A Classical Theory of Value after Sraffa

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(A Draft)

October 15, 2014

1 Introduction

Sraffa's statement on the Classical theory of value appears in Appendix D 2 of Sraffa[1960]. Sraffa states, "The conception of a standard measure of value as a medium between two extremes (§17 ff.) also belongs to Ricardo and it is surprising that the Standard commodity which has been evolved from it here should be found to be equivalent to something very close to the standard suggested by Adam Smith, namely 'labour commanded' (§ 43), to which Ricardo himself was so decidedly opposed". This remark suggested that the labour commamded is not different from the labour embodied. Fujitsuka [1952] have the same view and it advocated that in *Wealth of Nation* the commanded labour theory of value and the embodied labour theory of value are the same theory but have different expressions. These views of the Classical theory of value may be different from a common view that the commanded labour theory of value is not equal to the embodied labour theory of value and sometimes the commanded labour theroy of value is a wrong theory. However, we consider that the commanded labour theory of value and the embodied labour theory of value are the double theory or the pair theory of value. Unfortunately, both Sraffa [1960] and Fujitsuka [1952] consider their ideas in a static framework. In this paper, we will start the von-Neumann-Smith growth model of Negishi [1989], because it is a simple model with only one product and labour. We will extend Negishi [1989] to the model with produced means of production and explain the characteristics of our understanding of the Classical theory of value. Then we will explain the relationship between the Sraffian multi-sector model and the aggregate model based on the Sraffa model.

2 A Model with One Commodity and Labour

2.1 The Negishi Model of a Pure Labour Economy

In this section, we will explain the analysis of Negishi (1989). In the case that production is not instantaneous, Negishi [1989] assumes that the period of production is a unit period for both sectors so that a units of labour must be expended one period to produce one unit of product, and b units of the product must be consumed one period to produce one unit of labour. Negishi [1989] refers to the stock of the product of 'time t' and the labour population of 'time t'. We will use the notation of $N(t)$ for the amount of labour power at the end of production period. And we will use the notation of $X(t)$ for the stock of the product at the end of production period t. The production coefficients a and b can be defined by

$$\begin{aligned} X(t) &= bN(t+1) && (1) \\ N(t) &= aX(t+1) && (2) \end{aligned}$$

The stationary state can be characterized by

$$\begin{aligned} X(t+1) &= X(t) && (3) \\ N(t+1) &= N(t) && (4) \end{aligned}$$

Negishi [1989] explained that a stationary solution exists in (1) and (2) if and only if ab=1(see Negishi [1989] p.85). From this, the price equations will be represented by

$$\begin{aligned} P(t+1) &= w(t)a && (5) \\ w(t+1) &= bP(t) && (6) \end{aligned}$$

Negishi [1989] characterized his Smithian growth model as the von-Neumann growth model of the production of commodities by means of commodities where the labour power is considered as a commodity. The von-Neumann model is a model which all the commodity stock expand at the constant common growth rate, relative prices remain constant, and the rate of interest equals the rate of growth. In a steady growth pass of Negishi's von-Neumann-Smith growth model, it is assumed that the natural price of product and the natural rate of wage and the natural rate of profits are prevailing. Negishi [1989] states that the natural rate of wages exceeds the subsistence level by the natural rate of profit. A steady growth pass will be characterized by

$$\begin{aligned} X(t+1) &= (1+g)X(t) && (7) \\ N(t+1) &= (1+g)N(t) && (8) \end{aligned}$$

By substituting (7)(8) into (1)(2), the equation $ab(1+g)^2 = 1$ will be obtained.

Negishi [1989] assumes that profits are all invested, all the stock of product is advanced as wages fund, i.e.,

$$X(t)P(t) = w(t)N(t) \quad (9)$$

The value of labour power at the end of production period t can be equal to the wage rate at the end of *Period* t. The amount $X(t)P(t)$ is the funds to be destined to the payments of wages which workers will consume in *Period* $t+1$. The product $X(t)$ is produced in *Period* t by the technology of *Period* t. In labour market at the end of *Period* t, since the market is in equilibrium, the quantity of labour power which is commanded by the product available at the end of *Period* t will become equal to the quantity of labour power which will be employed at the end of *Period* t (or the initial point of *Period* $t+1$).

In a growing economy, the natural price of the product at the end of *Period* $t+1$ is given by

$$P(t+1) = (1+r)w(t)a \tag{10}$$

where $P(t+1)$ is the natural price of the product at the end of *Period* t +1, r is the natural rate of profit, and $w(t+1)$ is the natural rate of wage at the end of *Period* t. The natural rate of wage at the end of *Period* t+1 is given by

$$w(t+1) = (1+r')bP(t) \tag{11}$$

In (11), if b units of the product have to be consumed during the production period $t+1$ to produce one unit of labour power at the end of *Period* $t+1$, r' is positive if r is positive. In other words, the natural rate of wage is higher than the subsistence level in a growing economy. Under the condition of $w = 1$, the equations (10)(11) are reduced to the following

$$\begin{aligned} P &= (1+r)a && (12) \\ w &= 1 = (1+r')bP = (1+r)(1+r')ab && (13) \end{aligned}$$

From (10)(11), the equation $ab(1+r)(1+r') = 1$ can be obtained. Since $ab(1+g)^2 = 1$, r and r' are positive, if g is positive. In a steadily growing economy, the rate of profit is positive and the natural rate of wage is higher than the subsistence level.

From (8)(9), we can derive $w = Pb(1+g)$. From (13) we have $w = (1+r')Pb$. Therefore we can obtain $r' = g$. If we denote the equilibrium rate of profit by r^*, from $(1+g)^2ab = 1$ and $(1+r)(1+r')ab = 1$, we can obtain

$$r^* = r = r' = g \tag{14}$$

From (12)(14), we have

$$P = (1+g)a = a + ga > a \tag{15}$$

The labour commanded by the product is larger than the labour embodied in the product. This is an important result of Negishi [1989].

2.2 A Model with Produced Means of Production

2.2.1 The Quantity System

We will proceed to the analysis of our model with produced means of production. We will consider the case of no scarcity of land. If we denote the total output which is obtained at the end of Period t by $X(t)$, the net output which is obtained at the end of *Period* t by $Y(t)$, the replacement of produced means of production at the end of *Period* t by $M(t)$,then we have

$$X(t) = Y(t) + M(t) \tag{16}$$

The net output at the end of will be divided into two components: the total wage $W(t)$ and the total profit $\Pi(t)$. This is represented as

$$Y(t) = W(t) + \Pi(t) \tag{17}$$

It is assumed that the wage is consumed by the workers and the profit is invested by the capitalist. If we denote the wage share at the end of *Period* t by $\omega(t)$,and the profit share at the end of *Period* t by $\pi(t)$, we will have the following,

$$1 = \omega(t) + \pi(t) \tag{18}$$

where $\omega(t) = W(t)/Y(t), \pi(t) = \Pi(t)/Y(t)$. Like (1)(2), the production of product and the production of labour itself will be defined by

$$\begin{aligned} Y(t) &= \beta N(t+1) && (19) \\ N(t) &= \alpha Y(t+1) && (20) \end{aligned}$$

In this paper, the amount of labour power is measured at the end of a production period. In addition to this measurement of labour amount, we will introduce the different measurement of labour, the amount of labour which is expended in the production of product *during* a production period. In place of the notation $N(t)$ for the amount of labour power, let us use the notation $L(t)$ for the labour which is expended in *Period* t or the amount of labour during *Period* t. Then it will become

$$L(t+1) = N(t) \tag{21}$$

Both $L(t+1)$ and $N(t)$ are measured in terms of the same unit of labour. For simplicity, the labour power is composed only of productive labour. The difference between $N(t)$ and $L(t)$ is explained in *Figure* 1.

In Negishi model, the wage fund $w(t)N(t)$ is expended one period before the output $X(t+1)$. On the other hand, we can consider that the output $Y(t+1)$ at the end of *Period* $t+1$ is produced by the labour amount $L(t+1)$ of the same production period $(t+1)$ with the technology of *Period* $t+1$.

By the introduction of two types of labour amount, i.e. $N(t)$ and $L(t)$, the definition of α will become the following,

$$\alpha = L(t+1)/Y(t+1) = N(t)/Y(t+1) \tag{22}$$

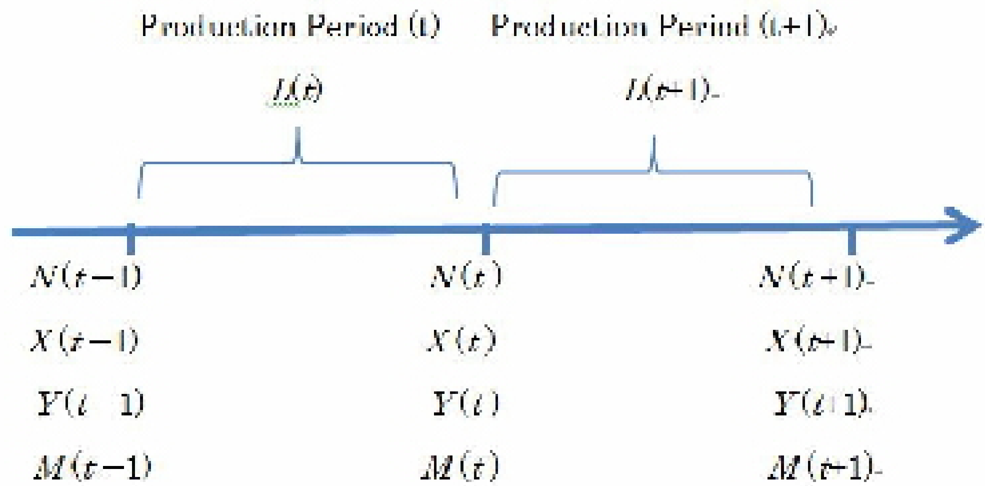
Since the inverse of α means the labour productivity, if we denote the labour productivity by Λ, then Λ will be given by

$$\Lambda = Y(t+1)/L(t+1) = Y(t+1)/N(t) \tag{23}$$

The growth rate of both sectors are given by g, and then we have

$$\begin{aligned} Y(t+1) &= (1+g)Y(t) && (24) \\ N(t+1) &= (1+g)N(t) && (25) \end{aligned}$$

Figure 1.



2.2.2 An Extension of Negishi's Growth Model

In a model with produced means of production, the wage rate $w(t)$ will be subject to the changes in distribution. Therefore we will introduce a notion of the value of labour and we denote it by v_L. In this section, v_L is the variable to evaluate labour value in the process of growth. On the other hand, since the product is distributed to the workers and the capitalists at the end of a production period, the wage rate $w(t)$ will be determined at the end of a production period, when the product and labour are exchanged in markets. We are considering two different sets of exchangeable values, i.e.$[P(t), v_L(t)]$ and $[P(t), w(t)]$. Corresponding to two different notions of exchangeable value of labour, there will be two different notions of labour commanded of product;

$$P_v(t) = P(t)/v_L(t) \tag{26}$$

and

$$P_w(t) = P(t)/w(t) \tag{27}$$

By using these notations, two sets of exchangeable values will be represented as $[P_v(t), 1]$ and $[P_w(t), 1]$.

At the end of production period, the value of product will become equal to the value of labour embodied in the production of product. Then we have

$$Y(t)P(t) = v_L L(t) \tag{28}$$

Like (10)(11), the price equations of product and labour will be given by

$$\begin{aligned} P(t+1) &= (1+r)v_L(t)\alpha & (29) \\ v_L(t+1) &= (1+r')\beta P(t) & (30) \end{aligned}$$

Let us consider the following condition of standard

$$v_L(t+1) \ = v_L(t) = 1 \tag{31}$$

In the case of a steady growth pass, as Negishi[1989] derived, we can obtain

$$r = r' = g \tag{32}$$

Because of $\alpha = N(t)/X(t+1)$, we can obtain

$$P_v(t+1) = \alpha + g\alpha > \alpha \tag{33}$$

This means that the labour commanded of the product is larger than the labour embodied in the product. But it is important to understand that the second term ($g\alpha$) of the equation (33) is the component part of the price which enable the economy to keep the ratio α constant in the next production period. Then the amount of labour embodied in the product will be kept constant in the next production period when the net output is growing at the rate of g. This will be represented by

$$\frac{N(t)}{Y(t+1)} = \frac{N(t+1)}{Y(t+2)} = \alpha \tag{34}$$

The equation (33) can be rewritten as

$$Y(t+1)P_v(t+1) = N(t) + gN(t) = N(t+1) \tag{35}$$

2.2.3 An Alternative View of the Steady Growth Pass

Now let us consider the following condition

$$v_L(t+1) \ = 1 \tag{36}$$

The value of labour at the end of *Period* t will be given by

$$v_L(t) = \frac{v_L(t+1)}{(1+r)} = \frac{1}{(1+r)} \tag{37}$$

We can rewrite the equation (37) as

$$r = \frac{1 - v_L(t)}{v_L(t)} \tag{38}$$

From (37), the price equation (29) can be rewritten as

$$P(t+1) = v_L(t+1)\alpha \tag{39}$$

Therefore we have

$$P_v(t+1) = \alpha \tag{40}$$

This means that the price of product of *Period* t+1 is equal to the labour embodied in the product. Therefore, in this case, the labour commanded is equal to the labour embodied.

From (40), we have

$$\frac{P_v(t+1)}{\alpha} = \frac{Y(t+1)P_v(t+1)}{N(t)} = \frac{Y(t+1)P_v(t+1)}{L(t+1)} = 1 \tag{41}$$

Because of $\alpha = 1/\Lambda$, we have

$$P_v(t+1) = 1/\Lambda \tag{42}$$

Or we have

$$Y(t+1)P_v(t+1) = L(t+1) \tag{43}$$

The equation (43) means that the value of net product is equal to the total labour expended during a production period. From (21)(35)(43), the difference is equal to

$$N(t+1) - L(t+1) = L(t+2) - L(t+1) = gL(t) \tag{44}$$

2.2.4 Resolution and Adding-up of Commodity Price

Let us consider an alternative standard condition that the wage rate measured in terms of net product (or the wage share) is set equal to unity as

$$\omega_Y(t+1) = 1 \tag{45}$$

The distribution ratio θ_Y is defined by

$$\theta_Y(t+1) = \pi_Y(t+1)/\omega_Y(t+1) \tag{46}$$

Or from (18)(46), we have

$$\theta_Y(t+1) = \frac{1-\omega_Y(t+1)}{\omega_Y(t+1)} \tag{47}$$

Because of $\omega_Y(t+1)+\pi_Y(t+1) = 1$, the price will be divided into two compornet parts as

$$P_v(t+1) = \omega_Y(t+1)P_v(t+1) + \pi_Y(t+1)P_v(t+1) \tag{48}$$

The price in terms of the wage can be defined by

$$P_\omega(t+1) = P_v(t+1)/\omega_Y(t+1) \tag{49}$$

Then the price equation is represented as

$$\begin{aligned} P_\omega(t+1) &= P_v(t+1) + \theta_Y(t+1)P_v(t+1) \\ &= \{1+\theta_Y(t+1)\}P_v(t+1) \end{aligned} \tag{50}$$

Under the condition of $v_L(t+1) \ = (1+r)\ v_L(t) \ = 1$, from (40)(50), we have

$$P_\omega(t+1) = \{1+\theta_Y(t+1)\}\alpha \tag{51}$$

The labour commanded is larger than the labour embodied.

2.2.5 Distribution in a Growing Economy

The equation (51) can be rewritten as

$$Y(t+1)P_\omega(t+1) = \{1+\theta_Y(t+1)\}L(t+1) \tag{52}$$

The right member of this equation means the amount of labour which the net product can purchase. Let us denote this labour commanded by $N^D(t{+}1)$. Therefore we have

$$N^D(t+1) \equiv \{1+\theta_Y(t+1)\}L(t+1) \tag{53}$$

On the other hand, the labour supply is given by the equation (8). Therefore, in equilibrium of labour market at the end of *Period* t+1, we have

$$N^D(t+1) \equiv \{1+\theta_Y(t+1)\}L(t+1) = N(t+1) \tag{54}$$

From (21)(25)(54) we have

$$\frac{N^D(t+1)}{N(t)} \equiv 1+\theta_Y(t+1) = 1+g \equiv \frac{N(t+1)}{N(t)} \tag{55}$$

From this we have

$$\theta_Y(t+1) = g \tag{56}$$

The Price of product will become

$$P_\omega(t+1) = (1+g)\alpha \tag{57}$$

From (47)(56) we have

$$g = \frac{1-\omega_Y}{\omega_Y} \tag{58}$$

From (18)(56) we can obtain

$$\omega_Y(t+1) = \frac{1}{1+g} \tag{59}$$

$$\pi_Y(t+1) = \frac{g}{1+g} \tag{60}$$

These are the characteristics of distribution in a steady growth pass.

However there remains a problem whether the produced means of production grows at the rate of g. If we denote the ratio between the aggregate value of the produced means of production and net output by $\gamma = M(t)/Y(t)$. It will not be assured that the ratio γ is kept constant. If the produced means of production does not grow at the rate of g, a steady growth will not be kept through time and an economy will fall in a Harrodian disequilibrium state.

2.2.6 Summary

We have shown a Smithian model with a produced means of production by using two price systems, $[P, v_L]$ and $[P, w]$,or $[P_v, 1]$ and $[P_w, 1]$,where v_L is the value of labour, w is the wage rate. We also introduced the distinction between the labour population at the end of a production period and the labour expended in a production period. In the case of a stationary growth pass, we have obtained the following results:

(i) Under the condition of $v(t+1) = v(t) = 1$, the price equation becomes $P_v(t+1) = a + ga$. The second term $(g\alpha)$ of the equation (33) is the component part of the price which enable the economy to keep the ratio α constant in the next production period when the net output is growing at the rate of g.

(ii) Under the condition of $v(t+1) = (1+r)v(t) = 1$, the commodity price will be represented as $P_v(t+1) = a$. This means that the labour commanded is equal to the labour embodied.

(iii) Under the condition of $\omega_Y(t+1) = 1$,the price is represented as $P_\omega(t+1) = \{1 + \theta_Y(t+1)\}\alpha = a + ga$ when the labour market at the end of a production period is in equilibrium.

(vi) The characteristics of distribution in a steady growth pass will be given by $g = \frac{1-\omega_Y}{\omega_Y}, \omega_Y(t+1) = \frac{1}{1+g}, \pi_Y(t+1) = \frac{g}{1+g}$.

(v) If the produced means of production does not grow at the rate of g, a steady growth will not be kept through time and an economy will fall in a Harrodian disequilibrium state.

In the next section, we will examine a Multi-sector model of Sraffa type and construct a Sraffian aggregate model. In our Sraffian aggregate model, the above properties of (i)-(iv) will hold, and in addition the problem of (v) will be cleared and the produced means of production will grow at the rate of g.

3 A Sraffian Multi-Sector Model

3.1 The Sraffa System

3.1.1 The Quantity System

Now we will proceed to the Sraffa System. We will discuss only the case of single product industries. There is no joint production. Land and fixed capital are excluded from the analysis. Each industry produces a single commodity by using a certain quantity of labour and certain quantities of commodities as a means of production. The number of industries and thus the number of products is equal to n.

In the case of production of a surplus, the total output is divided into two components: the part which is required for the replacement, and the part of the surplus produced, which is over and above the replacement for reproduction. If we denote the total output vector by $\mathbf{x}$, the net product vector by $\mathbf{y}$, the input coefficient matrix by $\mathbf{A}$, then we have

$$\mathbf{x} = \mathbf{y} + \mathbf{x}\mathbf{A} \tag{61}$$

We assume $\mathbf{x}$ >0, $\mathbf{y}$ >0, and that $\mathbf{A}$ is a semi-positive and indecomposable matrix. From (61), we also obtain

$$\mathbf{y} = \mathbf{x}(\mathbf{I} - \mathbf{A}) \tag{62}$$

where $\mathbf{I}$ is the identity matrix. Let us define $\mathbf{m}_y$ as

$$\mathbf{m}_y = \mathbf{x}\mathbf{A} > 0 \tag{63}$$

Then the equation (61) can be rewritten as

$$\mathbf{x} = \mathbf{y} + \mathbf{m}_y \tag{64}$$

Let us assume that there is no heterogeneity in labour, and the amount of total labour is equal to L_A. However, Sraffa assumes that the total amount of labour is set equal to unity. Let us denote the total labour normalized to one by L_S. Let us denote the labour coefficient vector corresponding to the normalized total labour by $\mathbf{l}_S$, then the vertically integrated labour coefficient vector is defined by

$$\mathbf{v}_S = [\mathbf{I} - \mathbf{A}]^{-1}\mathbf{l}_S \tag{65}$$

And the normalized total labour is represented by

$$L_S = \mathbf{x}\mathbf{l}_S = \mathbf{x}[\mathbf{I} - \mathbf{A}][\mathbf{I} - \mathbf{A}]^{-1}\mathbf{l}_S = \mathbf{y}\mathbf{v}_S = 1 \tag{66}$$

The standard net product is defined as the standard commodity with an additional assumption introduced in §26 of Sraffa[1960]. The standard net product is the standard commodity when the total labour of the standard system is equal to the total labour of the actual system. The vector $\mathbf{q}$ is defined as the vector that is given by

$$\mathbf{q} = (1 + \mathit{\Pi})\mathbf{q}\mathbf{A} \tag{67}$$

and, in addition, which satisfies the following assumption

$$\mathbf{q}\mathbf{l}_S = \mathbf{x}\mathbf{l}_S \tag{68}$$

Like (62), from (67), we can obtain the following definition of Sraffa's standard net product

$$\mathbf{s} = \mathbf{q}(\mathbf{I} - \mathbf{A}) = \Pi\mathbf{q}\mathbf{A} \tag{69}$$

The vector of the produced means of production of the standard system can be defined by

$$\mathbf{m}_S = \mathbf{q}\mathbf{A} \tag{70}$$

In the multi-sector production model of the Sraffa type, the produced means of production are reduced to the dated quantities of labour (the indirect labour input). The quantity relation of the standard system will be represented as

$$\mathbf{q} = \mathbf{s} + \mathbf{m}_S \tag{71}$$

The normalized total labour can be represented by

$$L_S = \mathbf{q}\mathbf{l}_S = \mathbf{q}[\mathbf{I} - \mathbf{A}][\mathbf{I} - \mathbf{A}]^{-1}\mathbf{l}_S = \mathbf{s}\mathbf{v}_S = 1 \tag{72}$$

3.1.2 The Reduction Equation of §§ 45-47 of Sraffa[1960]

We call the exchange-ratios, which enable the system to be economically viable, the prices of commodities or simply the prices. Let us denote the column vector of the commodity prices, which corresponds to the labour coefficient vector $\mathbf{l}_S$ by $\mathbf{p}_S$. And let us denote the rate of profits by r, which is assumed to be uniform all over the economic system. Moreover let us denote the maximum rate of profits by R. Then the rate of profits will take real numbers ranging from 0 to R ($0 \leq r \leq R$). Similarly, a uniform rate of wage (post factum) is assumed to be prevailing in the economy. It is denoted by w_S. Then, the price system can be written as

$$\mathbf{p}_S = (1 + r)\mathbf{A}\mathbf{p}_S + w_S\mathbf{l}_S \tag{73}$$

When the rate of profits is equal to zero, i.e. $r = 0$, the wage becomes equal to unity, i.e. $w_S = 1$, and the price vector can be represented as

$$\mathbf{p}_S = \mathbf{v}_S = [\mathbf{I} - \mathbf{A}]^{-1}\mathbf{l}_S \tag{74}$$

When the wage is equal to zero, the price equation (73) will reduce to

$$\mathbf{p}_S = (1 + R)\mathbf{A}\mathbf{p}_S \tag{75}$$

This implies that the price vector $\mathbf{p}_S$ is the right-hand eigenvector of the matrix $\mathbf{A}$ and $1/(1+R)$ is its eigenvalue. On the other hand, in the equation (67), the vector $\mathbf{q}$ is the left-hand eigenvector of the matrix $\mathbf{A}$ and $1/(1+\Pi)$ is its eigenvalue. From the Perron-Frobenius theorem, we have

$$\Pi = R \tag{76}$$

Substituting (76) into (69), we obtain

$$\mathbf{s} = \mathbf{q}(\mathbf{I} - \mathbf{A}) = R\mathbf{qA} \tag{77}$$

This is a useful expression of the standard net product.

From the price system (73), for all r $(0 \leq r < R)$, we can derive [1)]

$$\mathbf{sp}_S = \mathbf{xl}_S \quad \Longleftrightarrow \quad r = R(1 - w_S) \tag{78}$$

This means that the following equation can be considered as a standard condition. Because of $\mathbf{xl}_S = 1$, we have

$$\mathbf{sp}_S/\mathbf{xl}_S = \mathbf{sp}_S = 1 \tag{79}$$

Now we will introduce the notion of the value of labour v_L into the Sraffa system. Let us denote the value of labour embodied in the standard national income by v_L, then it can be defined as

$$\mathbf{sp}_S/\mathbf{xl}_S = v_L \tag{80}$$

where v_L is a scalar.

When a set of data ($\mathbf{x}$, $\mathbf{A}$, $\mathbf{l}_S$, $\mathbf{s}$, $R = \Pi$) is given, we have the following Evaluation System

[**Evaluation System 1**]

$$\mathbf{sp}_S = v_L\mathbf{xl}_S \tag{81}$$

$$\mathbf{p}_S = (1 + r)\mathbf{Ap}_S + w_S\mathbf{l}_S \tag{82}$$

In this system, there are $(n+1)$ independent equations and $(n+3)$ unknowns(i.e. v_L, $\mathbf{p}_S$, w_S, r). Then, if the rate of profits is given exogenously, the price system will become determinate.

In Evaluation System 1, for all r $(0 \leq r < R)$, we have

$$v_L = 1 \quad \Longleftrightarrow \quad r = R(1 - w_S) \tag{83}$$

The reduced form of $\mathbf{p}_v$ of the Sraffa price system (82) can be represented as

$$\mathbf{p}_v = \mathbf{p}_S/v_L = (1 - r/R)[\mathbf{I} - (1 - r)\mathbf{A}]^{-1}\mathbf{l}_S \tag{84}$$

This is the Reduction equation of Sraffa [1960], which is measured in terms of quantities of labour. The prices of this equation are subject to the changes in the rate of profits and the changes in the technological condition of production. But, since the value of labour embodied in the standard national income is invariant, there is no effect of the changes in the value of the chosen standard on the prices of commodities. Sraffa's theory can be considered not as a simple theory of price determination but as a theory of value.

The wage rate under the condition of $v_L = 1$ of (83) is represented as

$$\omega_v = w_S/v_L = 1 - r/R \tag{85}$$

3.1.3 The Standard in §43 of Sraffa[1960]

The quantity of labour that can be purchased by a commodity will be obtained by dividing the price of a commodity by the wage. It is the relative price between the price of a commodity and the wage. If we denote the vector of the relative prices divided by the wage by $\mathbf{p}_w$, it can be represented by

$$\mathbf{p}_w = \mathbf{p}_S / w_S = [\mathbf{I} - (1-r)\mathbf{A}]^{-1}\mathbf{l}_S \tag{86}$$

In (86), the normalization condition of the prices is represented as

$$w_S = 1 \tag{87}$$

The quantity of labour purchased by the standard net product will be obtained by dividing the standard national income by w_S,

$$\mathbf{s}\mathbf{p}_w = \mathbf{s}\mathbf{p}_S / w_S \tag{88}$$

From (83)(85), the equation (88) can be rewritten as

$$\mathbf{s}\mathbf{p}_w = \frac{1}{w_S} = \frac{R}{R-r} \tag{89}$$

This is the same as the equation shown in §43 of Sraffa[1960]. This is the quantity of labour commanded by the standard national income. In this equation, the value of the standard national income is measured in terms of the quantity of labour commanded by the standard net product.

3.1.4 Distribution in the Sraffa System

The condition of $r = R(1-\omega_v)$ can be applied to the analysis of the actual economy, because the difference between the actual income $\mathbf{y}\mathbf{p}_v$ and the standard income $\mathbf{s}\mathbf{p}_v$ corresponds only to the profit which comes from the difference between actual capital and the standard capital. The actual income is represented as

$$\mathbf{y}\mathbf{p}_v = r\mathbf{m}_y\mathbf{p}_v + \omega_v\mathbf{x}\mathbf{l}_S \tag{90}$$

The standard income is represented as

$$\mathbf{s}\mathbf{p}_v = r\mathbf{m}_S\mathbf{p}_v + \omega_v\mathbf{x}\mathbf{l}_S \tag{91}$$

Subtracting (91) from 90), we have

$$\mathbf{y}\mathbf{p}_v - \mathbf{s}\mathbf{p}_v = r(\mathbf{m}_y\mathbf{p}_v - \mathbf{m}_S\mathbf{p}_v) \tag{92}$$

This equation means that the difference between the actual national income and the standard national income is equal to the profit which can be obtained by the capitalist from the difference of two notions of capital.From this analysis, we can obtain the result that the linear wage curve can be applied to the analysis of an actual economy.

From (70)(77), we can obtain

$$\mathbf{s}\mathbf{p}_v = R\mathbf{m}_S\mathbf{p}_v \tag{93}$$

Then, we have

$$\mathbf{m}_S\mathbf{p}_v/\mathbf{s}\mathbf{p}_v = 1/R \tag{94}$$

It is important to understand that the value of standard capital is independent of the variation of the prices and distribution. It is given simply as the inverse of R.

The profit share to the standard national income π_v can be represented as

$$\pi_v = r\mathbf{m}_S\mathbf{p}_v/\mathbf{s}\mathbf{p}_v \tag{95}$$

From (94)(95), we have

$$\pi_v = r/R \tag{96}$$

The distribution ratio θ_v is defined as [2)]

$$\theta_v = \pi_v/\omega_v \tag{97}$$

From (85)(96)(97), for all r ($0 \leq r < R$), we can obtain

$$\theta_v = \frac{r}{R-r} = \frac{1-\omega_v}{\omega_v} \tag{98}$$

$$\frac{1}{\omega_v} = 1 + \theta_v \tag{99}$$

When the rate of profits r is positive, from (84)(86), the relationship between $\mathbf{p}_v$ and $\mathbf{p}_w$ is represented as

$$\mathbf{p}_v = (1 - r/R)\mathbf{p}_w \tag{100}$$

It is possible to consider that both the relative prices of (84) and (86) are expressed in terms of labour for all r ($0 \leq r < R$). From (97), we can rewrite the equation (100) as

$$\mathbf{p}_w = (1+\theta)\mathbf{p}_v \tag{101}$$

From this equation, we have

$$\mathbf{s}\mathbf{p}_w = (1+\theta)\mathbf{s}\mathbf{p}_v = (1+\theta)L_S \tag{102}$$

3.2 A Sraffian Aggregate Model

3.2.1 Index Numbers

The notations given data of *Period* t and *Period* t+1 are shown as follows:

[**Notations of Given Data**]
$\mathbf{A}^t$: the (transposed) Leontief input coefficient matrix of each period. The input coefficient matrixes are assumed to be a semi-positive and indecomposable matrix, $\mathbf{A}^t \geq 0$.
L^t_A : the actual total labour of each period, $L^t_A > 0$.
L^t_S: the standard total labour or in short standard labour of each period, which is normalized by the actual total labour L^t_A. They are calculated as $L^{t+1}_S = (1/L^t_A) L^{t+1}_A$.
$\mathbf{l}^t_A$: the Leontief labour input coefficient vector corresponding to the actual total labour L^t_A by $\mathbf{l}^t_A > 0$.
$\mathbf{l}^t_S$: the standard labour input coefficient vector corresponding to the standard total labour L^t_S respectively, $\mathbf{l}^t_S > 0$. The standard labour input coefficient vectors are calculated as $\mathbf{l}^{t+1}_S = (1/L^t_A)\ \mathbf{l}^{t+1}_A$.
$\mathbf{x}^t$: the actual output vector of each period, $\mathbf{x}^t > 0$.
$\mathbf{y}^t$: the actual net product of each period, $\mathbf{y}^t = \mathbf{x}^t\ [\mathbf{I}\text{-}\mathbf{A}^t] > 0$.
$\mathbf{q}^t$: the output vector of the standard system of each period, $\mathbf{q}^t > 0$. The vectors $\mathbf{q}^t$ is an eigenvector of the matrix $\mathbf{A}^t$ and they are assumed to satisfy $\mathbf{q}^t\mathbf{l}^t_S = \mathbf{x}^t\mathbf{l}^t_S = L^t_S$.
$\mathbf{s}^t$: the standard net product of each period, $\mathbf{s}^t = \mathbf{q}^t\ [\mathbf{I}\text{-}\mathbf{A}^t] > 0$.

The produced means of production can be reduced to the indirect labour inputs by calculating the vertically integrated labour coefficient vector. If we denote the Leontief coefficient matrix by $\mathbf{A}$, and if we denote the vertically integrated labour vectors corresponding to the standard labour coefficient vectors $\mathbf{l}^t_S$ by $\mathbf{v}^t_S$, then they can be represented as

$$\mathbf{v}^t_S = [\mathbf{I} - \mathbf{A}^t]^{-1}\mathbf{l}^t_S \tag{103}$$

From the above notations and the vertically integrated labour coefficient vectors of (103), the standard total labour of each period is represented respectively as

$$L^t_S = \mathbf{x}^t\mathbf{l}^t_S = \mathbf{y}^t\mathbf{v}^t_S = \mathbf{s}^t\mathbf{v}^t_S = \mathbf{s}^t\mathbf{p}^t_v = 1 \tag{104}$$

$$L^{t+1}_S = \frac{L^{t+1}_A}{L^t_A} = \mathbf{x}^{t+1}\mathbf{l}^{t+1}_S = \mathbf{y}^{t+1}\mathbf{v}^{t+1}_S = \mathbf{s}^{t+1}\mathbf{v}^{t+1}_S = \mathbf{s}^{t+1}\mathbf{p}^{t+1}_v \tag{105}$$

From this, we can consider that the vertically integrated labour coefficient vectors can be considered as the indicator of division of labour. The price vectors to construct index numbers are the vertically integrated labour coefficient vector $\mathbf{v}^t_S$. The quantity vectors are the standard product vectors $\mathbf{s}^t$. A set of data needed will be [$\mathbf{A}^{t+1}$, $\mathbf{l}^{t+1}_A$,L^{t+1}_A; $\mathbf{A}^t$, $\mathbf{l}^t_A$, L^t_A]. Let us denote the price index of Fisher type by P^{t+1}_v, the output index of Fisher type by Y^{t+1}_E, the input cost index by L^{t+1}_S. Then the indexes will be represented as

$$P_v^{t+1} = \sqrt{\frac{\mathbf{s}^t \mathbf{v}_S^{t+1}}{\mathbf{s}^{t+1} \mathbf{v}_S^t} \cdot L_S^{t+1}} \qquad Y_E^{t+1} = \sqrt{\frac{\mathbf{s}^{t+1} \mathbf{v}_S^t}{\mathbf{s}^t \mathbf{v}_S^{t+1}} \cdot L_S^{t+1}} \qquad L_S^{t+1} = \frac{\mathbf{s}^{t+1} \mathbf{v}_S^{t+1}}{\mathbf{s}^t \mathbf{v}_S^t} \tag{106}$$

From these, we can define the productivity index as

$$\Lambda_{sv} = 1/P_v^{t+1} = Y_E^{t+1}/L_S^{t+1} = \sqrt{\frac{\mathbf{s}^{t+1} \mathbf{v}_S^t}{\mathbf{s}^t \mathbf{v}_S^{t+1}} \cdot \frac{1}{L_S^{t+1}}} \tag{107}$$

We will call Λ_{sv} the *Standard productivity index*, or more precisely the *Standard physical productivity index*. The Standard productivity index takes out the effect of social productivity changes resulted from the difference in the structures of division of labour. It should be stressed that Λ_{sv} is defined both by $1/\ P_v^{t+1}$ in price side(or cost side), and by Y_E^{t+1}/L_S^{t+1} in quantity side, and therefore, Λ_{sv} is considered as a useful measure of the productivity change rate of the entire economy. The reason why we use the Fisher type index number is that by using Fisher type index we can derive the double definitions of Λ_{sv} of (107) (see Yagi[1998][2007a][2007b]).

3.2.2 Search for an Invariable Measure of Value

The search for an invariable measure of value is one of important problems for the Classical economists, especially for D.Ricardo. In case that the labour should be considered as an invariable measure, the value of labour should be kept constant even if the changes in social productivity and the changes in distribution occur. There may be another view point as for the problem of the search for an invariable measure. D.Ricardo has an interest in the price stability problem. In this case, the general price level should be constant even if he changes in social productivity and the changes in distribution occurs. Let us consider these problems. Yagi [2012] analyses these problems in a multisector model of Sraffa type. In this paper, we will consider these problem in our Sraffian aggregate model.

Productivity Changes From (107), we can obtain

$$P_v^{t+1} = 1/\Lambda_{sv} \tag{108}$$

This is the same as (42). The general price level will change as the social productivity changes. From (107), we can represent the standard output index as follows

$$Y_E^{t+1} = \Lambda_{sv} L_S^{t+1} \tag{109}$$

The left member is the product of the Standard productivity index and the standard labour. From this we can obtain

$$Y_E^{t+1}/L_S^{t+1} = \Lambda_{sv} = 1/\alpha \tag{110}$$

The ratio α is the same as the equation (20) or (22). From (108)(110), we can obtain

$$\frac{Y_E^{t+1}P_v^{t+1}}{L_S^{t+1}} = 1 \tag{111}$$

This is the same as the equation (28). We can consider that the condition of (111) is equivalent to

$$v_L^{t+1} = 1 \tag{112}$$

Therefore if the general price level is given by the standard indexes of (107), we can obtain the result that the value of labour becomes equal to unity. This is one of important result of our analysis.

If we define the effective labour by

$$L_E^{t+1} = \Lambda_{sv} L_S^{t+1} \tag{113}$$

then we can obtain

$$Y_E^{t+1} = L_E^{t+1} \tag{114}$$

Corresponding to the effective labour L_E^{t+1}, the effective labour coefficient vector will be defined as

$$\mathbf{l}_E^{t+1} = \Lambda_{sv}\ \mathbf{l}_S^{t+1} \tag{115}$$

From (104)(105) (113)(115), we obtain

$$\mathbf{x}^{t+1}\mathbf{l}_E^{t+1} = \Lambda_{sv} L_S^{t+1}\mathbf{x}^t\mathbf{l}_S^t \tag{116}$$

This means that the effective labour of *Period* t+1 can be compared with the standard labour of *Period* t. The equation (116) is an important basis for our intertemporal comparisons (See Yagi[2007a][2012]).

Changes in Distribution Let us consider that the price system of *Period* t+1 is

given by the effective labour coefficient vector $\mathbf{l}_E^{t+1}$. By multiplying both members of the price system of *Period* t+1 by Λ_{sv}, for all r^{t+1} $(0 \leq r^{t+1} < R^{t+1})$, we have

$$\mathbf{p}_E^{t+1} = (1 + r^{t+1})\mathbf{A}^{t+1}\mathbf{p}_E^{t+1} + w_S^{t+1}\mathbf{l}_E^{t+1} \tag{117}$$

The vertically integrated labour coefficient vector given by the effective labour coefficient vector $\mathbf{l}_E^{t+1}$ can be defined as

$$\mathbf{v}_E^{t+1} = [\mathbf{I} - \mathbf{A}^{t+1}]^{-1}\mathbf{l}_E^{t+1} \tag{118}$$

We can consider the following Evaluation System:

[**Evaluation System 2**]

$$\mathbf{s}^{t+1}\mathbf{p}_E^{t+1} = v_L^{t+1}\mathbf{x}^{t+1}\mathbf{l}_E^{t+1} \tag{119}$$

$$\mathbf{p}_E^{t+1} = (1 + r^{t+1})\mathbf{A}^{t+1}\mathbf{p}_E^{t+1} + w_S^{t+1}\mathbf{l}_E^{t+1} \tag{120}$$

In this system, there are (n+1) independent equations and (n+3) unknowns. If the condition of standard for *Period* t+1 is given, the above system will become determinate. From the price equation of the above Evaluation System, for all $0 \leq r^{t+1} < R^{t+1}$, we have

$$(1 - r^{t+1}/R^{t+1})\ \mathbf{s}^{t+1}\mathbf{p}_E^{t+1} = w_S^{t+1}\mathbf{x}^{t+1}\mathbf{l}_E^{t+1} \tag{121}$$

Therefore, for all $0 \leq r^{t+1} < R^{t+1}$, we have

$$v_L^{t+1} = 1 \iff r^{t+1} = R^{t+1}(1 - w_S^{t+1}) \tag{122}$$

This is also an important result of our analysis. Even if the changes in distribution happens, the value of labour will be kept constant.

Under the condition of $r^{t+1} = R^{t+1}(1 - w_S^{t+1})$, the prices and wages of both periods are measured in terms of unit of effective labour. The reduced forms of $\mathbf{p}_{Ev}^{t+1}$ are represented as

$$\mathbf{p}_{Ev}^{t+1} = \frac{\mathbf{p}_E^{t+1}}{v_L^{t+1}} = (1 - r^{t+1}/R^{t+1})[\mathbf{I} - (1 + r^{t+1})\mathbf{A}^{t+1}]^{-1}\mathbf{l}_E^{t+1} \tag{123}$$

$$w_v^{t+1} = \frac{w_S^{t+1}}{v_L^{t+1}} = 1 - r^{t+1}/R^{t+1} \tag{124}$$

Under the condition of $r^{t+1} = R^{t+1}(1 - w_S^{t+1})$, the aggregate value of standard net product is constant. And we have

$$Y_E^{t+1} = \mathbf{y}^{t+1}\mathbf{v}_E^{t+1} = \mathbf{s}^{t+1}\mathbf{v}_E^{t+1} = \mathbf{s}^{t+1}\mathbf{p}_{Ev}^{t+1} \tag{125}$$

The General Price Level If we want to obtain the condition which can keep the general price level constant, we can set the value productivity of product as

$$\frac{Y_E^{t+1}P^{t+1}}{L_S^{t+1}} = \Lambda_{sv} \tag{126}$$

In this case, we can obtain

$$v_L^{t+1} = \Lambda_{sv} \tag{127}$$

$$P^{t+1} = 1 \tag{128}$$

Pasinetti [1981][1993] introduce a notion of the dynamic standard commodity, which can keep the general price level constant when the sectoral productivities changes. In our multi-sector model, the social productivity is measured by our standard productivity index given by (107). The dynamic standard commodity $\hat{\mathbf{s}}^{t+1}$ and the condition which can keep the general price level constant can be defined in our discrete as

$$\hat{\mathbf{s}}^{t+1} = \mathbf{s}^{t+1}/\Lambda_{sv}L_S^{t+1} \tag{129}$$

$$\hat{\mathbf{s}}^{t+1}{}^{\mathfrak{t}}\mathbf{p}_{Ev}^{t+1} = 1 \tag{130}$$

These results are explained in Yagi [2012].

3.2.3 Reproduction Process of a Natural Economy

Let us consider the problem which we indicated in the last part of Section 1 of this paper. From (99)(108)(111), we have the equation of the general price level of the aggregate model.

$$P_\omega^{t+1} = P_v^{t+1}/\omega_v^{t+1} = (1+\theta_v^{t+1})\alpha \tag{131}$$

This corresponds to the euqtion (51). From this we have

$$Y_E^{t+1}P_\omega^{t+1} = (1+\theta_v^{t+1})L_S^{t+1} \tag{132}$$

This aggregate equation corresponds to the equation obtained from the sectoral model as

$$(1+\theta_v^{t+1})\mathbf{s}^{t+1}\mathbf{p}_{Ev}^{t+1} = (1+\theta_v^{t+1})L_S^{t+1} \tag{133}$$

From (108)(125), $\mathbf{s}^{t+1}\mathbf{p}_{Ev}^{t+1}$ of (133) becomes equal to $\mathbf{s}^{t+1}\mathbf{p}_v^{t+1}$ of (105). Therefore the equation (133) becomes

$$(1+\theta_v^{t+1})\mathbf{s}^{t+1}\mathbf{p}_v^{t+1} = (1+\theta_v^{t+1})L_S^{t+1} = (1+\theta_v^{t+1})(1+g)L_S^t \tag{134}$$

Let us define the labour demand measured in terms of labour commanded at the end of *Period* t +1. Then, because of $L_S^t = 1$ and $\theta_v^{t+1} = \frac{r^t}{R^t - r^t}$, for all r^t $(0 \leq r^t < R^t)$, then it will be given by

$$N_{t+1}^D \equiv (1+\frac{r^t}{R^t - r^t})(1+g) \tag{135}$$

On the other hand, because of $L_S^t = N_S^{t-1} = 1$, we have the labour supply at the end of *Period* t

$$N_S^{t+1} = (1+g)^2 \tag{136}$$

From the above, we can consider the labour market at the end of production period. When we have

$$N_{t+1}^D(r) < N_{t+1}^S \Leftrightarrow \frac{r^{t+1}}{R^{t+1} - r^{t+1}} < g \tag{137}$$

The rate of profits r^{t+1} will increase. When we have

$$N_{t+1}^D(r) > N_{t+1}^S \Leftrightarrow \frac{r^{t+1}}{R^{t+1} - r^{t+1}} > g \tag{138}$$

The rate of profits r^{t+1} will decrease. When the labour market is in equilibrium, for all r^{t+1} $(0 \leq r^{t+1} < R^{t+1})$, we have

$$N_{t+1}^D(r) = N_{t+1}^S \Leftrightarrow \frac{r^{t+1}}{R^{t+1} - r^{t+1}} = g \tag{139}$$

In equilibrium, from (139), we can obtain an interesting property in distribution in a growing economy as

$$\pi_v^{t+1} = \frac{r^{t+1}}{R^{t+1}} = \frac{g}{1+g} \tag{140}$$

$$\omega_v^{t+1} = \frac{1}{1+g} \tag{141}$$

This is the same results of Section 2.

Moreover, we can obtain the following equation.

$$r^{t+1} = \frac{g}{1+g} R^{t+1} \tag{142}$$

This is an interesting equation. It should be noted that in the equation (142) the rate of profit is not equal to the Harrodian natural rate of growth.

Under the condition of $v_L^{t+1} = (1+r^t)v_L^t = 1$,we can obtain the equation (42). The equation (42) is the same as the equation (108). Therefore in our multi-sector model we have

$$P_v^{t+1} = \alpha \tag{143}$$

The general price of *Period* $t+1$ becomes equal to the labour embodied.

Let us consider the growth of national income. We are considering *Period* t as base year, and from (104) we have

$$Y_S^t P_v^t = L_S^t \tag{144}$$

$$P_v^t = 1 \tag{145}$$

From (143), we can obtain

$$Y_E^{t+1} P_v^{t+1} = L_S^{t+1} \tag{146}$$

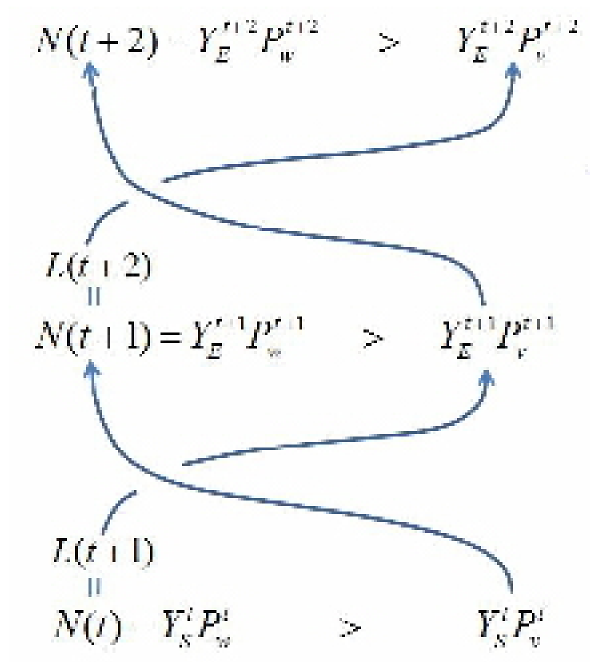
This means that the national income will grow at the rate of g, becuase the labour amount grows at the rate of g.

Let us consider the growth of the produced means of production. From (72)(105), we have

$$\begin{aligned}
L_S^{t+1} &= \mathbf{x}^{t+1}\mathbf{l}_S^{t+1} && (147)\\
&= \mathbf{x}^{t+1}[\mathbf{I}-\mathbf{A}^{t+1}][\mathbf{I}-\mathbf{A}^{t+1}]^{-1}\mathbf{l}_S^{t+1} && (148)\\
&= [\mathbf{x}^{t+1} - \mathbf{x}^{t+1}\mathbf{A}^{t+1}]\mathbf{v}_S^{t+1} && (149)\\
&= \mathbf{x}^{t+1}\mathbf{v}_S^{t+1} - \mathbf{m}_S^{t+1}\mathbf{v}_S^{t+1} &&\\
&= (1+g)\mathbf{x}^t\mathbf{v}_S^t - (1+g)\mathbf{m}_S^t\mathbf{v}_S^t && (150)
\end{aligned}$$

This means that, by the growth of labour amount by g, the produced means of production $\mathbf{m}_S^t\mathbf{v}_S^t$ will be added by $g\frac{\mathbf{m}_S^t\mathbf{v}_S^t}{L_S^t}$, which is equal to the breakeven investment as to the produced means of production. Therefore the produced means of production per worker will be kept constant and the produced means of production grows at the rate of g.

The reproduction process of a Natural Economy can be shown in the following figure.



4 Conclusion

We have characterized the Classical theory of value as the double helix theory of value and reproduction, or the double helix theory of commodities and labour.

It is true that in a steady growth pass the amount of labour, the output, and the income grows at the same rate. But in our theory, the labour embodied, labour commanded and distribution enable the system to be viable and reproduce. The standard net product and the standard productivity index enable us to grasp the truly real values of product and the properties of distribution and to keep the value of labour constant. The dynamic standard commodity can keep the general price level constant.

[**Acknowledgement**] This research is supported by Grant-in-Aid for Scientific Research(A), the Japan Society for Promotion of Science, Research Project Number: 22243019.

[**Notes**]

1) From (73), we can obtain

$$(1 - r/R)\mathbf{s}\mathbf{p}_S = w_S \mathbf{x}\mathbf{l}_S$$

From this equation, we can obtain the equation (78).

2) If the rate of profits is given exogenously, then the wage measured in terms of labour is determined. Let us denote the surplus value by δ. Since δ means the unpaid labour, it is represented by

$$\delta = 1 - w_v$$

from this, the rate of surplus value (or the rate of exploitation) can be represented by

$$\sigma = \delta / w_v = (1 - w_v)/w_v$$

Then we have

$$\sigma = r/(R - r)$$

This means that the rate of surplus value (the rate of exploitation) become determinate when the rate of profits is given exogenously. From (4.45)(4.85), the relationship between the rate of surplus value and the distribution ratio can be represented as

$$\sigma = \theta_v$$

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