## The Dynamic Properties of Alternative Assumptions on Price Adjustment in Fixed Prices and Predetermined Prices Models

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#### Abstract

We present a classification of the different new Phillips curves existing in the literature as a set of choices over three assumptions: the choice of the structure of price adjustments (Calvo or Taylor), the presence of backward indexation, and the type of price contracts (fixed prices or predetermined prices). We study the dynamic properties of each specification, following different monetary shocks on the growth rate of the money stock. We develop the analytical form of the price dynamics, and we display graphics for the responses of prices, output, and inflation. We show that the choice made for each of the three assumptions has a strong influence on the dynamic properties. Notably, the choice of the price structure, while often considered as unimportant, is indeed the most influential choice concerning dynamic properties of these models.

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## 1 Introduction

One of the central questions in the recent literature concerning inflation dynamics has been the ability of the New Keynesian Phillips curve (NKPC thereafter) to correctly reproduce the empirical impact of monetary shocks. This specification is attractive for several reasons: its simplicity, its tractability and the existence of microfoundations. Moreover, as shown by Roberts (1995), the different assumptions on price adjustment existing in the literature (Taylor, 1980, Rotemberg, 1982, and Calvo, 1983) lead to the same structural relation between inflation and output. For these reasons, this specification, based on the hypothesis of fixed prices (FP thereafter), has been for some years something close to a "standard specification" (McCallum, 1997). However, since the work of Ball (1994a) and Fuhrer and Moore (1995), the empirical plausibility of the NKPC has been strongly questioned. The main reason is linked to the forward-looking dynamics of the NKPC, which implies that inflation behaves like a "jump-variable". As a consequence, this model predicts an absence of inflation persistence, and a real neutrality of disinflation policies, while in reality, inflation is very persistent and disinflations are costly<sup>1</sup>.

One way to add some empirical accordance to the NKPC is to introduce lagged inflation in the dynamics, for example via the existence of backward-looking agents (Gali and Gertler, 1999), or with a backward-looking indexation mechanism (Woodford, 2003, Christiano, Eichenbaum and Evans, 2005). Recently, these "hybrid" Phillips curves, combining forward and backward inflation terms, have replaced the forward-looking NKPC as the new standard specification. The theoretical foundations of the introduction of this lagged inflation term are unclear, but it is often considered as a necessary condition to reproduce plausible inflation dynamics.

However, some authors, like Ball, Mankiw and Reis (2005) criticize all models built on the FP hypothesis. Deploring the current "sorry state" of the literature, these authors say that forward-looking New Keynesian models are at odds with the facts, and that hybrid models are even worse. In response to the apparent failures, they propose to replace the assumption of FP by the one of sticky information (Mankiw and Reis, 2002), which formally is a resurgence of the Fischer (1977) hypothesis of predetermined prices (PP thereafter). They show that this alternative outperforms the hypothesis of FP. Notably, predetermination of prices prevent inflation to jump immediately after shocks.

However, the superior performance of PP models has been questioned by some recent papers. Analyzing the responses of FP and PP models built on the structure of price adjustment of Calvo, some authors conclude that the PP model of Mankiw and Reis displays indeed a performance similar to the forward-looking NKPC (Devereux and Yetman, 2003) and to the "hybrid" Phillips curve (Trabandt, 2005). Woodford (2003) also uses forward and "hybrid" versions of the NKPC and find that these specifications correctly fit the facts. Collard and Dellas (2003), and Dupor and Tsuruga (2005) show that leaving the price structure of Calvo significantly lowers the performance of the PP model.

Given the importance of the choice of the structural form of the economy for the recommendations of monetary policy, the actual controversy and the presence of many contradicting declarations is embarrassing. Despite the apparent similarities of these models, their predictions in terms of responses to monetary shocks are quite different and their is no agreement on their relative performances. One reason that could explain these contrasted

<sup>&</sup>lt;sup>1</sup>Critics about the inflation dynamics implied by the NKPC are extensive in the literature (see among others Roberts, 1998, Walsh, 1998, Mankiw, 2001, Mankiw and Reis, 2002, Rudd and Wheelan, 2005).

results is the absence in the literature of an exhaustive synthesis summarizing the dynamics properties of these different models. Some comparative works exist, but either they do not compare models within a common framework (Nelson, 1998, Walsh, 1998, Jondeau and LeBihan, 2001), giving hardly comparable results, or the comparison only takes in account a limited set of alternatives<sup>2</sup>(Jeanne, 1998, Pereau, 2001, Mankiw and Reis, 2002, Devereux and Yetman, 2003, Trabandt, 2005). Another reason is the frequent use of simulations methods (for example Dupor and Tsuruga, 2005, or De Walque, Smets and Wouters, 2005). The absence of analytical results do not clearly highlights the properties of each hypothesis made on price adjustment.

In this paper, we try to evaluate the relative performance of the most important Phillips curves of the recent literature, within a common framework, for different money shocks We present both an analytical representation of the price dynamics (given the hypothesis made on the formation of expectations), and graphical illustrations. This permits to understand more clearly the dynamic properties of the different Phillips curves.

To do this comparative work, we adopt a classification of the different hypothesis presented in the literature around three points: the first is the choice of the nature of the nominal rigidity (either PP or FP), the second is the choice the price adjustment rule (either Calvo or Taylor), and the third is the presence or the absence of indexation. This permits to obtain comparable Phillips curves and to attribute the differences of response between each specification exclusively to the assumptions made on the pricing rules. The other elements of the economy are voluntarily simplified in order to highlight the properties of the price equation.

In addition to the survey, our work permits to show the following elements:

a) The choice of the price structure (Calvo or Taylor) has very important implications for price and output dynamic. The influential paper of Roberts (1995) concluding to the unimportance of the specific type of price rigidity has been misleading. For example, in the forward-looking FP models, the structure of Taylor implies structural expectations errors and then a positive cost of disinflation absent from the structure of Calvo.

b) Except for the case of the disinflation, the introduction of indexation does not by itself add significantly more persistence. The choice of the price structure (Calvo or Taylor) is more important. Said differently, hybrid sticky-prices models have very different dynamic properties concerning persistence.

c) Contrary to the affirmation of Ball, Mankiw and Reis (2005), the hybrid sticky price model built on the structure of Calvo produces plausible responses to all the shocks we consider. The assumption of PP needs the presence of the Calvo pricing rule to generates sufficient persistence.

This paper is organized as follows. In section 2, we present the derivation of the different Phillips curves. In section 3, we present the responses of the model to an auto-correlated shock on money growth. In section 4, we study the dynamic properties of each model consecutively to a disinflation policy. In section 5, we study the case of a pre-announced disinflation. In section 6, we compare our results with those of important papers in the literature. In Section 7, we conclude.

 $<sup>^{2}</sup>$ Most of the time, these papers focus on only scheme of price rigidity (Calvo or Taylor). The justification for this limitation comes from a general feeling of homogeneity in the reduced form of the models using different hypotheses of price rigidity (Roberts, 1995).

# 2 PRESENTATION OF THE PHILLIPS CURVES

#### 2.1 The common set-up

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We follow a standard two-step procedure. In the first step, we present the level at which prices would be set if they were entirely flexible. We note  $p_t^*$  as the optimal price of a firm during period t (all firms are identical). Following Romer (2001) or Kiley (2002a), in a monopolistic competition setup, absent wage rigidites, this price is given by:

$$p_t^* = p_t + \phi y_t \tag{1}$$

where  $p_t$  is the overall price level,  $y_t$  is the output gap and  $\phi$  is a measure of real rigidities. All variables are in logarithms.

In the second step we introduce nominal rigidities, with the standard assumption that firms can take new decisions on prices only when they receive signals of price changes. Each time a firm receives a signal, she sets a entire path of future prices. As signals of price changes are infrequent, we note  $\lambda_j$  the expected probability of having a new signal of price change j periods after setting the price path. We note  $x_{t,t+j}$  is the price set at time t for the period t + j. The objective of the firm setting new prices is to minimize the sum of the differences between  $x_{t,t+j}$  and  $p_{t+j}^*$ . We also assume that prices can be indexed to past inflation (Woodford, 2003). We note  $\gamma$  the degree of indexation. Up to a linear approximation, a firm having a signal of price change during period t tries to minimize the following loss function<sup>3</sup>:

$$\underset{x_{t,t+j}}{Min} L_t = (x_{t,t} - p_t^*)^2 + \sum_{j=1}^{\infty} (1 - \lambda_j)^j E_t \left( x_{t,t+j} - p_{t+j}^* + \gamma \sum_{i=0}^{j-1} \pi_{t+i} \right)^2$$
(2)

Future price gaps are weighted by the probability of not having a new price signal until this date.

Most of the models presented in the literature can be seen as imposing some restrictions on three elements of equation (2):  $\lambda_j$ ,  $x_{t,t+i}$  and  $\gamma$ .

## 2.2 The choice of the structure of price adjustments $(\lambda_j)$

The first distinction is about the choice of the rule governing price path adjustments. Each time a firm receives a new signal of price change, she can immediately set a new path of prices. We assume that the average expected duration between two signals is equal to N periods of the model (i.e. a firm changing its price path at the beginning of period t should have on average a new signal at the beginning of period t + N). Two assumptions have retained much attention in the literature. The first builds on the staggered prices structure of Taylor (1980). In this specification, each price lasts exactly N periods, with an individual probability of price change of 1 every N periods and 0 otherwise. Firms in the economy are divided between N cohorts of equal size, each cohort being differentiated by the dates of its price changes. The alternative, which is more used, builds on the partial adjustment structure of Calvo (1983). Each period a firm has a constant probability equal to 1/N to change its price, this probability being independent of the date of the last price change. The possible values of  $\lambda_i$  are given in Table 1.

<sup>&</sup>lt;sup>3</sup>The discount rate is equal to one. Its introduction is straightforward but unimportant for our analysis.

$\begin{array}{c c} CALVO & j \in [1; +\infty[  \Rightarrow \lambda_j = \frac{1}{N} \\ \hline TAYLOR & \begin{cases} j \in [1; N-1[  \Rightarrow \lambda_j = 0 \\ j = N & \Rightarrow \lambda_j = 1 \end{cases} \end{cases}$	Structure	Probabilities of pric	e signal
$TAYLOR \left\{ \begin{array}{c} j \in [1; N-1[ \implies \lambda_j = 0] \\ j = N \implies \lambda_i = 1 \end{array} \right\}$	CALVO	$j \in [1; +\infty[$	$\Rightarrow \lambda_j = \frac{1}{N}$
	TAYLOR	$\begin{cases} j \in [1; N - 1[\\ j = N \end{cases}$	$ \begin{array}{c} \Rightarrow \lambda_j = 0 \\ \Rightarrow \lambda_j = 1 \end{array} $

TAB. 1. Assumptions on the arrival of price signals

## 2.3 The nature of price rigidity $(x_{t,t+j})$

The second distinction concerns the restrictions imposed on the value of the prices set for each period of the price contract. We call a price contract the vector of prices set between two adjustments of the price path. In the literature, the prices specified in price contracts have taken two forms: fixity or predetermination. Following Blanchard and Fischer (1989), a price is predetermined for t to t + j if its path from t to t + j is predetermined as of time t. A price is fixed if it is predetermined and constant during that length of time. This is equivalent to impose  $x_{t,t+j} = x_t$  during all the duration of the contract.

## 2.4 The degree of indexation $(\gamma)$

Some authors have introduced a mechanism of indexation of the price set  $x_t$  to lagged inflation. Each period, a fraction  $\gamma$  of the inflation rate observed during the last period can be added to the value of the price if there is no new signal of price change. Several values of  $\gamma$  are considered in the literature. Christiano, Eichenbaum and Evans (2005) assume that indexation is complete ( $\gamma = 1$ ), and Woodford (2003) uses partial indexation.

The motivation for indexation in FP models (apart from raising the degree of persistence) is that for a positive trend inflation, there is a growing gap between the optimal price and the price effectively set, which is inefficient. However, as under PP, all expected inflation is integrated in the price path, this scheme of indexation is of little interest<sup>4</sup>. Consequently, we shall only study the impact of this kind of indexation in FP models. We shall also restrict our analysis to the polar cases of full indexation (noted I thereafter) and no indexation (noted FL, for forward-looking, thereafter).

### 2.5 Presentation of inflation dynamics

Combining the different assumptions, it is possible to derive the most important Phillips curves discussed in the recent literature.

#### 2.5.1 Fixed prices in the model of Calvo

First, we consider the most popular combinations: the FP version of the model of Calvo (with and without indexation). The loss functions reduces to the following forms (respectively for

<sup>&</sup>lt;sup>4</sup>It is possible to imagine an indexation rule that would introduce a correction mechanism if realized inflation differs from expected inflation, but this issue is beyond the scope of our paper.

the forward-looking model, then the model with full indexation):

$$\begin{aligned}
&M_{x_t} L_t = (x_t - p_t^*)^2 + \sum_{j=1}^{\infty} \left(\frac{N-1}{N}\right)^j E_t \left(x_t - p_{t+j}^*\right)^2 \\
&M_{x_t} L_t = (x_t - p_t^*)^2 + \sum_{j=1}^{\infty} \left(\frac{N-1}{N}\right)^j E_t \left(x_t - p_{t+j}^* + \sum_{i=0}^{j-1} \pi_{t+i}\right)^2
\end{aligned}$$

FOC give the following optimal prices (respectively with, then without indexation):

$$x_t = \left(\frac{1}{N}\right) \sum_{j=0}^{\infty} \left(\frac{N-1}{N}\right)^j E_t p_{t+j}^* \tag{3}$$

$$x_t = \left(\frac{1}{N}\right) \left[ \left(\sum_{j=0}^{\infty} \left(\frac{N-1}{N}\right)^j E_t p_{t+j}^*\right) - E_t \left(\sum_{j=0}^{\infty} \left(\frac{N-1}{N}\right)^{j+1} \sum_{i=0}^j \pi_{t+i}\right) \right]$$
(4)

Up to an approximation, the aggregate price level is a weighted average of the value of prices coexisting at time t, given that a fraction (1/N) of the prices are modified each period:

$$p_t = \left(\frac{1}{N}\right) x_t + \left(\frac{N-1}{N}\right) p_{t-1} \tag{5}$$

$$p_t = \left(\frac{1}{N}\right)x_t + \left(\frac{N-1}{N}\right)\left(p_{t-1} + \pi_{t-1}\right) \tag{6}$$

Given these equations and the value of  $p_{t+i}^*$  (equation 1), it is possible to derive a Phillips curve linking current inflation to its past and expected values, and the output gap:

$$\pi_t = E_t \pi_{t+1} + \frac{\phi}{N(N-1)} y_t$$
(7)

$$\pi_t = \frac{1}{2} \left( \pi_{t-1} + E_t \pi_{t+1} \right) + \frac{\phi}{2N \left( N - 1 \right)} y_t \tag{8}$$

Equation (7) corresponds to the widely used forward-looking Phillips curve of Calvo, and equation (8) represents a simplified version of the hybrid Phillips curve presented by Christiano, Eichenbaum and Evans (2005).

#### 2.5.2 Fixed prices in the model of Taylor

We consider now the structure of price adjustments of Taylor, under the hypothesis of FP. Given the probabilities of price adjustment, the loss functions reduce to the following forms (respectively for the FL version, then with full indexation):

$$\begin{aligned}
&M_{in} L_t = (x_t - p_t^*)^2 + \sum_{j=1}^{N-1} E_t \left( x_t - p_{t+j}^* \right)^2 \\
&M_{in} L_t = (x_t - p_t^*)^2 + \sum_{j=1}^{N-1} E_t \left( x_t - p_{t+j}^* + \sum_{i=0}^{j-1} \pi_{t+i} \right)^2
\end{aligned}$$

The FOC respectively give the following optimal prices:

$$x_{t} = \left(\frac{1}{N}\right) \sum_{j=0}^{N-1} E_{t} p_{t+j}^{*}$$
(9)

$$x_{t} = \left(\frac{1}{N}\right) \left[ \left(\sum_{j=0}^{N-1} E_{t} p_{t+j}^{*}\right) - E_{t} \left(\sum_{j=0}^{N-2} \sum_{i=0}^{j} \pi_{t+i}\right) \right]$$
(10)

The price level is again approximated by an average of the value of existing prices:

$$p_t = \frac{1}{N} \sum_{j=0}^{N-1} (x_{t-j})$$
(11)

$$p_t = \frac{1}{N} \left[ \sum_{i=0}^{N-1} x_{t-j} + \sum_{j=1}^{N-1} \sum_{i=1}^{N-j} \pi_{t-i} \right]$$
(12)

Given these equations and the value of  $p_{t+i}^*$ , it is possible to obtain a Phillips curve. However, while with the structure of Calvo, the form of the Phillips curve does not depend on the assumption made on N, the average length of contracts, this is not the case under the structure of Taylor. In most of the literature, 2-periods contracts are considered. For numerical applications, we use a semi-annual model, in order to match a length of contracts equal to one year, which is a standard and realistic assumption (see Taylor, 1999). For N = 2, the differences between the Phillips curves resulting from the structure of Taylor and Calvo are minimized. This gives the following Phillips curves (respectively for the FL version, then with full indexation):

$$\pi_t = \frac{1}{2} \left( E_{t-1} \pi_t + E_t \pi_{t+1} \right) + \frac{\phi}{2} \widetilde{y}_t$$
(13)

$$\pi_t = \frac{1}{3} \left( \pi_{t-1} + E_{t-1} \pi_t + E_t \pi_{t+1} \right) + \frac{2\phi}{3} \widetilde{y}_t$$
(14)

where  $\tilde{y}_t = y_t + y_{t-1} + E_{t-1}y_t + E_t y_{t+1}$ .

Equation (13) corresponds to the Phillips curve of the model of Taylor, and equation (14) to the Phillips curve of the model of Fuhrer and Moore. This presentation of inflation dynamics is important because it differs from the well-known presentations of the same models made by Roberts (1995), Fuhrer and Moore (1995) or Walsh (1998). In those papers, equations (13) and (14) are presented with the same form as equations (7) and (8), representing their equivalent under the structure of Calvo, with the addition of a neglected expectation error. This error term is neglected in most papers and the choice concerning the structure of Calvo or Taylor is often considered as equivalent (Roberts, 1995, Mankiw, 2001). However, as Ben Aïssa and Musy (2006) and Musy (2006) have shown, this error term is crucial to properly understand the dynamic properties of the price structure of Taylor. This point will be clearly illustrated in the following sections.

One should also note that the foundations of the model of Fuhrer and Moore do not build on the hypothesis of indexation, but on a hypothesis of relative contracting. We show in the Appendice that for 2-periods contracts, the Phillips curve resulting from the assumption of indexation in the model of Taylor and the Phillips curve derived by Fuhrer and Moore are exactly equivalent. However, we prefer to use indexation because the foundations assumptions used by Fuhrer and Moore are hard to reconcile with microeconomic foundations (Holden and Driscoll, 2003)

#### 2.5.3 Predetermined Prices

As indicated below, we consider only the case without zero indexation. The loss functions of a firm setting predetermined prices are the following (respectively for the structures of Calvo and Taylor):

$$\begin{array}{lcl}
& \underset{x_{t,t},\ldots,x_{t,t+j}}{Min} L_t &= & E_t \sum_{j=0}^{\infty} \left(\frac{N-1}{N}\right)^j \left(x_{t,t+j} - p_{t+j}^*\right)^2 \\
& \underset{x_{t,t},\ldots,x_{t,t+j}}{Min} L_t &= & E_t \sum_{j=0}^{N-1} \left(x_{t,t+j} - p_{t+j}^*\right)^2
\end{array}$$

In both case, the optimal price sequence is the same:

$$x_{t,t+j} = E_t p_{t+j}^*$$

The price level is an average of current prices predetermined at different dates, that is, with different sets of information. The price levels are the following (respectively under the hypotheses of Calvo and Taylor):

$$p_t = \frac{1}{N} \sum_{j=0}^{\infty} \left(\frac{N-1}{N}\right)^j x_{t-j,t}$$
$$p_t = \frac{1}{N} \sum_{j=0}^{N-1} x_{t-j,t}$$

Even if the sequence of prices is identical in both cases, the different definitions of the price level leads to very different Phillips curves. To be in accordance with previous section, we assume 2-periods contracts for the price structure of Taylor:

$$\pi_t = \frac{1}{N} \sum_{j=0}^{\infty} \left( \frac{N-1}{N} \right)^j E_{t-1-j} \left( \pi_t + \phi \Delta y_t \right) + \frac{\phi}{N-1} y_t$$
(15)

$$\pi_t = \pi_t + E_{t-1}\pi_t - E_{t-2}\pi_{t-1} + \hat{y}_t \tag{16}$$

where  $\Delta y_t = y_t - y_{t-1}$  and  $\hat{y}_t = \phi y_t - \phi y_{t-1} + E_{t-1}y_t - E_{t-2}y_{t-1}$ . Equation (15) is equivalent to the Phillips curve derived by Mankiw and Reis, and equation (16) represents a Phillips curve closely related to the model of Fischer (1977). Contrary to the models with FP, current inflation do not include any expectations of future variables. The expectations terms are related only to past expectations of current variables. The number of expectation lags is equal to maximum length of contracts, that is, N under the structure of Taylor, and infinity under the structure of Calvo.

#### 2.6 Summary of the Phillips Curves

In Table 2, we summarize the different Phillips curves presented. We now assume 2-periods contracts for all specifications. We note C for the hypothesis of Calvo, T for Taylor, FL for the forward-looking version (i.e. zero indexation) and I for full indexation. For each combination, we indicate the main paper of the literature using these hypotheses, and the associated Phillips curve.

Hypotheses	Associated Model	Phillips Curve
C/FP/FL	Calvo (1983)	$\pi_t = E_t \pi_{t+1} + \frac{\phi}{2} y_t$
C/FP/I	Christiano, Eichenbaum and Evans (2005)	$\pi_{t} = \frac{1}{2} \left( E_{t} \pi_{t+1} + \pi_{t-1} \right) + \left( \frac{\phi}{4} \right) y_{t}$
T/FP/FL	Taylor $(1980)$	$\pi_{t} = \frac{1}{2} \left( E_{t-1} \pi_{t} + E_{t} \pi_{t+1} \right) + \phi \widetilde{y}_{t}$
T/FP/I	Fuhrer and Moore (1995)	$\pi_{t} = \frac{1}{3} \left( \pi_{t-1} + E_{t-1} \pi_{t} + E_{t} \pi_{t+1} \right) + \left( \frac{2\phi}{3} \right) \tilde{y}$
C/PP	Mankiw and Reis (2002)	$\pi_t = \frac{1}{2} \sum_{j=0}^{\infty} \left(\frac{1}{2}\right)^j E_{t-1-j} \left(\pi_t + \phi \Delta y_t\right) + \kappa_t$
T/PP	Fischer (1977)	$\pi_t = \pi_{t-1} + E_{t-1}\pi_t - E_{t-2}\pi_{t-1} + \hat{y}_t$

TAB.	2.	The	Phillips	Curves
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where  $\tilde{y}_t = y_{t-1} + E_{t-1}y_t + y_t + E_ty_{t+1}$  and  $\hat{y}_t = \phi y_t - \phi y_{t-1} + E_{t-1}y_t - E_{t-2}y_{t-1}$ . We assume a simple output function, which depends on the level of real balances:

$$y_t = m_t - p_t \tag{17}$$

The path of money is exogenous and we study different money shocks in the following sections. As our aim is to understand the implications of each assumption on price rigidity, the simplified economy we study has the virtue to derive all dynamics from the nominal rigidity<sup>5</sup>.

# 3 AN AUTO CORRELATED SHOCK ON MONEY GROWTH

#### 3.1 Presentation

We assume that the growth of the money stock follows an AR(1) process:  $\Delta m_t = \rho \Delta m_{t-1} + \varepsilon_t$ , where  $\Delta m_t \equiv m_t - m_{t-1}$ . The main criteria of evaluation in the literature is the presence of a delayed and gradual response of output and inflation to the monetary innovation (Mankiw, 2001, Mankiw and Reis, 2002). As stated by Kiley (2002b), the response of inflation should be more delayed than the response of output. Other elements of evaluation, such as the reproduction of the "acceleration phenomenon", are also used to evaluate the relative performances of models of inflation dynamics (Mankiw and Reis, 2002, and Trabandt, 2005).

#### 3.2 Price dynamics

Economy begins on its steady-state. The money shock occurs at the beginning of period t = 1. We assume that the initial value of m is equal to  $m_0$ . The price dynamics<sup>6</sup> consecutive to the shock are presented in Table 3.

<sup>&</sup>lt;sup>5</sup>Trabandt (2005) studies the implications of some alternative assumptions of price rigidity in dynamic stochastic general equilibrium models.

<sup>&</sup>lt;sup>6</sup>The resolution of the PP model under the structure of Calvo is given by Mankiw and Reis (2002). The Taylor model with PP is straightforward to resolve once the values of expectations are given. The resolution of FP models is obtained by the method of factorization (see Blanchard and Fischer, 1989, or Mankiw and Reis, 2002 for an application to the FL model of Calvo).

Hypotheses	Price Dynamics
Calvo/Fixed Prices Forward-looking	$p_t = \theta p_{t-1} + (1-\theta) m_t + \frac{(1-\theta)\theta\rho}{1-\theta\rho} \Delta m_t$
Calvo/Fixed Prices Indexation	$p_{t} = (\theta_{1} + \theta_{2}) p_{t-1} - \theta_{1} \theta_{2} p_{t-2} + \frac{B}{\theta_{3}-1} m_{t} + \frac{B\rho}{(\theta_{3}-1)(\theta_{3}-\rho)} \Delta m_{t}$
Taylor/Fixed Prices Forward-looking	$p_{t} = \theta p_{t-1} + \frac{(1-\theta)}{2} \left( m_{t} + m_{t-1} \right) + \frac{(1-\theta)(1+\theta)\rho}{4(1-\theta)} \left( \Delta m_{t} + \Delta m_{t-1} \right)$
Taylor/Fixed Prices Indexation	$p_{t} = (\theta_{1} + \theta_{2}) p_{t-1} - \theta_{1} \theta_{2} p_{t-2} + \frac{A}{\theta_{3}-1} (m_{t} + m_{t-1}) + \frac{A(1+\theta_{3})\rho}{2(\theta_{3}-1)(\theta_{3}-\rho)} (\Delta m_{t} + \Delta m_{t-1})$
Calvo/Predetermined Prices	$p_t = m_0 + \sum_{j=0}^{\infty} \left[ \frac{\phi(1-\lambda^{j+1})(1-\rho^{j+1})}{(1-\rho)[1-(1-\lambda^{j+1})(1-\phi)]} \right] \varepsilon_{t-j}$
Taylor/Predetermined Prices	$p_{t} = p_{t-1} + \frac{1}{1+\phi} \left[ \phi \Delta m_{t} + (1+\rho) \Delta m_{t-1} - \rho \Delta m_{t-2} \right]$

TAB. 3. Price dynamics (auto correlated shock on money growth)

with  $A = [4\phi/(1-2\phi)]$  and  $B = \phi/4$ .  $\theta$  is the stable root of forward-looking FP models. In models with indexation,  $\theta_1$  and  $\theta_2$  are two complex roots, with a modulus below 1, and  $\theta_3 > 1$  is a real root.

Before seeing graphical illustrations, one can see some properties of each assumption on price adjustement.

• In FP models: - Indexation introduces a second price lag, with a negative influence;

- The price structure of Taylor implies, even in its purely forward-looking form, the presence of one lag of the value of the money stock, and one lag of the value of money growth<sup>7</sup>. This results from the predetermination at time t of the expectations of one cohort of firms.

• In PP models, dynamics are very dependent on the choice of the structure of price adjustment. A unique and permanent shock on  $\Delta m$  has a very short impact in the PP model built on the structure of Taylor. With the structure of Calvo, dynamics include an infinity of predetermined expectations. Consequently, persistence is larger.

## 3.3 The response of output and inflation

For numerical applications, we assume  $\rho = 0.5$ . Small deviations from this value don't alter the conclusions of our comparative analysis. This value means that the cumulative effect on inflation in the long run is twice the initial value of the monetary innovation. We also assume  $\phi = 0.1$ , which corresponds to an important degree of strategic interactions between firms. While there is a strong debate on the plausible values of  $\phi$ , Woodford (2003) shows that values of  $\phi$  around are plausible. To make comparisons with the literature easier, we use this small value, in accordance with an important part of the literature<sup>8</sup> (see Taylor, 1999).

In Figure 1, we present the reaction of inflation.

<sup>&</sup>lt;sup>7</sup>When N rises, the number of money lags also rises.

<sup>&</sup>lt;sup>8</sup>For the impact of variations of  $\phi$  on persistence, see Kiley (2002) and Dixon and Kara (2005b).



The forward-looking FP model of Calvo is the only one to reproduce an immediate response of inflation. All other specifications, including the forward-looking model of Taylor, reproduce a hump shaped response. The absence of delay in the inflation response is one argument used by Mankiw and Reis (2002) to criticize FP models. Indeed, it seems to be only a special feature of the FP forward-looking model of Calvo. Ball, Mankiw and Reis (2005, p.5) argue that even the hybrid FP model reproduce a jump-response of inflation. This is not exact, and the inflation response of the Calvo hybrid model is even more delayed than the response of the PP model of Mankiw and Reis (2002).

In Figure 2 we present the response of the output.



FIG. 2. Response of output (autocorrelated shock)

The three models built on the structure of Calvo display a hump-shaped response of output. None of the models built on the structure of Taylor can reproduce such output behavior<sup>9</sup>. Since they reproduce a hump-shaped response of inflation, that means that they all respect the condition of a more delayed response of inflation than output, a criterion advanced by Kiley (2002b). The only model that reproduce a faster response of inflation (the peak of inflation occurs before the peak of output) is the forward-looking model of Calvo.

The behavior of output is of interest because it can summarize the degree of persistence implied by nominal rigidities. Absent such rigidites, we would have each period m = p. The degree of persistence can be measured as the cumulative deviation between m and p, which is equal to y. If we adopt this measure, it is interesting to note that the strongest degree of persistence is displayed by the FP model of Calvo (see Table 4).

Model	Calvo/FP/FL	Calvo/PP	Calvo/FP/I	Taylor/FP/FL	Taylor/FP/I	Taylor/PP
Deviation	1 0.046 0.034 0.030 0.015 0.008 0.006					0.006
	TAB 4 Cumulative deviation between $m$ and $p$ (over 25 periods)					

It is interesting to note that the forward-looking versions of the FP models imply a more

 $<sup>^9{\</sup>rm This}$  result is dependent of the use of 2-periods contracts. If contract length is longer, hump shaped dynamics can be reproduced.

important total output deviation than then counter-part with indexation. This results from the cyclical responses of the models with indexation. If we consider only the absolute value of deviations, this strongly raises the cumulative deviation of the FP model of Calvo with indexation (to 0.057), but the impact is low with the model of Taylor (from 0.008 to 0.013), which is still lower than with the forward-looking version.

One could note that the degree of persistence is much lower with the structure of Taylor. However, as shown by Dixon and Kara (2005), the difference should be lower if we allow for a motivated change in the calibration of  $\lambda$ , which should be higher with the models using the structure of Calvo, lowering the persistence of output and inflation. We do not adopt this calibration here, despite the persasive arguments of Dixon and Kara, in order to facilitate the comparisons between our results and previous papers in the literature.

#### **3.4** The acceleration phenomenon

As stated by Mankiw and Reis (2002), the acceleration phenomenon, which represents the correlation between the output gap<sup>10</sup>  $y_t$  (log from trend) and one-year inflation change  $(\pi_{t+1} - \pi_{t-1})$ , centered around the observation date is a well-documented macroeconomic fact.

In Figure 3, we present the correlation between output gap  $y_t$  and the annual change of inflation<sup>11</sup> ( $\pi_{t+1} - \pi_{t-1}$ ) (the acceleration phenomenon), assuming that the output gap and the output coïncide (real shocks are held constant).



FIG. 3. The acceleration phenomenon

All correlations are positive except the one of the FL model of Calvo. Contrary to the assertion of Mankiw and reis (2002), a FP forward looking model can reproduce a positive correlation if we depart from the Calvo assumption. Models with backward looking

<sup>&</sup>lt;sup>10</sup>For this calculation, output  $y_t$  is the deviation of log real GDP from trend, where trend is calculated using the Hodrick-Prescott filter.

<sup>&</sup>lt;sup>11</sup>We consider the US inflation based on the Consumer Price Index (CPI) from the FRED II Database of the Federal Reserve of Saint Louis. As we use a semi-annual model, we consider  $corr(y_t, \pi_{t+1} - \pi_{t-1})$ , and not  $corr(y_t, \pi_{t+2} - \pi_{t-2})$  as in Mankiw and Reis, who use a quarterly model.

component predict a higher correlation than the initial models, but the low value for the model of Taylor is not necessary a weakness since it is in line with the empirical values found with semi-annual US data (Table 5).

Periods	1960:1-2005:2	1970:1-2005:2	1980:1-2005:2	1990:1-2005:2
The acceleration phenomenon	0.1472	0.1853	0.2017	0.3392

TAB. 5. The empirical acceleration phenomenon

# 4 A DISINFLATION POLICY

#### 4.1 Presentation

A special case of the previous money process, where  $\rho = 1$ , is also often regarded as a criterion of evaluation. This particular value of  $\rho$ , while counterfactual, is of great interest because with a negative monetary innovation, it corresponds to the case of a "cold-turkey" disinflation policy engineered by the central bank. This kind of disinflation policy has often been used as a test to dismiss the validity of the forward-looking New Keynesian model of inflation dynamics (see among others Ball, 1994a, Fuhrer and Moore, 1995, Mankiw and Reis, 2002). The stylized facts that this model is supposed to be unable to reproduce is the presence of a positive output cost of the disinflation (Ball, 1994b), and the delayed response of inflation following the slowing of money growth (Mankiw and Reis, 2002).

#### 4.2 The money process

The central bank sets the growth rate of money  $\Delta m_t$  to a level compatible with her inflation target  $\pi_t^*$ , that is, each period  $\Delta m_t = \pi_t^*$ . Then, expectations are given by:  $E_t \Delta m_{t+i} = \pi_t^* = \Delta m_t, \forall i \geq 0.$ 

Assume that the economy is initially at its steady-state, defined by  $\Delta m_t = \pi_t = \pi_1^*$  and  $y_t = 0$ . A sudden and unexpected disinflation policy is equivalent to a negative shock on the inflation target, e.g. from  $\pi_1^*$  to  $\pi_2^*$  with  $\pi_2^* < \pi_1^*$ . Let the disinflation begin in period 1. This policy is credible and permanent, but is not expected by the agents. For numerical applications, we assume  $m_0 = 0$ , and the evolution of  $m_t$  is given by  $m_t = m_{t-1} + \pi_t^*$ , with the following path:

$$\left\{\begin{array}{l}t\in \left[-\infty,0\right], \pi_t^*=0.025\\t\in \left[1,+\infty\right], \pi_t^*=0\end{array}\right\}$$

#### 4.3 Price dynamics

Given the expectations of the money path, we can derive the analytic form of price dynamics.

Hypotheses	Price Dynamics	
Calvo/Fixed Prices	$p_t = \theta p_{t-1} + (1 - \theta) m_t + \theta \Delta m_t$	
Forwara-tooking		
Calvo/Fixed Prices Indexation	$p_{t} = (\theta_{1} + \theta_{2}) p_{t-1} - \theta_{1} \theta_{2} p_{t-2} + \frac{B}{\theta_{3} - 1} m_{t} + \frac{B}{(\theta_{3} - 1)^{2}} \Delta m_{t}$	
Taylor/Fixed Prices Forward-looking	$p_{t} = \theta p_{t-1} + \frac{(1-\theta)}{2} \left( m_{t} + m_{t-1} \right) + \frac{(1+\theta)}{4} \left( \Delta m_{t} + \Delta m_{t-1} \right)$	
Taylor/Fixed Prices Indexation	$p_{t} = (\theta_{1} + \theta_{2}) p_{t-1} - \theta_{1} \theta_{2} p_{t-2} + \frac{A}{\theta_{3} - 1} (m_{t} + m_{t-1}) + \frac{A(1 + \theta_{3})}{2(\theta_{3} - 1)^{2}} (\Delta m_{t} + \Delta m_{t-1})$	
Calvo/Predetermined Prices	$p_t = \frac{\phi \left[1 - (1/2)^{t+1}\right] + 0.025 (1+t) (1/2)^{t+1}}{1 - (1-\phi) \left[1 - (1/2)^{t+1}\right]}$	
Taylor/Predetermined Prices	$p_{t} = p_{t-1} + \frac{1}{1+\phi} \left(\phi \Delta m_{t} + 2\Delta m_{t-1} - \Delta m_{t-2}\right)$	
TAB. 6. Price dynamics (Disinflation)		

## 4.4 Graphical illustrations

Once we assume 2-periods contracts, the only free parameter is  $\phi$ . For numerical illustrations, we assume  $\phi = 0.1$  (Taylor, 1999). Figure 4 represents the reaction of the price level.



FIG. 4. Response of the price level (unexpected disinflation)

Altough variables of interest are mostly inflation and output, we present price dynamics because there are important to understand clearly the origins of the real costs implied by the disinflation. Given equation demand (17), in order to reproduce a positive cost of disinflation, models have to display a mechanism of price over-shooting (i.e., the price level must continue to rise despite the stability of the money growth). The only model unable to reproduce this over-shooting mechanism is the forward-looking FP model of Calvo. As stated by Musy (2006), this is not a general feature of forward-looking FP models, but only a characteristic of the very specific rule of adjustment of Calvo. Even in its simplest form, the structure of Taylor can reproduce this price over-shooting.

Figure 5 represents the reaction of inflation.



FIG. 5. Response of inflation (unexpected disinflation)

The behavior of prices implies that after reaching a peak, the price level has to fall. Consequently, inflation also displays an over-shooting response. For all models built on the structure of Taylor, this mechanism begins very quickly<sup>12</sup> (the second period consecutive to the shock), while for the models built on the structure of Calvo (excepted the FL version), the peak response of inflation is more delayed.

Figure 6 represents the reaction of output.

<sup>&</sup>lt;sup>12</sup>The very quick response presented here is dependent on the very short length of contracts assumed. For the forward-looking version, the peak response of inflation occurs N periods after the shock, where N is the length of contracts.



The magnitude of the response of output depends on the degree of over-shooting presented in Figure 1. The models of Christiano, Eichenbaum and Evans (Calvo hybrid) and Mankiw Reis (Calvo and PP) display comparable responses. The two models with indexation (Fuhrer and Moore, and Christiano, Eichenbaum and Evans) reproduce oscillatory dynamics due to the presence of complex roots. It is important to note that the introduction of indexation in the structure of Taylor does not significantly increase the output cost of disinflation. This can be more clearly illustrated by the calculation of sacrifice ratios<sup>13</sup> (Figure 7).

 $<sup>^{13}</sup>$ The sacrifice ratio is defined as the cumulative reduction in output required to achieve a one percentage point reduction in the rate of inflation.



FIG. 7. Sacrifice ratio (unexpected disinflation)

To have an order of comparison, Ball (1994b) has estimated sacrifice ratios ranging from 0.0 to 3.6, with quarterly data.

To summarize the results, as it is well known, the forward-looking model of Calvo does not produce any inflation persistence or output costs. However, contrary to a common view (among others Roberts, 1998, Walsh, 1998), this is a special feature of the probabilistic price structure of Calvo and not a property shared by all FP forward-looking models. Even with the simplest deterministic structure (Taylor uniform contracts for two periods), inflation displays some persistence, and the output cost of disinflation is positive. This result is general and does not depend on the specific value chosen for  $\phi$ . The sacrifice ratio in the model of Taylor presented is always positive and is equal to  $(1/4\sqrt{\phi})$  (sect next section for a demonstration). While often neglected, the expectation errors present in the structure of Taylor have very important implications for disinflations. When looking at the Phillips curves of the models of Taylor and Mankiw and Reis, one can see that the sources of persistence are indeed similar: both models rely on the presence of predetermined expectations to reproduce a delay in the adjustment of inflation. Even in the sticky prices model, past information sets of agents have an influence of current variables because they are included in the prices which are still in effect in the current period. Then, as in the sticky-information model, past money growth has an influence on current output. There is only a special case for which this is not the case: the assumption of Calvo of a constant probability of price changes among all firms (see next section and Musy, 2006).

The introduction of indexation in the structure of Taylor does not significantly rise the amount of persistence. As shown by Ben Aïssa and Musy (2006), when taking in account the expectation errors, the models of Taylor and Fuhrer and Moore have close dynamics properties, the latter producing only small additional persistence than the former. However, introducing indexation in the model of Calvo significantly alters the dynamics. Inflation is very persistent and the output cost is the strongest of all the specifications considered. The corollary of the previous remark is that FP hybrid models have very different dynamic properties.

Properties of models with PP are also sensitive to the choice of the pricing structure. As it is well known for this price rigidity, a monetary shock has real effects only as long as all contracts have not been modified (Blanchard and Fischer, 1989). Then, persistence depends of the length of the longest contract. The particular structure of Calvo, allowing for some contracts of infinite length, produce strong persistence and asymptotic convergence. With finite length of contracts, persistence is much lower and there is strict convergence. Another presentation of this point is made by Collard and Dellas (2003), and Dupor and Tsuruga, (2005).

#### 4.5 The sacrifice ratio in fixed price forward-looking models

We show that the presence of a positive real cost following a disinflation is possible, even with fully rational forward-loooking agents facing sticky prices, which is a result often contested in the literature. We propose a general demonstration of this result for a "cold-turkey" disinflation, considering only the two models of Taylor and Calvo, with sticky prices and no indexation. Let disinflation begins in period j. We derive the general form of inflation dynamics in the model of Calvo<sup>14</sup>, given the output equation (17):

$$\pi_t = \theta^C \pi_{t-1} + \left(1 - \theta^C\right) \Delta m_t + \left(1 - \theta^C\right) \theta^C \sum_{i=0}^{\infty} \left(\theta^C\right)^i \left(E_t \Delta m_{t+i+1} - E_{t-1} \Delta m_{t+i}\right)$$

where  $\theta^{C}$  is the stable root of the dynamics. Inflation dynamics in the model of Taylor are given by:

$$\pi_t = \theta^T \pi_{t-1} + \left(\frac{1-\theta^T}{2}\right) \left(\Delta m_t + \Delta m_{t-1}\right) + \frac{\left(1-\theta^T\right)\left(1+\theta^T\right)}{4} \left(E_t \Delta m_{t+1} - E_{t-2} \Delta m_{t-1}\right) \\ + \frac{\theta^T \left(1-\theta^T\right)\left(1+\theta^T\right)}{4} \left[\sum_{i=0}^{\infty} \left(\theta^T\right)^i \left(E_t \Delta m_{t+i+2} - E_{t-2} \Delta m_{t+i}\right)\right]$$

where  $\theta^T$  is the stable root of the dynamics. Expected terms on money growth are different between the two structures. Given the money process, expectations are given by:  $E_t \Delta m_{t+i} = \pi_t^* = \Delta m_t, \forall i \geq 0$ . Inflation dynamics reduce in both cases to the following forms:

$$\pi_t = \theta^C \left( \pi_{t-1} - \Delta m_{t-1} \right) + \Delta m_t \tag{18}$$

$$\pi_t = \theta^T \pi_{t-1} + \frac{1 - \theta^T}{2} \left( \Delta m_t + \Delta m_{t-1} \right) + \frac{1 + \theta^T}{4} \left( \Delta m_t - \Delta m_{t-2} \right)$$
(19)

We note  $\pi_1^*$  the inflation rate before the disinflation. Initial state of the economy is given by:  $\Delta m_t = \pi_t = \pi_1^*$ , and  $y_t = 0$ . When the disinflation policy begins, the inflation target is changed permanently to  $\pi_2^*$ , with  $\pi_2^* < \pi_1^*$ . Let the disinflation begin during period j.

In the model of Calvo, equation (18) implies that as soon as the economy begins on its steady-state  $(\pi_{j-1} = \Delta m_{j-1})$ , we have  $\pi_{j+i} = \Delta m_{j+i} = \pi_2^*$ , and  $y_{j+i} = 0, \forall i \geq 0$ . In the model of Taylor, due to the presence of lagged expectations, we have by contrast:

$$\pi_{j} = \frac{3 - \theta^{T}}{4} \pi_{2}^{*} + \frac{1 + \theta^{T}}{4} \pi_{1}^{*}$$
$$\pi_{j+1} = \frac{5 - \theta^{T}}{4} \pi_{2}^{*} - \frac{1 - (\theta^{T})^{2}}{4} \pi_{1}^{*}$$
$$\pi_{j+i} = \theta^{T} \pi_{j+i-1} + (1 - \theta^{T}) \pi_{2}^{*}, \quad \forall i \geq 2$$

<sup>&</sup>lt;sup>14</sup>In this appendix, to make the comparison simpler, we index by  $^{C}$  the variables relative to the model of Calvo, and we index by  $^{T}$  the variables relative to the model of Taylor.

The condition  $\pi_j > \pi_2^*$  is always verified<sup>15</sup>. This means that there is some inflation persistence at the beginning of the disinflation. After, the inflation rate overshoots its long run value ( $\pi_{j+1} < \pi_2^*$ ). During subsequent periods, the inflation rate converges to this longrun value. Convergence is gradual and monotone if  $\phi \prec 1$ , immediate if  $\phi = 0$ , and oscillatory if  $\phi \succ 1$ .

In the model of Taylor, output is always negative during the process, and its dynamics are given by:

$$y_{j+i} = -\frac{\left(\theta^{T}\right)^{i} \left(1 + \theta^{T}\right)}{4} \left[\pi_{1}^{*} - \pi_{2}^{*}\right]$$

for  $i \ge 0$ . The total cost of disinflation (TCD), given by  $\sum_{i=0}^{\infty} y_{j+i}$ , is equal to (respectively in the model of Calvo and in the model of Taylor):

$$TCD^{C} = 0$$
  

$$TCD^{T} = -\frac{\left(1+\theta^{T}\right)}{4\left(1-\theta^{T}\right)} \left[\pi_{1}^{*}-\pi_{2}^{*}\right]$$

As  $\theta^T \in [-1; 1]$ , we have  $TCD^T < 0$ . In the forward-looking model of Taylor, a coldturkey disinflation always implies a positive real cost of the disinflation. This result does not depend on the value of  $\phi$ . This is an important point, because there are some debates on the plausible values of  $\phi$  (see Chari, Kehoe and McGrattan, 2000 and Woodford, 2003). However, the sacrifice ratio (SR) is decreasing in  $\phi$ :

$$SR^T = \frac{1}{4\sqrt{\phi}}$$

A presentation of the mechanics underlying this positive cost is given by Musy (2006). It is important to note that the source of this cost is the presence of expectation errors. If the disinflation is announced in advance, these error terms disappear and the disinflation presented doesn't play any role, and disinflation is not costly. But in this case, PP under the structure of Taylor can also imply a costless disinflation (see Taylor, 1983).

## **5 COMPARISON WITH THE LITERATURE**

Our paper is close in spirit to the one of Nelson (1998), who shows that many alternative sticky-price specifications may be written as special cases of a log-linear model of prices dynamics. Among others, he concludes that the hybrid model of Fuhrer and Moore is better suited to reproduce empirical inflation serial correlations than the forward-looking model of Taylor. However, his conclusions are distorted for two reasons. The first is the absence of expectations errors in the dynamics, while the lecture of equations (13) and (14), and the results presented below show the importance of these errors. Another problem is the absence of a common framework to derive the implications of the alternative assumptions on price adjustement. Consequently, in his presentation, the numerical value of some parameters appears to be artificially different from one model to another. As an exemple, the value of  $\phi$  is estimated when he considers the model of Fuhrer and Moore, which gives a very small value of 0.008, which is common in empirical studies. On the contrary, when analyzing the

<sup>&</sup>lt;sup>15</sup>Because  $\phi$  is always positive and  $\theta^T$  is given by:  $\theta^T = (1 - \sqrt{\phi}) / (1 + \sqrt{\phi}).$ 

model of Taylor, he uses the calibration of  $\phi$  proposed by Chari, Kehoe and McGrattan (2000), who show that  $\phi$  cannot be smaller than 1.2 in a general equilibrium framework. For a great part, this can explain his contrasted conclusions about the relevance of the two models.

Another relevant paper is the paper of Kiley (2002a). Our paper is complementary and exhibits another difference between the forward-looking models of Calvo and Taylor. Kiley, considering a shock on the level of the money stock, shows that the different distributions of price between the two models has an impact in terms of output persistence, the model of Calvo being more persistent. Here we focus on shocks on the growth rate of the money stock. The different assumptions on price adjustment have other implications. Notably, in the model of Taylor, price changes are made by the firms charging the oldest prices in the economy. This feature has important implications and inflation and output dynamics, and this effects are absent from the paper of Kiley because the shock on the level of the money stock he considers do not produce such effects.

Our paper adopts the same criteria of evaluation than the paper of Mankiw and Reis (2002), that is the reproduction of stylized facts. Their conclusion is that the sticky information model is consistent with "accepted views of how monetary policy works", while the forward-looking sticky price models fails on three points: disinflations are not contractionary; money shocks have their maximum effect on inflation with a substantial delay (this concerns also the hybrid model, see Ball, Mankiw and Reis, 2005); the model is unable to explain the acceleration phenomenon that vigorous economic activity is positively correlated with rising inflation (Mankiw and Reis, 2002, p. 1318). Their study concerns only the Calvo version of the forward-looking sticky price model. Interestingly, we show that the model of Taylor can successfully reproduce all these three points, even in its fully forward-looking version : the sacrifice ratio is positive, the correlation between output gap and the annual change of inflation is positive (the acceleration phenomenon), and the maximum impact of the money shock on inflation does not occur immediatly. The conclusion of Mankiw and Reis is biased toward the sticky information model, because they compare it only to the forward-looking model of Calvo, which implies very specific dynamic properties. Even simple departures from this special rule can greatly improve the fit of the macro implications of the sticky price assumption to the "accepted views of how monetary policy works".

## 6 CONCLUSION

Our main conclusion is that the most important element for dynamic properties is the choice of the price structure. In this paper, we have considered only the simplest forms of the structures of Calvo and Taylor, but even with these restrictions, differences are crucial. This result is surprising because, since the influential paper of Roberts (1995), this choice has often been considered as unimportant. As a consequence, the conclusion of a total absence of sources of inflation inertia in forward-looking FP models, as stated by Ball, Mankiw and Reis (2005), is not exact. The frequent negligence of expectation errors inherent to the structure of Taylor explains this result. Indeed, the absence of disinflation costs is a very special characteristic attributable to the structure of Calvo. For critics finding that the simplest structure of Taylor can produce only a small degree of inflation persistence, the introduction of a small part of longer contracts seems able to significantly rise persistence [see Dixon and Kara, 2005a]. Then, when using the New Keynesian Phillips curve, it seems more important to not systematically use the model of Calvo, rather than to apologize in a

footnote, as suggested by Mankiw (2005).

Another surprising result is the fact that the presence of lagged inflation in the dynamics improves only slightly the degree of inflation persistence produced by each price structure, excepted in the case of the Calvo structure submitted to a disinflation shock.

Concerning the choice of PP versus FP, we show that this choice by itself is not sufficient to determine dynamic properties of inflation and output. Both hypotheses can produce an important degree of persistence. In PP models, the choice of the price structure is even more important than in FP models, because under the Taylor structure, the degree of persistence is very low.

# **APPENDICE :** Equivalence between equation (14) and the Phillips curve of Fuhrer and Moore (1995)

The general formulation of the model of Fuhrer and Moore consists to introduce a relative term in the structure of Taylor. Equations (1), (9) and (11) of the model of Taylor can be re-written under the following form:

$$x_{t} = \frac{1}{N^{2}} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} (x_{t+i-j}) + \frac{\phi}{N} \sum_{i=0}^{N-1} E_{t}(y_{t+i})$$
(20)

There is no reference to the price level. Fuhrer and Moore (1995) criticize this point and argue that contracts should be written in relative terms. They propose the following rule for the evolution of  $x_t$ :

$$x_t - p_t = \left(\frac{1}{N}\right) \sum_{i=0}^{N-1} E_t \left(v_{t+i} + \phi y_{t+i}\right)$$
(21)

where  $v_t$  is the mean real price of existing other contracts coexisting during period t:

$$v_t = \left(\frac{1}{N}\right) \sum_{i=0}^{N-1} (x_{t-i} - p_{t-i})$$
(22)

Then, (21) becomes:

$$x_{t} - p_{t} = \sum_{i=1}^{N-1} \frac{N-i}{N(N-1)} \left( x_{t-i} - p_{t-i} \right) + \sum_{i=1}^{N-1} \frac{N-i}{N(N-1)} E_{t} \left( x_{t+i} - p_{t+i} \right) + \frac{\phi}{N-1} E_{t} y_{t+i}$$
(23)

For N = 2, this gives:

$$x_t - p_t = \frac{1}{2} \left( x_{t-1} - p_{t-1} + E_t x_{t+1} - E_t p_{t+1} \right) + \phi(y_t + E_t y_{t+1})$$
(24)

The corresponding Phillips curve is:

$$\pi_t = \frac{1}{2} \left( \pi_{t-1} + E_t \pi_{t+1} \right) + \phi \left( \tilde{y}_t \right) + (1/2) \eta_t \tag{25}$$

where  $\tilde{y}_t = y_t + y_{t-1} + E_{t-1}y_t + E_t y_{t+1}$  and  $\eta_t^{\pi} = E_{t-1}\pi_t - \pi_t$ . When we take into account explicitly the expectation error, the last equation corresponds exactly to the equation (14) of the text:

$$\pi_t = (1/3) \left( \pi_{t-1} + E_{t-1} \pi_t + E_t \pi_{t+1} \right) + (2\phi/3) \left( \widetilde{y}_t \right)$$

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