Nonlinear Adjustment of the Real Exchange Rate Towards its Equilibrium Value: a Panel Smooth Transition Error Correction Modelling[*]

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Abstract

We study the nonlinear dynamics of the real exchange rate towards its behavioral equilibrium value (BEER) using a Panel Smooth Transition Regression model framework. We show that the real exchange rate convergence process in the long run is characterized by nonlinearities for emerging economies, whereas industrialized countries exhibit a linear pattern. Moreover, there exists an asymmetric behavior of the real exchange rate when facing an over- or an undervaluation of the domestic currency. Finally, our results suggest that the real exchange rate is unable to unwind alone global imbalances.

JEL Classification: F31, C23.
Keywords: Equilibrium exchange rate, BEER model, Panel Smooth Transition Regression, Panel Vector Error Correction Model.

1   Introduction

The assessment of equilibrium values for the real exchange rate has always been an important issue in international macroeconomics, especially in the current context of global imbalances. Between the short-run market view and the PPP attractor supposed to

hold at a remote time horizon, a wide range of intermediate approaches have been developed.[1] Among them, there is the BEER or "Behavioral Equilibrium Exchange Rate" model which was introduced by Clark and MacDonald (1998) and has proved to be a consistent framework to derive equilibrium exchange rate values.[2] This approach consists in the estimation of a long-run or cointegration relationship between the real effective exchange rate and a set of economic fundamentals. The BEER value is then calculated by predicting the real effective exchange rate from the estimated long-run equation. Vector error correction models (VECM) are subsequently perfectly accurate to assess the speed at which the real exchange rate converges towards its equilibrium value.

In this context, according to the standard macroeconomic view, any deviation from the equilibrium level is considered as temporary since there are forces ensuring quickly mean-reverting dynamics. However, in many countries, the experience of real exchange rates over the last two decades has been characterized by substantial misalignments, with time lengths much higher than suggested by the theoretical models (Dufrénot, Lardic, Mathieu, Mignon and Péguin-Feissolle, 2008). The fact that exchange rates can spend long periods away from their fundamental values implied a revival of interest in the study of exchange rate misalignments. Our aim is to contribute to this literature by investigating the dynamics of the adjustment process of the exchange rate towards its equilibrium value in a nonlinear panel framework.

The nonlinear cointegration support allows us to investigate the slowness of the adjustment process towards the long-run equilibrium. Numerous factors may explain such a nonlinear dynamics: transaction costs (Dumas, 1992; Sercu, Uppal and Van Hulle, 1995; O'Connell and Wei, 1997; Obstfeld and Taylor, 1997; Imbs, Mumtaz, Ravn and Rey, 2003), heterogeneity of buyers and sellers (Taylor and Allen, 1992), speculative attacks on currencies (Flood and Marion, 1999), presence of target zones (Krugman, 1991; Tronzano, Psaradakis and Sola, 2003), noisy traders causing abrupt changes (De Long, Shleifer, Summers and Waldmann, 1988), heterogeneity of central bank interventions (Dominguez, 1998). All these factors imply, either a nonlinear relationship between the exchange rates and the economic fundamentals, or a nonlinear adjustment mechanism with time-dependence properties. We consider here a smooth transition model for the adjustment process which can be viewed as a reduced form of structural models of fundamental exchange rate accounting for nonlinearities such as transaction costs, changing-

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[1] For recent surveys, see MacDonald (2000) and Driver and Westaway (2004).

[2] See Bénassy-Quéré, Béreau and Mignon (2008b) for a detailed study on the robustness of the BEER approach.

regimes fluctuations,...Moreover, such models help modelling asymmetries inherent to the adjustment process. This is particularly interesting since these asymmetries may explain, for instance, the unequal durations of undervaluations and over-valuations.

While numerous contributions have applied this nonlinear cointegration methodology in time series[3], this has not be done so far in the panel context. This constitutes a lack since we think that, to derive consistent equilibrium values of exchange rates, it seems important to work with a large panel of countries. Indeed, as noticed by Bénassy-Quéré, Duran-Vigneron, Lahrèche-Révil and Mignon (2004) among others, the large literature on equilibrium exchange rates has typically focused on country-by-country estimations of equilibrium exchange rates (Clark and MacDonald, 1998) or on consistent estimations of equilibrium exchange rates for a set of industrial economies (Williamson, 1994; Wren-Lewis and Driver, 1998). Until the mid-1990s, this approach was in line with a two-tier international monetary system, the first tier consisting in a small number of key currencies (the dollar, the Deutschemark, the yen and the British pound) and the second tier consisting in all other currencies. Since the mid-1990s, the rising share of emerging countries in global imbalances has made such divide no longer adequate and calls for the estimation of consistent sets of equilibrium exchange rates for a large number of currencies. To account for this evolution, we consider the G-20 in deriving our estimates of equilibrium exchange rates, a group that covers both industrial and emerging economies.

To sum up, the goal of this paper is to investigate the nonlinear behavior of the real exchange rate's adjustment process towards its equilibrium value in a panel framework by estimating a Panel Smooth Transition Error Correction Model. To this end, the rest of the paper is organized as follows. Section 2 briefly sketches out methodological aspects relating to panel nonlinear models. Section 3 discusses our approach, data and their properties. Section 4 contains the estimation results and related comments. Section 5 concludes.

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[3]See, for instance, Michael, Nobay and Peel (1997); Ma and Kanas (2000); Chen and Wu (2000); Taylor, Peel and Sarno (2001); Baum, Barkoulas and Caglayan (2001); Dufrénot and Mignon (2002); Dufrénot, Mathieu, Mignon and Péguin-Feissolle (2006); Dufrénot et al. (2008); López Villavicencio (2008).

2 Panel nonlinear models

2.1 PTR and PSTR models

In his seminal paper, Hansen (1999) introduced the panel threshold regression (PTR) model to allow regression coefficients to vary over time.

Let $\{y_{i,t}, s_{i,t}, x_{i,t}; t = 1, ..., T; i = 1, \ldots, N\}$ be a balanced panel with t denoting time and i the individual. Denoting $y_{i,t}$ the dependent variable, μ_i the individual fixed effects, $s_{i,t}$ the threshold variable and $x_{i,t}$ a vector of k exogenous variables, the PTR model can be written as follows:

$$y_{i,t} = \left\{ \begin{array}{ll} \mu_i + \beta_1' x_{i,t} + \varepsilon_{i,t}, & s_{i,t} \leq c \\ \mu_i + \beta_2' x_{i,t} + \varepsilon_{i,t}, & s_{i,t} > c \end{array} \right\} \tag{1}$$

In this model, the observations in the panel are divided into two regimes depending on whether the threshold variable is lower or larger than the threshold c. The error term $\varepsilon_{i,t}$ is independent and identically distributed. As in the time series context, the transition from one regime to another is abrupt and the model implicitly assumes that the two groups of observations are clearly identified and distinguished, which is not always feasible in practice.

To account for possible smooth and gradual transitions, González, Teräsvirta and van Dijk (2005) have introduced the panel smooth transition regression (PSTR) model.[4] Considering, as for the PTR model, the case of two regimes, the PSTR model is given by:

$$y_{i,t} = \mu_i + \beta_0' x_{i,t} + \beta_1' x_{i,t} g\left(s_{i,t}; \gamma, c\right) + \varepsilon_{i,t} \tag{2}$$

where $g\left(s_{i,t}; \gamma, c\right)$ is the transition function, normalized and bounded between 0 and 1, $s_{i,t}$ the threshold variable which may be an exogenous variable or a combination of the lagged endogenous one[5] (see van Dijk, Teräsvirta and Franses, 2002), γ the speed of transition and c the threshold parameter. Following Granger and Teräsvirta (1993) and Teräsvirta (1994) in the time series context or González et al. (2005) in a panel framework, the logistic specification can be used for the transition function:

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[4]See also He and Sandberg (2004) and Fok, van Dijk and Franses (2005) who have introduced dynamic nonlinear panel models through the development of PLSTAR (panel logistic smooth transition autoregressive) models.

[5] As Fouquau (2008) reminds us, the endogenous variable must be lagged to avoid simultaneity problems.

$$g\left(s_{i,t};\gamma,c\right) = \left[1 + \exp\left(-\gamma \prod_{j=1}^{m}\left(s_{i,t} - c_j\right)\right)\right]^{-1} \tag{3}$$

with $\gamma > 0$ and $c_1 \leq c_2 \leq ... \leq c_m$. When $m = 1$ and $\gamma \to \infty$, the PSTR model reduces to a PTR model. González et al. (2005) mention that from an empirical point of view, it is sufficient to consider only the cases of $m = 1$ or $m = 2$ to capture the nonlinearities due to regime switching.[6] Note that it is possible to extend the PSTR model to more than two regimes:

$$y_{i,t} = \mu_i + \beta_0' x_{i,t} + \sum_{j=1}^{r} \beta_j' x_{i,t} g_j\left(s_{i,t}^{(j)};\gamma_j,c_j\right) + \varepsilon_{i,t} \tag{4}$$

where $r+1$ is the number of regimes and the $g_j\left(s_{i,t}^{(j)};\gamma_j,c_j\right), j = 1,...,r$, are the transition functions (see Equation (3)).

2.2 Methodology

Following the methodology used in the time series context, González et al. (2005) suggest a three step strategy to apply PSTR models: (i) specification, (ii) estimation, (iii) evaluation and choice of the number of regimes (choice of r). Let us give some explanations about each of these steps.

The aim of the **identification** step is to test for homogeneity against the PSTR alternative. This can be done by testing the null hypothesis $\gamma = 0$. Due to the presence of unidentified nuisance parameters under the null, a first-order Taylor expansion around zero is used for the function g (see Lüükkonen, Saikkonen and Teräsvirta, 1988, or González et al., 2005):

$$y_{i,t} = \mu_i + \beta_0'^* x_{i,t} + \beta_1'^* x_{i,t} s_{i,t} + ... + \beta_m'^* x_{i,t} s_{i,t}^m + \varepsilon_{i,t}^* \tag{5}$$

where $\beta_1'^*,...\beta_m'^*$ are multiple of γ and $\varepsilon_{i,t}^* = \varepsilon_{i,t} + r_m \beta_1' x_{i,t}$, r_m being the remainder of the Taylor expansion. Testing the null hypothesis of linearity is then equivalent to test $\beta_1'^* = ... = \beta_m'^* = 0$ in Equation (5). To this end, González et al. (2005) provide a LM-test statistic that is asymptotically distributed as a $\chi^2(mk)$ under the null.

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[6] Note that the case $m = 1$ corresponds to a logistic PSTR model and $m = 2$ refers to a logistic quadratic PSTR specification.

As in the time series context, this test can be used to select (i) the appropriate transition variable as the one that minimizes the associated p-value and (ii) the appropriate order m in Equation (3) in a sequential manner.

Turning to the **estimation** step, nonlinear least squares are used to obtain the parameter estimates, once the data have been demeaned. It should be noticed that unlike the within transformation in linear models, demeaning the data in the nonlinear context is not straightforward due to the presence of parameters from the transition function, namely γ and c, in the expression of the second regime coefficients. Indeed, those parameters are reestimated at each iteration of the procedure and demeaned values are recomputed as well (see Hansen, 1999, González et al., 2005 or Colletaz and Hurlin, 2006 for details).

The **evaluation** step consists in (i) applying misspecification tests in order to check the validity of the estimated PSTR model and (ii) determining the number of regimes. González et al. (2005) propose to adapt the tests of parameter constancy over time and of no remaining nonlinearity introduced by Eitrheim and Teräsvirta (1996) in the time series context. The test of no remaining nonlinearity, which is interpreted as a test of no remaining heterogeneity in panel data context, can be useful for determining the number of regimes of the PSTR model. To this end, González et al. (2005) suggest a sequential procedure starting by estimating a linear model, then a PSTR model if the homogeneity hypothesis is rejected, a PSTR model with 3 regimes is the no remaining heterogeneity hypothesis is rejected in the PSTR 2 regimes model, and so on.

3   Data and their properties

3.1   The model

As mentioned in the introduction, our aim is to study the possible nonlinear convergence process of the real exchange rate towards its long-run equilibrium value given by a BEER specification. Numerous explanatory variables may be used in the BEER model.[7] Here we rely on the parsimonious specification developed by Alberola, Cervero, Lopez and Ubide (1999) which has proved to be consistent to numerous robustness checks as showed by Bénassy-Quéré et al. (2008b). Combining the BEER approach with the modelling of

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[7]See among others Faruqee (1994) and MacDonald (1997) for a general review of the real exchange rate determinants or Egert, Halpern and MacDonald (2006) for a survey on equilibrium exchange rate models applied to transition economies.

the short term dynamics and using the former notations for the PSTR model in Section 2, our complete model can be written as follows:

$$\Delta q_{i,t} = \mu_i + \underbrace{\theta z_{i,t-1} + \beta_1 \Delta nfa_{i,t} + \beta_2 \Delta rpi_{i,t}}_{\text{Regime 1}} +$$

$$\underbrace{\left[\theta^* z_{i,t-1} + \beta_1^* \Delta nfa_{i,t} + \beta_2^* \Delta rpi_{i,t} \right]}_{\text{Regime 2}} g\left(s_{i,t}; \gamma, c \right) + \varepsilon_{i,t} \tag{6}$$

with:

$$g\left(s_{i,t}; \gamma, c \right) = \left[1 + exp\left(-\gamma \prod_{j=1}^{m} (s_{i,t} - c_j) \right) \right]^{-1} \text{for } m = 1, 2 \tag{7}$$

and:

$$z_{i,t} = q_{i,t} - \underbrace{\widehat{c}_i - \widehat{\beta}_1^{LT} nfa_{i,t} - \widehat{\beta}_2^{LT} rpi_{i,t}}_{BEER_{i,t}} \tag{8}$$

where $q_{i,t}$ is the logarithm of the real effective exchange rate of country i (an increase in $q_{i,t}$ corresponds to a real depreciation of currency i), $nfa_{i,t}$ the net foreign asset-to-GDP ratio, and $rpi_{i,t}$ the logarithm of the relative productivity differential proxy (see the following sub-section for further details). $\widehat{c}_i$, $\widehat{\beta}_1^{LT}$ and $\widehat{\beta}_2^{LT}$ respectively stand for the estimated long-run fixed effect and coefficients from the linear cointegrating relationship between the real effective exchange rate and the explanatory variables (namely the linear panel BEER equation).

Here, $s_{i,t} \in S = \{\Delta q_{i,t-1}, z_{i,t-1}, nfa_{i,t-j}, ca_{i,t-j}, nfag_{i,t-j}, cag_{i,t-j}\}$ for $j = 0, 1$, with $ca_{i,t}$ the observed current account value of country i at year t, $cag_{i,t}$ (resp. $nfag_{i,t}$) the gap between the observed value of the current account (resp. the net foreign asset position) of country i at year t and its target value $\overline{ca}_{i,t}$ (resp. $\overline{nfa}_{i,t}$). By selecting the set S for $s_{i,t}$, we assume that what determines the adjustment speed of the real exchange rate towards equilibrium may be either the fact that the currency appreciates or depreciates (through the sign of $\Delta q_{i,t}$), the size of the past currency misalignement ($z_{i,t}$), or the magnitude of the current account or of the net foreign asset position disequilibrium (through $nfa_{i,t-j}$, $ca_{i,t-j}$ or $nfag_{i,t-j}$ and $cag_{i,t-j}$ respectively, as mentioned below).

It has to be noticed that at any time, the coefficients of the explanatory variables in Equation (6) are given by: $c_x = \beta_x + \beta_x^* g(s_{i,t}; \gamma, c)$ with $\beta_x = \theta, \beta_1, \beta_2$. When $g(s_{i,t}; \gamma, c) = 0$, then $c_x = \beta_x$ and the estimated coefficients correspond to those of Regime 1. At the

other extreme, i.e. when $g(s_{i,t}; \gamma, c) = 1$, then $c_x = \beta_x + \beta_x^*$. Between those two points, c_x takes a continuum of values depending on the realization of the nonlinear transition function $g(s_{i,t}; \gamma, c)$.

3.2 Data

Here we concentrate on 15 countries or areas belonging to the Group of Twenty (G-20), a country grouping created in 1999 to tackle financial stability issues that has sometimes been viewed as a possible substitute for the G-7 on international monetary issues. [8]

Regarding the long-run BEER equation, the dependent variable is the real effective exchange rate (q) and the explanatory variables are the stock of net foreign assets (nfa) and the productivity differential proxied here by the relative CPI-to-PPI ratio (rpi). All series are in logarithms except nfa which is expressed as share of GDP in percentage points. Data are annual and cover the period 1980 to 2005.

The real effective exchange rate for each country i is calculated as a weighted average of real bilateral exchange rates against each j trade partner. Bilateral real exchange rates are derived from nominal rates and consumer price indices (CPI); they are based in 2000.[9] The weights have been calculated as the share of each partner in imports and exports of goods and services in 2005.[10] Intra-Eurozone flows have been excluded and trade weights have been normalized to sum to one across the partners included in the sample.

Denoting the variables in logarithms in lower cases, we can write:

$$q_{i,t} = \sum_{j \neq i} \omega_{ij} \left(e_{i,t} - e_{j,t} \right) = \sum_{j \neq i} \omega_{ij} e_{ij,t} \text{ where } \sum_{j \neq i} \omega_{ij} = 1 \tag{9}$$

where $e_{i,t}$ denotes the real bilateral exchange rate of currency i vis-à-vis the USD, $e_{ij,t}$ the one against the j currencies and ω_{ij} the trade weights. When $q_{i,t}$ rises (resp. falls), it corresponds to a depreciation (resp. appreciation) of currency i vis-à-vis the j currencies.

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[8]See, e.g., O'Neill and Hormats (2004). The exact composition of the G-20 sample is given in Appendix A.

[9]Source: World Bank *World Development Indicators* (WDI) for nominal exchange rates and CPI data except for the EUR/USD exchange rate which was extracted from Datastream and China's real exchange rate which was calculated with GDP deflator (WDI).

[10]Source: IMF *Direction of Trade Statistics* (DOTS).

The net foreign asset position is built using the Lane and Milesi-Ferretti database from 1980 to 2004.[11] The 2005 data is calculated by adding the current account position to the 2004 NFA value.[12] Regarding the CPI-to-PPI ratio, data were extracted from WDI and IFS (IMF *International Financial Statistics*) databases. We take the difference between the value for country i and the weighted average of its j partners' values as follows:

$$rpi_{i,t} = ln\left(\frac{CPI}{PPI}\right)_{i,t} - \sum_{j \neq i} \omega_{ij} ln\left(\frac{CPI}{PPI}\right)_{j,t} \tag{10}$$

Current account data were extracted from the WDI database. They are also expressed in proportion of GDP in absolute terms (the sum is supposed to be equal to zero, which is not the case in practice due to a large world discrepancy). To account for the impact of current account or net foreign asset position disequilibria on the real exchange rate convergence speed towards its long-run value, we need to define measures of the distance between the observed current account or net foreign asset position values and their respective long-run targets. We named those measures the current account and net foreign asset position gaps respectively. They are defined as follows:

$$cag_{i,t} = ca_{i,t} - \overline{ca}_{i,t} \text{ with } \overline{ca}_{i,t} = \phi(\overline{nfa}_{i,t})$$
$$nfag_{i,t} = nfa_{i,t} - \overline{nfa}_{i,t} \text{ with } \overline{nfa}_{i,t} = \psi(dem_{i,t}, gdebt_{i,t}, gdppc_{i,t})$$

where $dem_{i,t}$, $gdebt_{i,t}$ and $gdppc_{i,t}$ respectively stand for the demographic structure, the public debt-to-GDP ratio and the logarithm of GDP per capita; ϕ and ψ being linear functions (see below).

As previously mentioned, $\overline{ca}_{i,t}$ and $\overline{nfa}_{i,t}$ denote the target values of the current account and the net foreign asset position-to-GDP ratio respectively. To assess those values, we rely on the long-run net foreign asset position model proposed by Lane and Milesi-Ferretti (2001) and derive target values for the current account that are consistent with the reach of the equilibrium net foreign asset positions in 5 years.[13]

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[11]Source: http://www.imf.org/external/pubs/cat/longres.cfm?sk=18942.0, see Lane and Milesi-Ferretti (2007).

[12]Source: IMF *International Financial Statistics* (IFS), March 2007. Unfortunately, valuation effects cannot be included in the 2005 figure because the composition of gross assets and liabilities was not available.

[13]See Bénassy-Quéré, Béreau and Mignon (2008*a*) for further details on the specification and estimation of ϕ and ψ.

4 Estimation results

As revealed from panel unit root and cointegration tests, all our series are cointegrated of order $(1,1)$.[14] The long-run relationship between the real exchange rate and the explanatory variables, estimated using the panel Dynamic OLS procedure, is given by:

$$\widehat{q}_{i,t} = \widehat{\mu}_i - 0.331 nfa_{i,t} - 0.829 rpi_{i,t} \tag{11}$$

The results from the panel cointegration estimation appear consistent with the theory: the real exchange rate appreciates (q falls) in the long run if the net foreign asset position rises and if the tradable-to-non-tradable productivity ratio increases compared to the rest of the world (as a Balassa-Samuelson effect would suggest[15]).

4.1 The linear error correction model

As a first approximation, and for comparative purposes, we have estimated linear error correction models (ECM) for the whole panel (G-20) and for different groups of countries. Four sub-groups of countries are considered: the G-7 group, emerging countries (non G-7 group), Asian developing countries (Asia group) and countries that have overcome a financial crisis during the 1990s (denoted as 'Crisis' in the following tables). [16] The estimated model is the following:

$$\Delta q_{i,t} = \mu_i + \rho \Delta q_{i,t-1} + \theta z_{i,t-1} + \beta_1 \Delta nfa_{i,t} + \beta_2 \Delta rpi_{i,t} + \varepsilon_{i,t} \tag{12}$$

where $z_{i,t-1}$ corresponds to the past deviation of the real exchange rate from its equilibrium value as calculated in Equation (8) (i.e. the misalignment of the real exchange rate at year t-1). Given that Equation (12) is a dynamic panel data model, we have estimated it by the Generalized Method of Moments (GMM), which provides a convenient framework for obtaining efficient estimators in this context.[17] The results show that the dynamic term (ρ) was not significant in the specification in any of the different panels. Therefore, we have dropped the lagged exchange rate variations in our final estimation, keeping only the short-run fundamentals and the error correction term.[18]

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[14] All the results are available upon request to the authors.

[15] An alternative interpretation of this effect is that a positive shock on productivity in the tradable sector leads to a rise in intertemporal income, hence on the demand for both tradables and non-tradables. Because non-tradables cannot be imported, their relative price rises, which amounts to an exchange-rate appreciation. See, e.g., Schnatz and Osbat (2003).

[16] The composition of each country group is detailed in Appendix A.

[17] See among others, the seminal papers of Anderson and Hsiao (1982) and Arellano and Bond (1991).

[18] We have also estimated Equation (12) by Instrumental Variables (IV), finding similar results. To avoid too many tables, IV specifications are not presented here, but are available upon request to the authors.

As mentioned before, we are particularly interested in the characteristics of the adjustment speed of the real effective exchange rate towards its long-run equilibrium value (i.e. θ in Equation (12)). The theory of cointegration predicts that, if the real exchange rate and its fundamental determinants are cointegrated, we may expect a later reversal in case of a misalignment. Indeed, if the error correction coefficient is significantly negative, then a past undervaluation of currency i (resp. over-valuation) will generate a current real appreciation (resp. depreciation) of currency i vis-à-vis the j currencies. In other words, if $z_{i,t-1}$ is positive (resp. negative), meaning that currency i is undervalued (resp. over-valued), a negative sign of θ will guaranty a current appreciation (resp. depreciation) of the current real exchange rate corresponding to a decrease (resp. increase) in $q_{i,t}$. Table 1 reports the GMM estimates of the error correction coefficient in our final linear specification for the whole G-20 panel and the different sub-groups of countries.

Table 1: **GMM estimates of the error correction coefficient - linear specification**

	G-20	G-7	Non G-7	Asia	Crisis
θ	-0.155	-0.156	-0.132	-0.129	-0.089
T-stat	-4.23	-5.27	-3.19	-3.78	-2.0

As expected, we find a negative and statistically significant error correction term in each case, implying that if the fundamentals in the last period dictate a lower (resp. upper) real exchange rate than that observed, then the real exchange rate will strictly depreciate (resp. appreciate) in the current period. The (average) error correction coefficients reported here show that between 9% and 16% of the adjustment takes place within a year.

4.2 Nonlinear error correction model

The linear ECM implicitly assumes that the adjustment speed towards equilibrium is both continuous and constant, regardless of the extent of the real misalignment. However, as mentioned before, we may imagine that the convergence speed increases with the size of the deviation from equilibrium, a feature that the previous linear model would not be able to capture. In that case, Equation (12) could be better approximated by a panel nonlinear model.

To formally analyze this possibility, we have tested linearity in model (12)[19] using the González et al. (2005) test with different possible transition variables. First, we use the lagged estimated cointegrating vector $(z_{i,t-1})$ as the appropriate threshold variable. This model is particularly attractive from an economic point of view as it implies the existence of a lower threshold (whether a logistic function is used in (6) with $m = 1$) or a band (whether the function is a logistic quadratic one, i.e. m=2 in Equation (6)) above or outside which there is a strong tendency for the real exchange rate to revert to its equilibrium value.[20] In addition, we have tested for nonlinearity using $\Delta q_{i,t-1}$, as the threshold variable. This specifications is also attractive, since it allows the adjustment speed to vary whether the real exchange rate appreciates (when $\Delta q_{i,t-1}$ is below a threshold, c) or depreciates (when $\Delta q_{i,t-1}$ is above c).

The results are summed up in Tables 2 and 3. They show that, when the past misalignment is used as the threshold variable (Table 2), linearity is strongly rejected for all groups of countries, except for the panel composed of industrialized countries alone (namely the G-7 countries), where linearity seems to be a pattern. Therefore, we estimated the corresponding panel smooth transition regression models for the G-20, the emerging markets (non G-7 countries), Asian emerging markets (Asia) and countries having overcome a recent financial crisis (crisis).

Table 2: **PSTR model with $z_{i,t-1}$ as the threshold variable**

| | Regime 1 | | Regime 2 | | | Transition | |
	θ	T-stat	θ^*	T-stat	$\theta + \theta^*$	γ	c
G-20	-0.031	-0.54	-0.214	-2.16	-0.245	17.461	-0.143
G-7	Linear						
Non G-7	0.024	0.404	-0.279	-2.06	-0.255	18.013	-0.092
Asia	0.037	0.63	-0.367	-2.90	-0.330	41.846	-0.018
Crisis	0.097	1.01	-0.337	-1.86	-0.240	16.955	-0.112

Notes: Model chosen according to BIC and the lowest p-value in the linear tests.

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[19]As mentioned in the previous section, the coefficient of the lagged endogenous term was not significant. That is why, we have dropped $\Delta q_{i,t-1}$ from our final specification, which allows us to apply the PSTR methodology since our model does not contain any dynamic component.

[20]We have discriminated between logistic and logistic quadratic panel smooth transition functions according to two criteria: we selected first those with the lowest p-value in the linear test and then selected the one that exhibited the lowest Schwarz information criterion (BIC).

Table 3: **PSTR model with $\Delta q_{i,t-1}$ as the threshold variable**

	Regime 1		Regime 2			Transition	
	θ	T-stat	θ^*	T-stat	$\theta + \theta^*$	γ	c
G-20	Linear						
G-7	1.029	3.77	-1.280	-4.24	-0.251	27.555	-0.145
Non G-7	Linear						
Asia	Linear						
Crisis	Linear						

Notes: Model chosen according to BIC and the lowest p-value in the linear tests.

The main parameters of interest here are the error correction coefficients in the two extreme regimes θ and $\theta + \theta^*$, the threshold parameter c and the speed of transition γ. [21] Regarding the results for the G-20, the threshold estimate is -0.143 (corresponding to an over-valuation of 14%) which is the lower band below which deviations from the real exchange rate equilibrium level (i.e. when $g(q_{i,t}; \gamma, c)=0$) are not corrected. Note that θ is not significant in the first regime, which means that there is no convergence process towards the BEER value for the real exchange rate in t when the over-valuation exceeds 14 pp. However, once the misalignment crosses this threshold, there is a strong tendency of the real exchange rate to go back to its equilibrium value ($\theta + \theta^*$ is significant and strongly negative in the second regime).

This result can be understood as a confirmation of the asymmetric property of the real exchange rate's adjustment towards equilibrium. Indeed, as the distribution of the threshold variable confirms (see Figures 1 and 2 in Appendix B), even if the threshold c is not fixed at 0[22], most of the points that are above the threshold are positive figures (i.e. there are more points above 0 than between the threshold and 0). This implies that the adjustment process is more effective in case of an undervaluation than when an overvaluation occurs. This result is particularly true for emerging economies and developing

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[21]In most of the cases, the logistic transition function shows better properties than the logistic quadratic one. This implies that the predominant type of asymmetry is that which distinguishes between positive or negative deviations from equilibrium. In other words, the short-term adjustment that occurs, being nonlinear, corrects deviations from the equilibrium positions by giving more weight to the sign of the deviations - whether it is an over-valuation or an undervaluation of the currency i - than to their magnitude. Our results are then based on these models.

[22]Recall that a negative (resp. positive) value for $z_{i,t-1}$ corresponds to an over (resp. under) valuation of the real exchange rate.

Asia sub-samples, with threshold variables estimated at −0.092 and −0.018 respectively. This is consistent with the fact that emerging countries' currencies, and especially those of China, Indonesia and India, appear rather undervalued (see Bénassy-Quéré et al. (2008a)). One may expect that the exact opposite applies for industrialized countries' exchange rates that are mainly characterized by over-valuations. Indeed, it would be expected a quicker adjustment in case of an over-valuation for G-7 countries since those over-valuations are somewhat the counterpart of developing economies currencies' under-valuations. But as we deal with effective misalignements, this analysis holds only if the weights of developing economies are equivalent to those of industrialized countries in our effective variable calculations. As revealed from Table 5 in Appendix A, this is not the case since developing economies' weights represent less than 30% in the calculation of G-7 countries' effective misalignments. In addition, we have already mentioned that the long-run real exchange rate dynamics for industrialized countries is rather characterized by a linear pattern. This may be due to the fact that the observed effective misalignments for industrialized countries are in absolute value of lesser magnitude than those observed for emerging economies (i.e., 9.74% versus 14.39% respectively, see Table 5 in Appendix A for more details).

It is important to notice that the convergence process in the nonlinear model is more pronounced than that in the linear specification, with a 24% of the adjustment taking place within a year corresponding to a half-life of 3.2 years versus 4.8 in the linear estimation for the G-20. In the other subgroups, the adjustment is even quicker both with respect to the nonlinear G-20 specification and to the figures obtained in the linear models. The correction is particularly crucial below an appreciation of 2% in emerging Asia (reaching 33% within a year, which corresponds to a half-life of 2.4 years versus 5.7 years in the linear estimation).

Figures 3, 4, 5, 6 and 7 report the values of both the threshold variable and the transition function against time for each country belonging to our different sub-panels. For all the considered groups, the movements of the disequilibrium error above (below) zero are associated with undervalued (overvalued) real exchange rate. As it can be noticed, undervaluations are corrected faster than over-valuations, confirming our former conclusions. Besides, the transition function changes from the lower to the higher regime quite often. As a result, the transition function is, indeed, smooth and we can observe several observations in each side of the threshold, with a relatively higher presence of observations above the threshold.

However, the case of the advanced economies alone is completely different from the rest of the panel. Indeed, the first interesting feature in this group is that linearity is not rejected when the previous misalignment is used as threshold variable (Table 2). Therefore, in industrialized countries, reversion to equilibrium is a characteristic that happens regardless of the size of the deviation from equilibrium, confirming previous studies in time series (see López Villavicencio (2008) among others). Second, when the selected transition variable is the real exchange rate variation ($\Delta q_{i,t-1}$), linearity cannot be rejected in any of the other panels but the G-7 (Table 3). For those groups of countries it is more past misalignments that matter than the magnitude of exchange rate variations.

The estimated parameters of the nonlinear model for the G-7 can be found in Table 3. As observed, reversion is much faster in the nonlinear model above a depreciation of 14% than in the linear specification, with associated half-lives of 3.1 and 5.6 years respectively. Yet, as observed on Figure 4, this acceleration has only been the case in Japan between 1987-88 and the euro zone in 1987. Therefore, the consistency of our results with respect to the nonlinear behavior in the short-run adjustment model seems to depend critically on the presence of just a few observations.[23] As expected, this is reflected in the transition function showing most of the observations to the right of the location parameter, where reversion to equilibrium is higher.

We also checked the linearity of the adjustment process with net foreign asset and current account gaps as threshold variables (see Section 2 for the construction of data.). Indeed, it could have been reasonable to think that, as the BEER corresponds to an exchange rate level consistent with the net foreign asset position being at an equilibrium value (characterized by $\overline{nfa}_{i,t}$), the adjustment speed would be fastened if the gap between the current and the equilibrium values had gone beyond a certain threshold. The same explanation holds for the current account gap, the stabilization of the stock implying that of the flow. However, our results show that linearity cannot be rejected in most of the cases or at least, lead to irrelevant results. [24] This implies that, whatever the distance between the observed values of the current account or the net foreign asset position and their respective long-run targets, the adjustment speed of the real exchange rate towards its equilibrium long-run value will remain the same. In other words, the adjustment process of the real exchange rate is not sensitive to the magnitude of the current account or net foreign asset imbalances. Our findings then corroborate those of Bénassy-Quéré

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[23]In order to check this, we eliminate Japan and in the euro zone from this group and proceed to linearity tests. The results confirm our intuitions since the null of linearity is not rejected.

[24]All results are available upon request to the authors.

et al. (2008a) showing that the real exchange rate may probably not be the key of global imbalances' unwinding.

5   Conclusion

In this paper, we have studied the nonlinear convergence process of the real exchange rate towards its equilibrium BEER value using a Panel Smooth Transition Regression model framework. We have shown that the real exchange rate dynamics in the long run is proved to be nonlinear for emerging economies, whereas industrialized countries exhibit a linear pattern, confirming previous studies in time series (see López Villavicencio (2008) among others). More especially, there exists an asymmetric behavior of the real exchange rate when facing an over- or an undervaluation of the domestic currency. The adjustment speed appears drastically accelerated in case of an undervaluation, which is consistent with the fact that developing economies and especially emerging Asian countries are more inclined to exhibit undervalued currencies. The converse does not hold for industrialized countries which mainly face over-valuations of their currencies. Two reasons may explain this difference between industrialized and emerging countries. First, the weight of emerging countries in the effective misalignments is relatively weak, implying that behaviors of those two sub-groups are rather disconnected. Second, misalignments in absolute values are of lesser magnitude for developed countries, i.e. the convergence process towards equilibrium is linear for industrialized countries because misalignments are more homogeneous. Another conclusion of our findings is that the convergence process towards the long-run equilibrium is independent from the magnitude of the current account or the net foreign asset imbalances, which confirms that the real exchange rate is probably not the key of global imbalances' unwinding as suggested by Bénassy-Quéré et al. (2008a).

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A Tables

Table 4: **Country samples**

G-20	Argentina (ARG), Australia (AUS), Brazil (BRA), Canada (CAN), China (CHN), United Kingdom (GBR), Indonesia (IDN), India (IND), Japan (JPN), Korea (KOR), Mexico (MEX), Turkey (TUR), United States (USA), South Africa (ZAF), and Euro area (ZZM)
G-7	Australia (AUS), Canada (CAN), United Kingdom (GBR), Japan (JPN), United States (USA), and Euro area (ZZM)
Non G-7	Argentina (ARG), Brazil (BRA), China (CHN), Indonesia (IDN), India (IND), Korea (KOR), Mexico (MEX), Turkey (TUR), and South Africa (ZAF)
Asia	China (CHN), Indonesia (IDN), India (IND), and Korea (KOR)
Crisis	Argentina (ARG), Brazil (BRA), Indonesia (IDN), Korea (KOR), Mexico (MEX), and Turkey (TUR)

Table 5: **Further details on calculated effective misalignements**

Country	G-7	Non G-7
ARG	43.35	56.65
AUS	62.53	37.47
BRA	66.34	33.66
CAN	89.73	10.27
CHN	77.36	22.64
GBR	87.11	12.89
IDN	67.03	32.97
IND	69.27	30.73
JPN	54.38	45.62
KOR	62.59	37.41
MEX	90.48	9.52
TUR	84.96	15.04
USA	60.32	39.68
ZAF	80.35	19.65
ZZM	66.93	33.07
G-7	**70.82**	**29.83**
Non G-7	**71.30**	**28.70**

(a) G-7 and non G-7 trade weights for each G-20 country (in %)

Country	Mean	Median
ARG	18.35	15.22
AUS	7.76	7.68
BRA	10.55	9.14
CAN	6.03	4.67
CHN	26.88	23.63
GBR	11.51	12.84
IDN	16.26	12.79
IND	17.20	16.59
JPN	17.11	14.56
KOR	12.10	11.98
MEX	12.19	12.26
TUR	6.13	5.01
USA	7.20	5.91
ZAF	9.60	9.01
ZZM	8.82	10.05
G-7	**9.74**	**8.11**
Non G-7	**14.39**	**11.92**

(b) Average and median misalignments (absolute values, in %)

B   Graphs

Figure 1: **Kernel density estimate of** $z_{i,t-1}$

G-20

Emerging economies
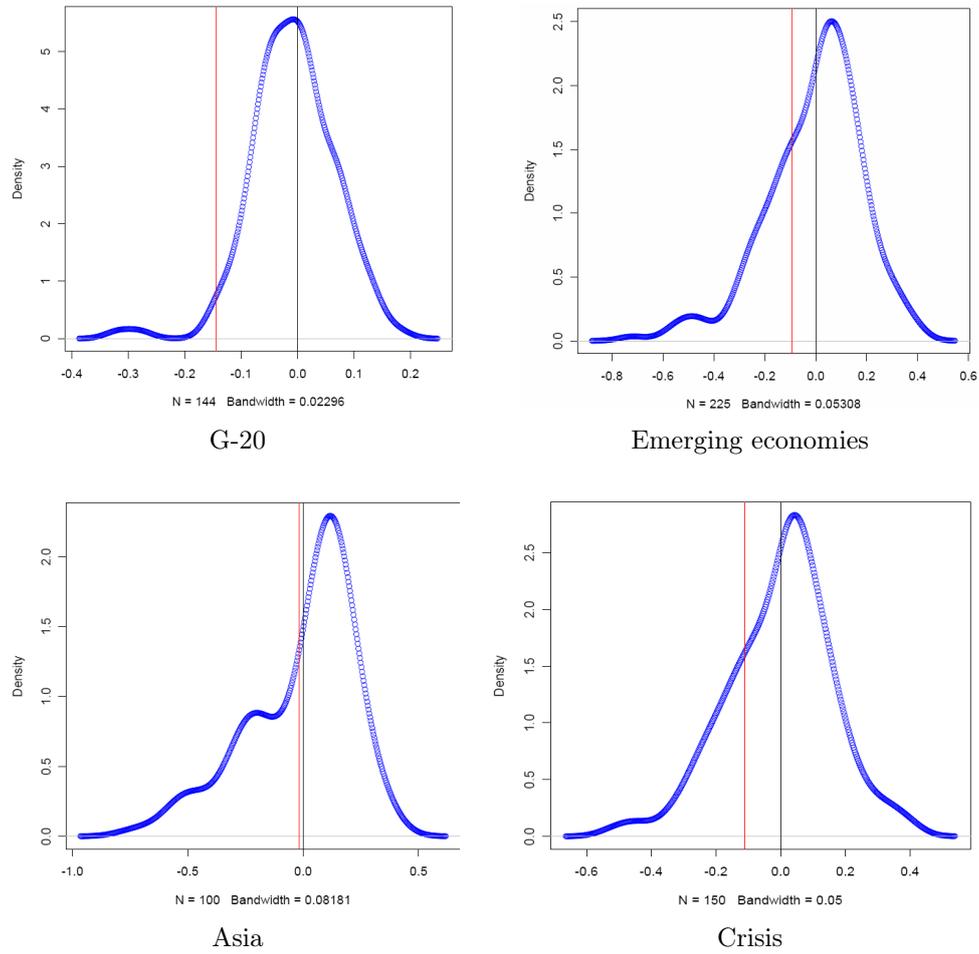
Asia

Crisis

Figure 2: **Kernel density estimate of** $\Delta q_{i,t-1}$
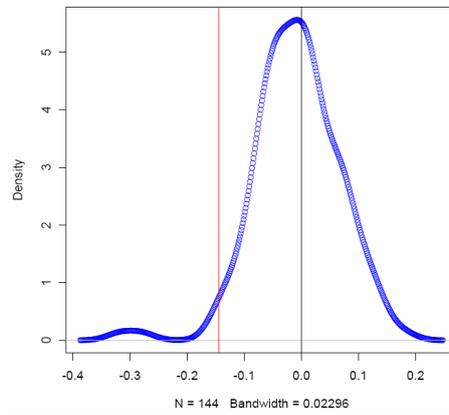
G-7

Figure 3: **G-20**
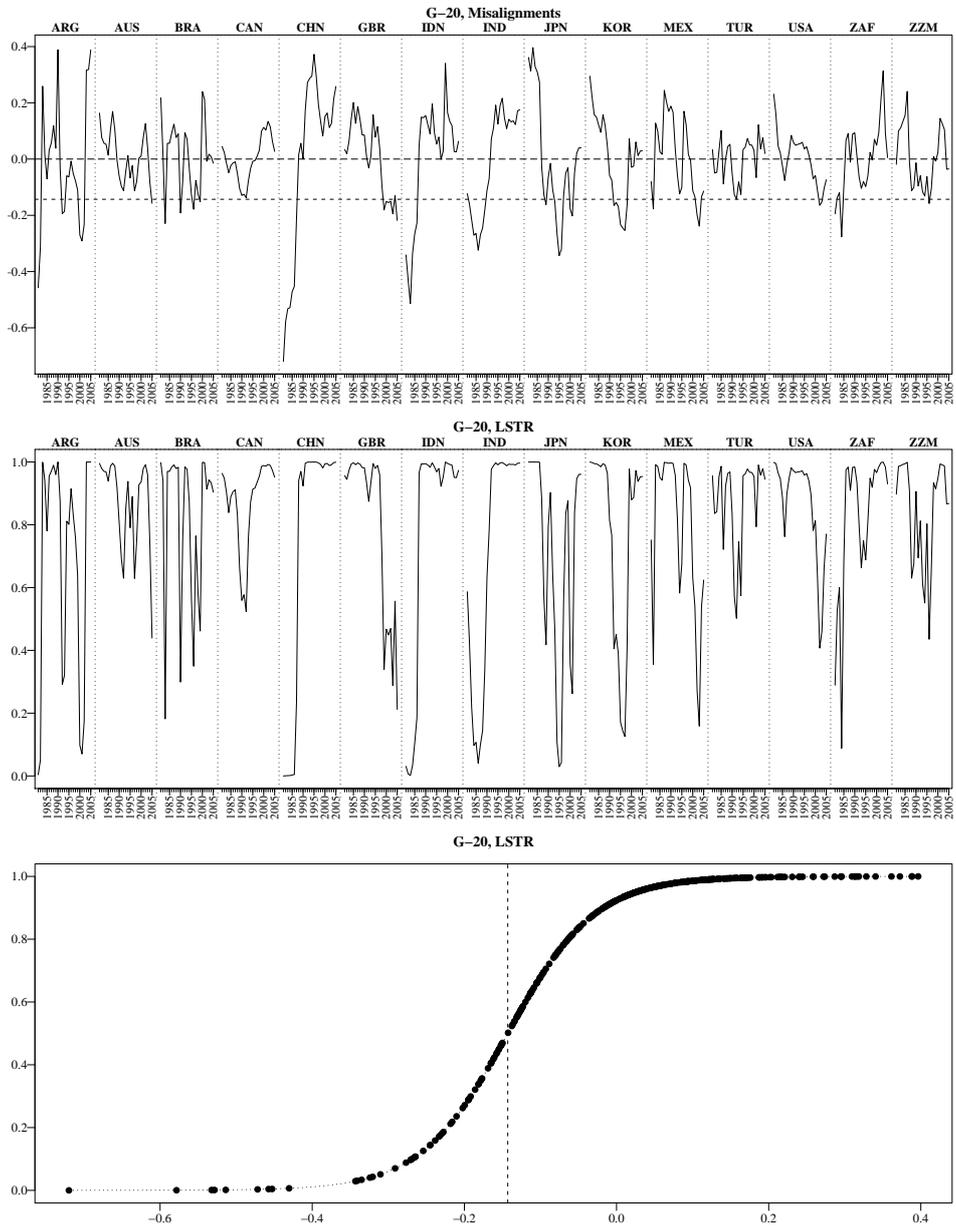
Figure 4: **G-7**
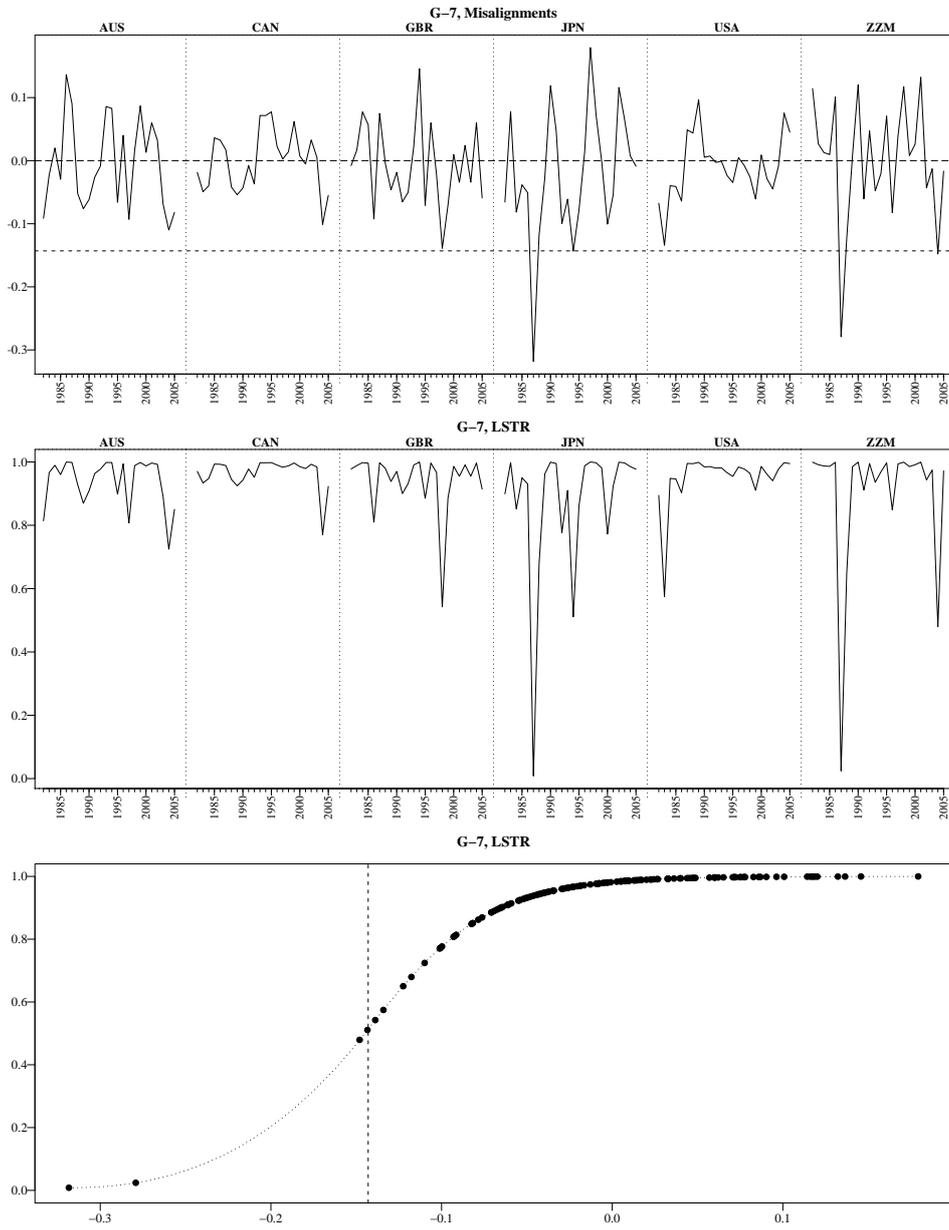
Figure 5: **Emerging economies (Non G-7)**
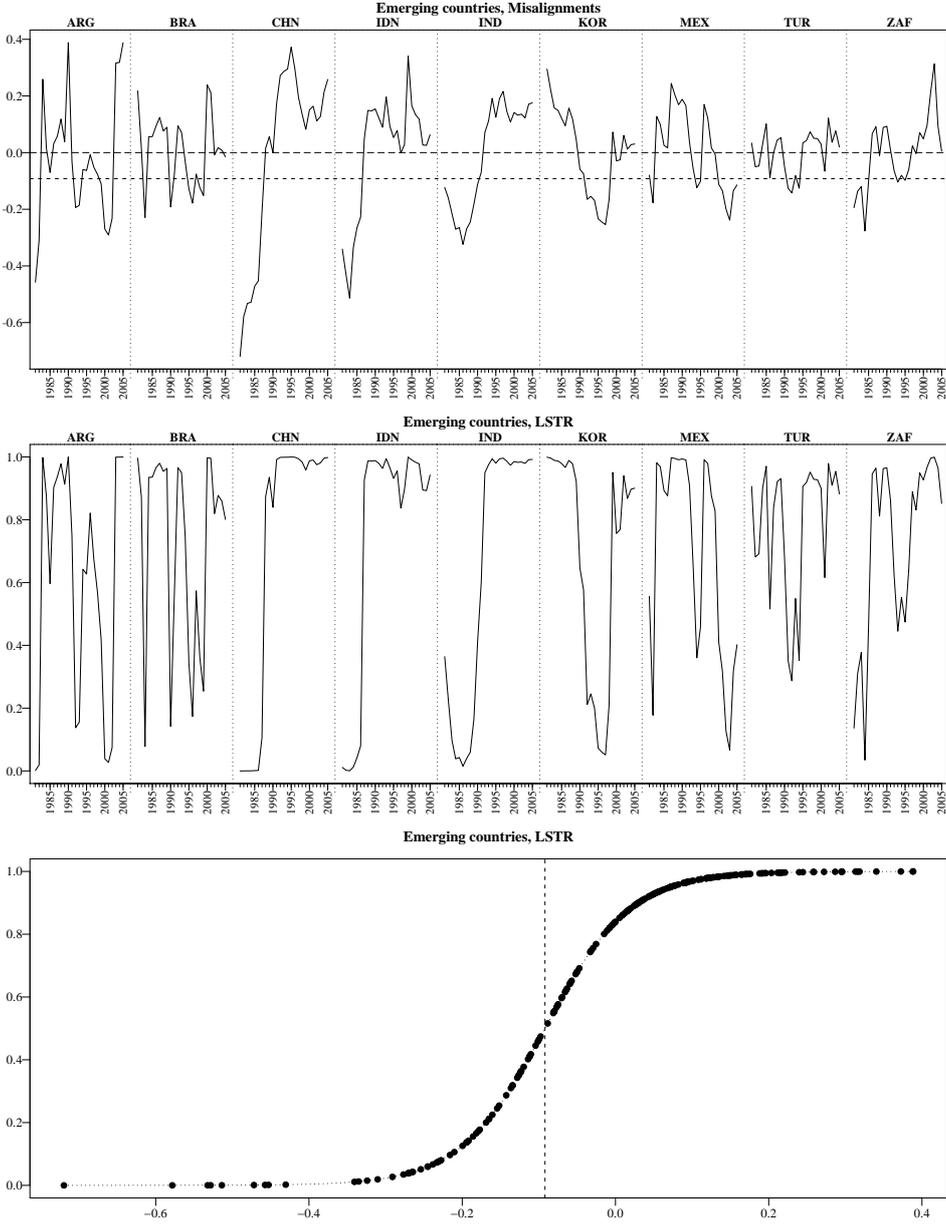
Figure 6: **Developing Asia**
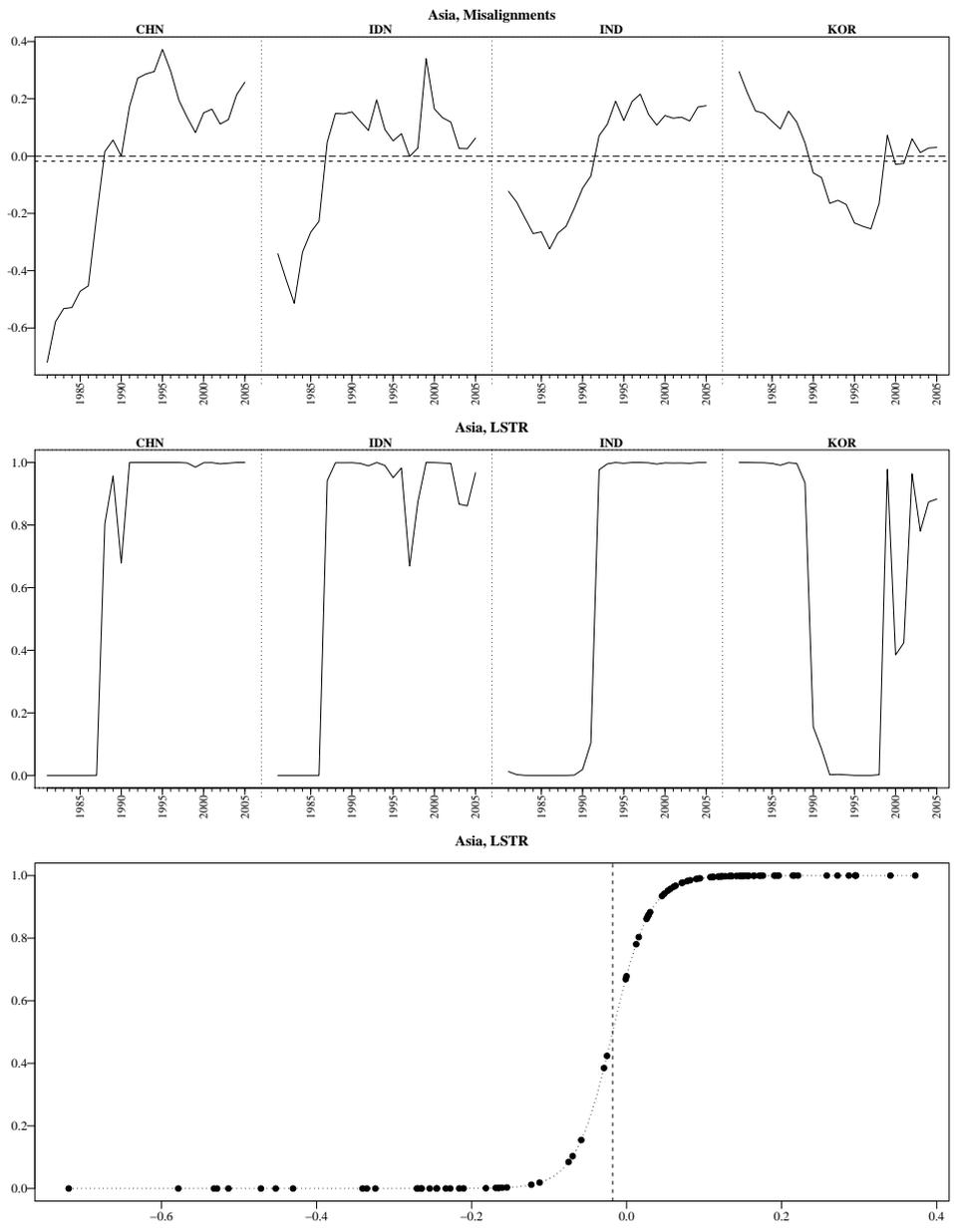
Figure 7: **Countries having overcome a recent financial crisis**