tilde{P}}{b_{42}-a_{42}} \quad (44)$$

$$y_5 = \frac{(1-b_{44})\left[d_2-k_1(L-\tilde{P})\tilde{P}\right]}{b_{42}-a_{42}} \quad (45)$$

The wage and prices are the same as in the Farmed Tuna System.

6.3 A Numerical Example (II)

Let us return to our previous example, and assume that the rate of profits is $r = 10\%$. The critical value of the catch per boat is $\tilde{b}_{22} = 10.575$. This implies that $\tilde{P} = 10575$, with the corresponding fleet size equal to $y_2 = 80$ boats and the sustainable catch of wild tuna equal to $\tilde{b}_{22}y_2 = 846$. Since we have $10386 < 10575 < 20772$, condition (40) is verified.

Suppose the net demands for corn and tuna are equal to $d_1 = 356$ and $d_2 = 1206$. Given that the supply of wild tuna is equal to 846 units, the net supply of farmed tuna must be $1206 - 846 = 360$ units, which requires an aquacultural park equal to $y_4 = 40$ ponds. In each period 8 vessels are replaced, as well as 4 ponds. Summarizing, we have:

$$y_1 = 62,\ y_2 = 80,\ y_3 = 8,\ y_4 = 40,\ y_5 = 4$$

With regard to prices and the wage, we have:

$$p_2 = 1.6,\ p_3 = 40.1,\ p_4 = 26.7,\ w = 7.8$$

7 Government Policies

7.1 An Unstable Situation

The Mixed Tuna System comes into being when the demand for tuna is too high to be satisfied by wild tuna only, and when the price of production of wild tuna happens to be lower than that of farmed tuna for sufficiently high levels of the population of tuna. Condition (40) is in fact crucial to have a stable equilibrium. Suppose that wild tuna remains cheaper to produce even at relatively low levels of the population of tuna, or more formally, that $\tilde{P} < L/2$. As long as the tuna population exceeds $\tilde{P}$, boat owners are in a position to capture rents, and hence incentives exist to increase the fishing fleet. Owning a boat may be a source of considerable wealth, and it can be expected that rent seeking will occur (Krueger, 1974) The expansion of the fishing fleet inevitably leads to capture levels exceeding the natural increase, and hence the tuna population will decrease. In principle, this process comes to an end when the population reaches the equilibrium level $\tilde{P}$, but since this is an unstable equilibrium, there is a very high risk that we end up in a situation where the population of tuna will steadily decline and eventually perish.

Put differently, if the population level for which wild and farmed tuna are equally costly is too low, mechanisms are at work which will lead to overfishing and extinction of tuna. In comparison to farmed tuna, wild tuna remains too cheap for too long. As long as there is cost advantage for wild tuna, the fishing fleet will expand. But when the cost advantage finally disappears, the remaining population of tuna will be low and fragile.

Governments or supranational organizations such as the *European Union* which are concerned about the survival of wild tuna might think about ways of preventing the system from entering the danger zone of overfishing and extinction. In what follows I present two possible policies, one working on quantities, the other on prices.

7.2 Limiting the Fleet Size

Since it is the expansion of the fleet size which pushes the system towards overfishing, an obvious policy consists of imposing a limit on the number of boats. Suppose that the government introduces a strict boat licensing system by which the maximum number of boats is fixed at the level $\bar{y}_2$. Evidently, in order to be effective this maximum number should be small enough to keep the system in the region of stable equilibria.

The presence of a constant and consistently enforced limitation of the fleet size gives rise to a variant of the Mixed Tuna System, characterized by a positive boat rent z. With the number of boats limited to $\bar{y}_2$, the associated population $\bar{P}$ can be found through the equation $\bar{y}_2 = \left[k_1(L-\bar{P})\right]/k_2$. Given $\bar{P}$, we can determine the catch per boat $\bar{b}_{22} = k_2\bar{P}$ and hence the sustainable catch $\bar{b}_{22}\bar{y}_2 = k_1(L-\bar{P})\bar{P}$. The systems of equations determining the prices and quantities can now be written as:

Mixed Tuna System with Fleet Limitation

$$\left.\begin{array}{c} y_1b_{11} = y_1a_{11} + \bar{y}_2a_{21} + y_3a_{31} + y_4a_{41} + y_5a_{51} + d_1 \\ \bar{y}_2\bar{b}_{22} + y_4b_{42} = y_4a_{42} + d_2 \\ \bar{y}_2b_{22} + y_3 = \bar{y}_2 \\ y_4b_{44} + y_5 = y_4 \end{array}\right\} \qquad \text{(MFL.Q)}$$

$$\left.\begin{array}{c} b_{11} = (1+r)(a_{11} + l_1w) \\ \bar{b}_{22}p_2 + b_{23}p_3 = (1+r)(a_{21} + p_3 + l_2w) + z \\ p_3 = (1+r)(a_{31} + l_3w) \\ b_{42}p_2 + b_{44}p_4 = (1+r)(a_{41} + a_{42}p_2 + p_4 + l_4w) \\ p_4 = (1+r)(a_{51} + l_5w) \end{array}\right\} \qquad \text{(MFL.P)}$$

The rent per boat is determined by the difference between the cost prices of wild and farmed tuna and equal to:

$$z = k_2\bar{P}(p_2^F - p_2^W) \qquad (46)$$

The selling price of tuna, whether it be wild or farmed, is equal to the cost price of farmed tuna.

7.3 Taxing Fishing Boats

The incentive to increase the number of vessels comes from the existence of rents. An alternative policy consists of reducing the rents by taxation. Suppose the government imposes a flat tax per boat equal to T. This obviously raises the cost of production of wild tuna. As in the case of a fleet limitation policy, the tax should be sufficiently high to to keep the system out of the region of unstable equilibria.

Boat owners earn a positive net rent as long as $z > T$. The incentive to increase the number of vessels therefore disappears when the size of the fishing fleet is such that $z = T$. In view of (46), the corresponding population level is determined by means of the following equation:

$$P = \frac{T}{k_2(p_2^F - p_2^W)} \tag{47}$$

Given P, it is easy to determine the size of the fishing fleet $y_2 = [k_1(L-P)]/k_2$, the catch per boat $b_{22} = k_2 P$, and the sustainable catch $b_{22}y_2 = k_1(L-P)P$.

The systems of equations determining the prices and quantities can now be written as:

Mixed Tuna System with Boat Taxation

$$\left.\begin{array}{c} y_1 b_{11} = y_1 a_{11} + y_2 a_{21} + y_3 a_{31} + y_4 a_{41} + y_5 a_{51} + d_1 \\ y_2 b_{22} + y_4 b_{42} = y_4 a_{42} + d_2 \\ y_2 b_{22} + y_3 = y_2 \\ y_4 b_{44} + y_5 = y_4 \end{array}\right\} \qquad \text{(MBT.Q)}$$

$$\left.\begin{array}{c} b_{11} = (1+r)(a_{11} + l_1 w) \\ b_{22} p_2 + b_{23} p_3 = (1+r)(a_{21} + p_3 + l_2 w) + T \\ p_3 = (1+r)(a_{31} + l_3 w) \\ b_{42} p_2 + b_{44} p_4 = (1+r)(a_{41} + a_{42} p_2 + p_4 + l_4 w) \\ p_4 = (1+r)(a_{51} + l_5 w) \end{array}\right\} \qquad \text{(MBT.P)}$$

7.4 Numerical Example (III)

Let us once again return to our numerical example, with one change: the maximum population of tuna is now $L = 30879$. Assume that the rate of profits is $r = 10\%$. The critical value of the catch per boat remains $\tilde{b}_{22} = 10.575$ and that of the population of tuna $\tilde{P} = 10575$. The corresponding fleet size is now $y_2 = 160$ boats and the sustainable catch of wild tuna $\tilde{b}_{22}y_2 = 1692$. Since we have $10575 < 15439.5$, condition (40) is not verified.

Assume that the fishing fleet is restricted to $\bar{y}_2 = 110$ vessels. The corresponding tuna population is $\bar{P} = 16920$, the catch per boat $\bar{b}_{22} = 16.92$ and the sustainable catch $\bar{b}_{22}\bar{y}_2 = 1861.2$. The activity levels corresponding to the net demands for corn and tuna $d_1 = 356$ and $d_2 = 2052$ are equal to:

$$y_1 = \frac{226987}{3600},\ \bar{y}_2 = 110,\ y_3 = 11,\ y_4 = \frac{949}{45},\ y_5 = \frac{949}{450}$$

For $r = 10\%$ the prices and wage are the same as in the case without limitation of the fleet size. Since the cost price of wild tuna is significantly different from the cost price of farmed tuna ($p_2^W = 1$, $p_2^F = 1.6$), the rent per boat is substantial: each boat earns a rent equal to $z = 10.152$.

Assume alternatively that a boat tax equal to $T = 10.152$ is imposed. Then the same solution will be obtained.

8 Discussion

8.1 Predictions of the Model

It may be useful to summarize what the corn-tuna model predicts under plausible assumptions about the evolution of the net demand for corn and tuna. To make things interesting, let us start the analysis at a point of time in which the world's oceans are plenty of wild tuna and the demand is relatively small.

At first, a small fishing fleet suffices to catch all the wild tuna that is required, at a low per unit cost. Wild tuna therefore dominates the market. As the demand for tuna increases, the fishing fleet becomes larger, the average catch per boat declines, and the price of wild tuna gradually increases. The rate at which the wild tuna price increases is the same as the rate at which the catch per boat decreases. Inevitably, one of two things will happen: either farmed tuna will enter into competition with wild tuna when the population of tuna is relatively large (higher than $L/2$), or it will do so when the population of tuna is relatively small (lower than $L/2$). In the former case, a stable equilibrium exists where the market for tuna is divided between wild and farmed tuna. Boat owners will not earn rents, and the risk of overfishing appears to be low. In the latter case, however, the existence of rents will push the system into the danger zone of unstable equilibria, with a considerable risk of overfishing and depletion. Without government intervention, the most probable outcome is that wild tuna will eventually become extinct, and the whole supply of tuna will consist of farmed tuna.

In the Good Scenario (i.e., the first case) the wild tuna price is initially lower than the farmed tuna price, but not too much. As the demand for tuna increases, the wild tuna price increases until it reaches the level of the farmed tuna price. From that moment on, the price of tuna remains constant and equal to the farmed tuna price, even though wild tuna continues to be supplied to the market. In the Bad Scenario (i.e., the second case) wild tuna is initially much cheaper than farmed tuna. But the problem is that it remains too cheap for too long: farmed tuna enters into competition with wild tuna only when the stocks of tuna have become very low. It is difficult to say what will be the evolution of the tuna price in this highly unstable environment.

The corn-tuna model therefore predicts that in a situation of growing demand the price of the renewable resource will steadily increase, until it reaches a steady equilibrium (the Good Scenario) or becomes unstable (the Bad Scenario). This may be thought of as a kind of Hotelling rule for renewable resources, but it must be kept in mind that the logic behind this rule is fundamentally different from the logic behind the Hotelling rule. The increase of the price of an exhaustible resource, such as guano in the corn-guano model, is explained by the profit-maximizing behaviour of the resource owners and occurs even when demand is not growing. In the corn-tuna model, by contrast, the price increase of the renewable resource is driven by a combination of biological and economic factors and occurs only when demand is growing.

The model also predicts that in the long run the renewable resource will either have a declining market share (the Good Scenario) or be replaced entirely by an industrially produced ersatz good (the Bad Scenario), unless a policy is implemented to prevent the collapse of the renewable resource industry. Such a policy can take the form of quantitative restrictions or of price incentives.

8.2 Limitations and Shortcomings of the Model

It remains to be seen whether the predictions of the corn-tuna model are in accordance with the stylized facts of the world's fisheries. It may very well be that this simple model misses out on some essential features of the real world. The model surely cannot be applied to all renewable resources. For instance, it is not always the case that the natural 'source' which 'breeds' the renewable resource (in the corn-tuna model the source is the sea) has no owner. Environmental economists know very well that different property regimes may lead to different outcomes (see, e.g., Chapter 10 of Pearce and Warford, 1993).

The model can be improved by establishing explicit links between crucial variables. Feedback effects, positive or negative, are absent. The model also ignores technical change, both in the traditional fishing industry and in aquaculture. These might be powerful enough to counteract the tendency of increasing wild tuna prices, or to bring about a more rapid change to aquaculture.

9 Conclusion

In this paper I have examined how renewable resources can be integrated into classical theory. Starting from the insights which can be obtained from the corn model and the corn-guano model, I have developed a corn-tuna model in which a valuable renewable resource (tuna) can be produced either by nature or by man. In the first case it is harvested by means of boats, in the second it is farmed using aquaculture ponds. Increasing levels of demand will entail the intensified use of the natural process, but eventually the man-made process will become more and more dominant. It could very well be that the natural process is operated for such a long time and at such a high level that the renewable resource becomes over-exploited, which will lead to the exhaustion of the resource. Since classical theory has a long-term perspective, I believe it should try to come to grips with this issue.

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