Optimal damages with court delay and heterogenous time preferences

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1 Introduction

Delay is a problem that undermines the functioning of court systems around the world.

Vereeck and Mühl (2000) distinguish four periods within the waiting time between conflict and court decision: the negotiation time between parties; the procedural time to prepare the trial; the waiting time before the actual start of the trial; and the time of the trial itself. They argue that while data on court delay often include the last three periods, only the third can be considered as genuine court delay since it depends on the performance of the courts.

Di Vita (2010) review four different explanations for the excessive duration of controversies. First, as emphasized by Buscaglia and Dakolias (1996), the courts' efficiency might be insufficient due to a lack of resources devoted to the justice and/or to its defective organization. Second, following Vereeck and Mühl (2000), the complexity and the diversity of the law can itself be a source of conflict and, indirectly, of the total amount of litigation. Third, as argued by Djankov et al. (2003), the trial procedures allow the litigants to lengthen the time required to reach a decision, thus prolonging civil disputes. Finally, there is the possibility that both the judges (Palumbo and Sette, 2006) and the lawyers (Dewatripont and Tirole, 1999; Djankov et al., 2003; Djankov, McLiesh and Ramalho, 2006; Emons, 2000; Marchesi, 2003; Miceli, 1994) may benefit from longer court delays.

In this paper, we analyse how court delay impact the functioning and efficiency of the justice. We consider a standard model of liability, with some agents (the defendants) engaging in dangerous activities, with potentially detrimental consequences on the welfare of third parties (the plaintiffs). The defendants can exert costly care to limit the risk of causing an accident. After an accident, the defendant and the plaintiff may enter or not in a pretrial negociation to reach a settlement. If they fail to do so, the plaintiff may claim damages for loss with a judge.

Following Gravelle (1990) and Vereeck and Mühl (2000), our framework departs from the standard model of liability, by considering the existence of a court delay. The key difference with Gravelle (1990) and Vereeck and Mühl (2000) lies in our assumption that the time preferences are assumed heterogeneous and private information ([1]). Our framework then allows to analyse how court delay and damages modify the litigants behaviors at each stage of the judicial process.

In this setting, we characterize the (optimal) damages that minimize the total social costs. We decompose the problem in three steps. The first one determines the damages that would induce the defendant to internalize the external costs of his activity, taking the expected social costs of an accident as given. The second one then investigates the way the damages impact the expected social costs of an accident. Qualitatively, we distinct two different effects. First, as expected, larger damages induce more plaintiffs to bring a suit, with larger litigation costs as a counterpart. Second, and more surprisingly, in present value, larger damages can either be costly or beneficial, depending on the litigants time preferences. More precisely, larger damages will contribute to larger (resp., smaller) expected social costs of an accident if the defendant's discount factor is smaller (resp., larger) than the plaintiff's discount factor. The two effects can either reinforce or contradict each other, generating two different shapes of expected social costs of an accident. Finally, the last step use these arguments to characterize the optimal damages.

Section 2 analyses the case where pretrial negociation is not possible. Section 3 extends the analyse to the case where the litigants can negociate a settlement. Most proofs are in the Appendix.

2 The basic model

An economic agent, referred to as the defendant (D), exerts a potentially dangerous activity. D can expend x in care to reduce the risk of causing an accident. The probability of accident is denoted $p(x)$. It is assumed that $p'(x) < 0 < p''(x)$. If an accident occurs, a third party, referred to as the plaintiff (P), bears a loss l. P may claim damages for loss with a judge (J). Then, J evaluates the harm l and sets the damages f to be paid by D to P. A judgment costs c_A to the judicial administration. A litigation costs c_D to D and c_P to P. Let $k_0 \equiv c_A + c_D + c_P$ be the total legal expenses. (The possibility of settlement, whereby the legal expenses could in large part be avoided, will be considered

[1] In fact, we assume that only the discount factor of the defendant is private information. This assumption is mainly made both for the sake of simplicity.

subsequently.) The social welfare criterion is assumed to be the minimization of total social costs.

For all $L \geq 0$, define $x^*(L)$ the level of care that minimizes $x + p(x)L$. Assuming an interior solution, $x^*(L)$ satisfies:

$$1 + p'(x^*(L))L = 0.$$

By the envelop theorem, we can show that $x^*(L)$ is increasing in L:

$$(x^*)'(L) = (p'(x^*(L)))^2 / p''(x^*(L)) > 0.$$

2.1 No court delay

Consider briefly the above standard model, where no court delay is considered.

Let f_0 be the damages the judge would set at trial. P will bring a suit if and only if $f_0 > c_P$. Otherwise, P will drop the case. Accordingly, D will choose $x = x^*(c_D + f_0)$ to minimize $x + p(x)(c_D + f_0)$, if $f_0 > c_P$, and will choose $x = 0$, otherwise ([2]).

Total social costs will be the sum of prevention costs, expected losses, and expected legal expenses. Thus, it will equal $x + p(x)(k_0 + l)$, if the litigants go to trial, and $x + p(x)l$, otherwise.

Suppose that the total legal expenses k_0 are not too large, so that $x^*(k_0 + l) + p(x^*(k_0 + l))(k_0 + l) < p(0)l$. Then, given the litigants' behavior, total social costs are minimized if and only if $f_0 = c_A + c_P + l$, inducing P to bring a suit and D to choose $x = x^*(k_0 + l)$.

2.2 Positive court delay

Below, our framework will depart from the above standard model by considering a court delay and heterogeneous time preferences. The court delay is defined as the waiting time between the accident and the verdict. The judicial administration discount factor is denoted δ_A. D's discount factor is denoted δ_D. It is assumed public information. P's discount factor is denoted δ_P. It is assumed private information. However, it is common knowledge that δ_P is ditributed on $[0, 1]$, according to the cumulative distribution $G(\delta_P)$. The density is denoted $g(\delta_P)$. The distribution elasticity is denoted $\varepsilon(\delta_P) = -\delta_P g'(\delta_P) / g(\delta_P)$. Let $k_1 = \delta_A c_A + c_D + c_P$ be the (present value) legal expenses.

To make clear the gap with the standard model, consider again the previous analysis, step by step.

[2] Notice that $x^*(0) = 0$.

Let f_1 be the damages the judge would set at trial. If $f_1 \leq c_P$, no plaintiff will bring a suit. Otherwise, P will bring a suit if and only if $\delta_P f_1 > c_P$. Alternatively, for all types $\delta_P > c_P/f_1$, P will bring a suit and, for all types $\delta_P \leq c_P/f_1$, P will drop the case. c_P/f_1 is the "marginal type of plaintiff". D anticipates P's behavior in choosing his care. However, as δ_P is private information to P, he only knows that P's discount factor is distributed according to $G(\delta_P)$. Therefore, if an accident occurs, D expects to pay at trial:

$$\begin{aligned} L_D(f_1) &= (1 - G(c_P/f_1))(\delta_D f_1 + c_D), \text{ if } f_1 > c_P, \\ &= 0, \text{ otherwise.} \end{aligned}$$

Thus, D will choose $x = x^*(L_D(f_1))$ to minimize $x + p(x) L_D(f_1)$.

Total social costs will be the sum of prevention costs, expected losses, and expected (present value) legal expenses. Thus, ex post, it will equal $x + p(x)((\delta_D - \delta_P) f_1 + k_1 + l)$, if the litigants go to trial, and $x + p(x) l$, otherwise. The difference with the standard model arises because c_A and f_1 are postponed and paid at the trial time.

Let $L_S(f_1)$ be the expected social costs given D caused an accident. As P goes to court if and only if $\delta_P > c_P/f_1$, we have:

$$\begin{aligned} L_S(f_1) &= \int_{c_P/f_1}^{1} ((\delta_D - \delta_P) f_1 + k_1) \, \mathrm{d}G + l, \text{ if } f_1 > c_P, \\ &= l, \text{ otherwise.} \end{aligned}$$

Ex ante, total social costs will be:

$$C(f_1) = x + p(x) L_S(f_1).$$

The social problem is to choose f_1 to minimize this objective, knowing that D chooses $x = x^*(L_D(f_1))$ to minimize:

$$x + p(x) L_D(f_1).$$

Below, to solve this problem, we analyse in turn two effects of the choice of the damages. We first analyse the damages that induce D to internalize the external costs of his activity. We then discuss the effect of the damages on the social costs arising given D caused an accident. Finally, we gather both points of view, to characterize the optimal damages.

2.3 Optimal care

Consider here D's incentives to take care.

D's incentive to care is driven by his private costs $L_D(f_1)$, while the social costs equal $L_S(f_1)$ if D causes an accident. Let $L_E(f_1)$ be the difference between

$L_S(f_1)$ and $L_D(f_1)$, which can be referred to as the external costs of D's activity. We have:

$$\begin{aligned} L_E(f_1) &= \int_{c_P/f_1}^{1} (\delta_A c_A + c_P - \delta_P f_1)\, \mathrm{d}G + l, \text{ if } f_1 > c_P, \\ &= l, \text{ otherwise.} \end{aligned}$$

Clearly, if they exist, damages f_1 such that $L_E(f_1) = 0$ are valuable, as they will induce D to internalize the external costs of his activity and to choose a socially optimal level of care. Formally, if $L_E(f_1) = 0$, then D's private objective coincides with the social objective, thereby inducing him to choose $x = x^*(L_D(f_1))$ to minimize $x + p(x) L_S(f_1)$.

We show in proposition 1 that damages f_1 inducing D to internalize the external costs of his activity exist and are unique under assumption 1 below.

Assumption 1. The distribution elasticity $\varepsilon(\delta_P)$ is bounded from above by $2 + c_P/\delta_A c_A$.

Let us denote f_1^* the (unique) damages such that $L_E(f_1^*) = 0$.

The following proposition aims at characterizing f_1^*.

Proposition 1. Under assumption 1, the liability f_1^* inducing D to internalize the external costs of his activity exists, is unique, is larger than $\delta_A c_A + c_P + l/E[\delta_P]$, and is increasing in the loss l.

Now, suppose the judge adopts the damages f_1^* satisfying (1).

For the sake of interpretation, the following implicit definition may be seen as more elegant and transparent:

$$f_1^* = \frac{1}{E[\delta_P \mid \delta_P > c_P/f_1^*]} \left(\delta_A c_A + c_P + \frac{l}{1 - G(c_P/f_1^*)} \right), \qquad (1)$$

where $E[\delta_P \mid \delta_P > c_P/f_1^*] \equiv \int_{c_P/f_1^*}^{1} \delta_P \mathrm{d}G / (1 - G(c_P/f_1^*))$ is the conditional expectation of δ_P, given that $\delta_P > c_P/f_1^*$.

According to this formulae, from the viewpoint of deterrence, D should be liable for the legal expenses of the other parties (in present value), for punitive damages and for compensatory interests. Punitive damages are required because D sometimes escapes his liability. As usual, punitive damages are equal to the harm, multiplied by the reciprocal of the probability of D escaping his liability (Polinsky and Shavell, 1997). The legal interest rate should be set to reflect the expected discount factor among the population of plaintiffs going to trial.

When f_1^* is used, as D internalizes the external costs of his activity, we have:

$$\begin{aligned} & x^*\left(L_D\left(f_1^*\right)\right)+p\left(x^*\left(L_D\left(f_1^*\right)\right)\right)L_S\left(f_1^*\right) \\ \leq \quad & x+p\left(x\right)L_S\left(f_1^*\right), \end{aligned}$$

for all x. In particular, when $x=x^*\left(k_0+l\right)$, we obtain:

$$\begin{aligned} & x^*\left(L_D\left(f_1^*\right)\right)+p\left(x^*\left(L_D\left(f_1^*\right)\right)\right)L_S\left(f_1^*\right) \\ \leq \quad & x^*\left(k_0+l\right)+p\left(x^*\left(k_0+l\right)\right)L_S\left(f_1\right). \end{aligned}$$

Now, remember that, in the standard model, where no court delay is considered, total social costs is minimized if the level of care is $x^*\left(k_0+l\right)$. Then total social costs equal:

$$x^*\left(k_0+l\right)+p\left(x^*\left(k_0+l\right)\right)\left(k_0+l\right).$$

From this, it should be clear that total social costs can be made smaller with a positive court delay if $L_S\left(f_1^*\right)\leq k_0+l$. The following proposition derives from this. The proof is relegated in the appendix.

Proposition 2. There exists $\alpha>1$ such that, if $\delta_D\leq\alpha E\left[\delta_P\right]$, for all damages f_0 that would prevail in the economy with no court delay, total social costs can be made smaller in the economy with a positive court delay, provided the judge adopts the damages f_1^* defined above.

2.4 The social costs of an accident

Consider now the effect of the damages on the social costs resulting if D causes an accident.

The expected social costs given D caused an accident, $L_S\left(f_1\right)$, depends on f_1, both because the damages determine P's incentive to go to court and because D and P value them differently in present values, due to heterogeneous time preferences. Formally, we have:

$$\begin{aligned} L_S\left(f_1\right) &= \int_{c_P/f_1}^{1}\left(\left(\delta_D-\delta_P\right)f_1+k_1\right)\mathrm{d}G+l,\text{ if } f_1>c_P, \\ &= l,\text{ otherwise.} \end{aligned}$$

Below, we will admit the following assumption.

Assumption 2. For all $f_1>c_P$, the expected social costs given D caused an accident, $L_S\left(f_1\right)$, is strictly concave in the liability f_1.

More general shapes could have been considered. However, this would not have brought any new insight, while seriously burdening the discussion. Besides, we show in the appendix that this assumption holds true, in particular, if the probability cumulative function is $G(\delta_P) = (\delta_P)^\lambda$, with $\lambda \geq 1$.

Using assumption 2, we are able to show the following property. The proof can be found in the appendix.

Property 1. The expected social costs given D caused an accident, $L_S(f_1)$, satisfies the following properties (In parts (b) and (c), only $f_1 > c_P$ are considered):

(a) An increase of l simply translates $L_S(f_1)$ upward.

(b) The slope of $L_S(f_1)$ is positive in a neighborhood of c_P and tends to $\delta_D - E[\delta_P]$ at infinity.

(c) Under assumption 1, if $\delta_D \geq E[\delta_P]$, $L_S(f_1)$ is strictly increasing. Otherwise, if $\delta_D < E[\delta_P]$, there exists $F > c_P$, satisfying $L'_S(F) = 0$, such that $L_S(f_1)$ is strictly increasing, when $f_1 < F$, and strictly decreasing, when $f_1 > F$.

The figure below illustrates our results. The left part deals with the case where $\delta_D \geq E[\delta_P]$. The right part deals with the case where $\delta_D < E[\delta_P]$.

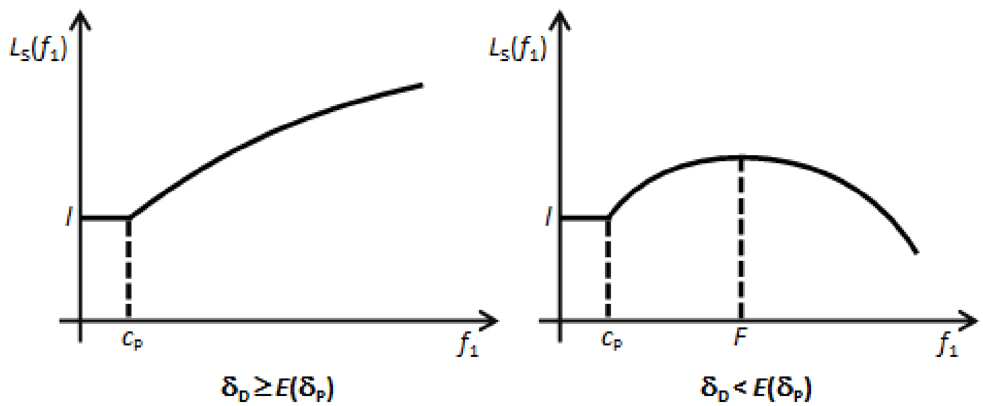


Figure 1 - Expected social costs of an accident.

This stricking result may require further explanations. To understant it, notice that the judge's delay in rendering his verdict, in a way, commands a *future* transaction between D and P. The court delay postpones the time when D will compensate the loss l with the payment of the damages f_1. In present value, this transfert costs $\delta_D f_1$ to D and reports $\delta_P f_1$ to P. Thus, depending on the time preferences, the transaction can be either costly or beneficial, meaning that the damages will impact the social costs of an accident. As property 1, part a, states, when D's discount factor is larger that P's expected discount factor, increasing the liability can only raise the social costs of an accident; otherwise, increasing the liability will eventually reduce the social costs of an accident.

2.5 The optimal policy

Consider finally the optimal policy, where all effects of the damages are taken into account, that is, on the one hand, its effect on D's incentive to take care and, on the other hand, its effect on the social costs resulting after D caused an accident.

The social problem is to choose f_1 to minimize:

$$C(f_1) = x + p(x) L_S(f_1),$$

knowing that D expends $x = x^*(L_D(f_1))$ in care to minimize:

$$x + p(x) L_D(f_1).$$

After substitution, total social costs equal:

$$C(f_1) = x^*(L_D(f_1)) + p(x^*(L_D(f_1))) L_S(f_1).$$

Therefore, for an interior solution, the optimal liability satisfies:

$$\begin{aligned} C'(f_1) = \; & [1 + p'(x^*(L_D(f_1))) L_S(f_1)] (x^*)'(L_D(f_1)) L'_D(f_1) \\ & + p(x^*(L_D(f_1))) L'_S(f_1) = 0 \end{aligned} \tag{2}$$

This condition shows that the choice of the judge should account for the two effects discussed previously. On the one hand, the damages determines the way D will internalize the external costs of his activity. This is represented by the term under brackets. By definition, it will vanish if and only if the damages are set at f_1^*. On the other hand, the damages influence the expected social costs resulting after D has provoked an accident. This is reflected by last term. From property 1, we know that this effect can go in any direction.

Let f_1^o denote the optimal liability. For its characterization, it will be instructive to compare it with f_1^*, i.e., the liability that would induce D to completely internalize the external costs of his activity (see proposition 1).

Since, by construction of f_1^*, we have:

$$1 + p'(x^*(L_D(f_1^*))) L_S(f_1^*) = 0,$$

if evaluated at $f_1 = f_1^*$, the first-order derivative above becomes:

$$C'(f_1^*) = p(x^*(L_D(f_1^*))) L'_S(f_1^*). \tag{3}$$

Assuming that the social problem is convex, this observation can be used to show that the optimal liability f_1^o will be smaller, equal, or larger than f_1^*, if $L_S(f_1)$ is respectively increasing, constant, or decreasing in a neighborhood of f_1^*.

Indeed, consider for example the case where $L_S'(f_1^*) > 0$. Then, (3) implies that $C'(f_1^*) > 0$. Besides, the optimal liability satisfies $C'(f_1^o) = 0$. Under the assumption that the social problem is convex, $C'(f_1)$ is an increasing function. Then, as $C'(f_1^o) < C'(f_1^*)$, it is necessary that $f_1^o < f_1^*$. The argument can easily be adapted to deal with the other cases.

This result is quite intuitive. In a standard model of liability, where the damages do not influence the cost of an accident, the social objective simply is to induce an optimal care by D. This goal is filled if the magnitude of the damages is set such that D will face the expected cost of an accident at the time he chooses his level of care. Here, where the damages can either increase or decrease the cost of an accident, including the loss and legal expenses (in present value), the same objective must be counterbalanced by that of reducing the cost of an accident. As expected, the damages should be reduced or increased, with respect to the standard model, if they respectively increase or decrease the cost of an accident.

Using Property 1, we can identify more explicitly the situations where f_1^o should be set smaller, equal, or larger than f_1^*.

In the case where $\delta_D \geq E[\delta_P]$, property 1 shows that $L_S(f_1)$ is always increasing. Consequently, the policy prescription in this case is that the optimal liability f_1^o should always be chosen smaller than f_1^*.

If $\delta_D < E[\delta_P]$, the policy prescription is less clear. Indeed, property 1 states that $L_S(f_1)$ is strictly increasing, when $f_1 < F$, and strictly decreasing, when $f_1 > F$. Hence, f_1^o will be smaller, equal, or larger than f_1^*, if f_1^* is respectively smaller, equal, or larger than F.

Below, we propose a construction to discuss the relative positions of f_1^* and F. To construct it, remember that the graph of $L_S(f_1)$ is an inverted U-shaped curve (for $f_1 > c_P$), attaining a maximum at $f_1 = F$. From property 1, an increase of the loss l simply translates $L_S(f_1)$ upward. The graph of $L_D(f_1)$ is an increasing curve. From proposition 1, $L_D(f_1)$ and $L_S(f_1)$ intersect only once when $f_1 = f_1^*$.

The first figure below deals with the general case. We consider three different level of losses, l', l_0, and l'', such that $l' < l_0 < l''$. They are chosen such that $L_D(f_1)$ and $L_S(f_1)$ respectively intersect before, at, and after F. Accordingly, f_1^* is smaller, equal, or larger than F, when the harm is respectively equal to l', l_0, and l''.

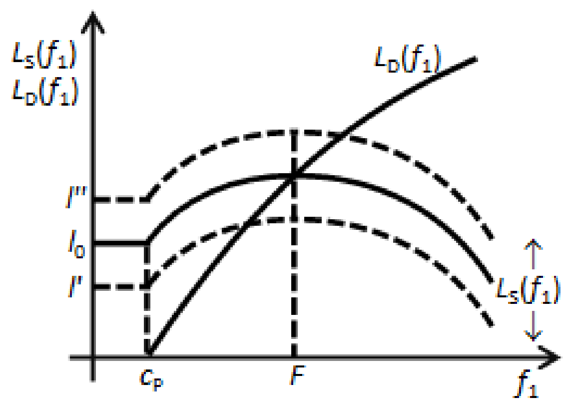
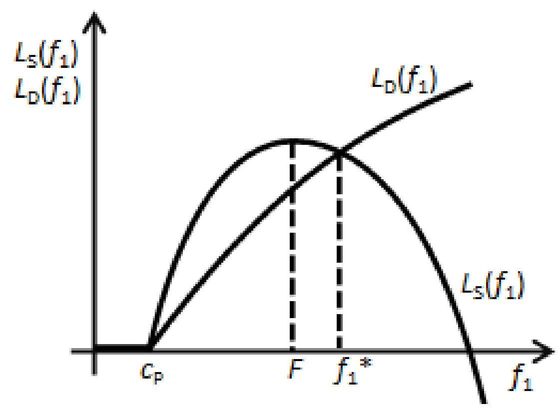


Figure 2 - Relative position of f_1^* and F.

The second figure below deals with the special case where $L_D(f_1)$ and $L_S(f_1)$ intersect at $f_1^* > F$, when $l = 0$. Then, f_1^* is larger than F, for all l.



The following proposition summarizes our results.

Proposition 3. If $\delta_D \geq E[\delta_P]$, the optimal liability f_1^o will be smaller than f_1^*. If $\delta_D < E[\delta_P]$, two cases need to be considered, depending on whether $f_1^* < F$ or not, when $l = 0$. In the first one, there exists $l_0 > 0$, such that the optimal liability f_1^o will be smaller, equal, or larger than f_1^*, if l is respectively smaller, equal, or larger than l_0. In the second one, the optimal liability f_1^o will be larger than f_1^*.

3 The extended model

We consider here the possibility of settlement.

Let s be the offer made by D to P, with $0 \leq s \leq f_1 - c_P$. P accepts D's offer if and only if $s \geq \delta_P f_1 - c_P$. Alternatively, for all types $\delta_P \leq d \equiv (s + c_P)/f_1$, P will accept the offer s and, for all types $\delta_P > d$, P will refuse D's offer. d is the "marginal type of plaintiff".

D anticipates P's behavior in choosing his offer s. The interim expected payoff to D who offers $s = df_1 - c_P$ is:

$$G(d)(df_1 - c_P) + (1 - G(d))(\delta_D f_1 + c_D).$$

The first term is the probability of offer s being accepted times the offer s. The second term is the probability that offer s is rejected times D's cost in court.

Assuming an interior solution (i.e., $0 < s < f_1 - c_P$ and $c_P/f_1 < d < 1$), the equilibrium marginal type of plaintiff function d^* is implicitly defined by:

$$[(d^* f - c_P) - (\delta_D f_1 + c_D)]\, g(d^*) + f_1 G(d^*) = 0.$$

The associated equilibrium offer function is $s^* = d^* f_1 - c_P$.

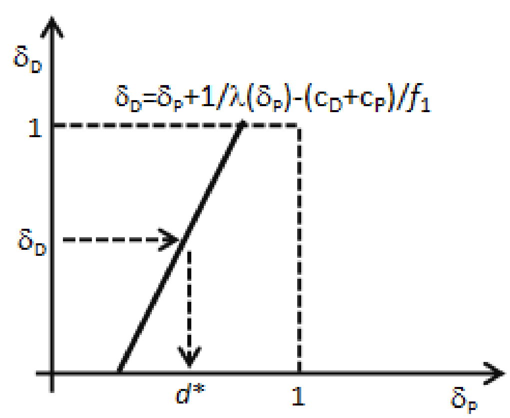
A minor amount of manipulation yields the following equivalent condition:

$$d^* + 1/\lambda(d^*) - (c_D + c_P)/f_1 = \delta_D, \tag{4}$$

where: $\lambda(\delta_P) \equiv g(\delta_P)/G(\delta_P)$.

Assumption 3. The hazard rate $\lambda(\delta_P)$ is non increasing.

The equilibrium settlement can be characterized using the figure below, representing (4). The curve actually depicts D's type δ_D as a function of his marginal type of plaintiff d^*. Under assumption 3, it is strictly increasing. The pairs of D and P, with types δ_D and δ_P, at the left of the frontier settle. The pairs of D and P, with types δ_D and δ_P, at the right of the frontier go to court.



Several properties follow directly. An interior solution exists, for all δ_D, if and only if $1/\lambda\left(c_P/f_1\right) \leq c_P/f_1$ and $1/\lambda\left(1\right) \geq \left(c_D + c_P\right)/f_1$. The marginal type of plaintiff d^* is decreasing in f_1, and increasing in δ_D, c and $\lambda\left(\cdot\right)$.

$<$ *In progress* $>$

4 Appendix

A.1. Proof of proposition 1.

The external costs of D's activity is:

$$\begin{aligned} L_E(f_1) &= \int_{c_P/f_1}^{1} (\delta_A c_A + c_P - \delta_P f_1)\, \mathrm{d}G + l, \text{ if } f_1 > c_P, \\ &= l, \text{ otherwise.} \end{aligned}$$

To see that damages f_1^* such that $L_E(f_1^*) = 0$ exist, first remark that:

$$\begin{aligned} L_E(f_1) &= l, \text{ for all } f_1 \leq c_P, \\ \lim_{f_1 \to +\infty} L_E(f_1) &= -\infty. \end{aligned}$$

As $L_E(f_1)$ is continuous, f_1^* exists and satisfies $c_P < f_1^* < \infty$.

To prove that f_1^* is unique, intuitively, we show below that, for all $f_1 > c_P$, the graph of $L_E(f_1)$ is an inverted U-shaped curve and that only its decreasing banch intersects the x-axis.

First-order differentiation yields:

$$L'_E(f_1) = \delta_A c_A \frac{c_P}{(f_1)^2} g\left(\frac{c_P}{f_1}\right) - \int_{c_P/f_1}^{1} \delta_P dG.$$

From this, we can show that ([3]):

$$\begin{aligned} \lim_{f_1 \to c_P} L'_E(f_1) &= \frac{\delta_A c_A}{c_P} g(1), \\ \lim_{f_1 \to +\infty} L'_E(f_1) &= -E(\delta_P). \end{aligned}$$

Second-order differentiation yields:

$$L''_E(f_1) = -\delta_A c_A \frac{c_P}{(f_1)^3} g\left(\frac{c_P}{f_1}\right)\left(2 + \frac{c_P}{\delta_A c_A} - \varepsilon\left(\frac{c_P}{f_1}\right)\right).$$

Under assumption 1, $L''_E(f_1) < 0$ and $L'_E(f_1)$ is decreasing. Together with the limits of $L'_E(f_1)$, this implies that there exists φ, satisfying $L'_E(\varphi) = 0$, such that $L'_E(f_1) > 0$, for all $f_1 < \varphi$, and $L'_E(f_1) < 0$, for all $f_1 > \varphi$. Now, as $L_E(f_1)$ is increasing, for all $f_1 < \varphi$, and $\lim_{f_1 \to c_P} L_E(f_1) = l$, $L_E(f_1)$ is larger than l, for all $f_1 \leq \varphi$. This shows that $f_1^* > \varphi$. Finally, as $L_E(f_1)$ is decreasing, for all $f_1 > \varphi$, given that $L_E(\varphi) > l$ and $\lim_{f_1 \to +\infty} L_E(f_1) = -\infty$, there exists a *unique* f_1^*, with $\varphi < f_1^* < \infty$, such that $L_E(f_1^*) = 0$.

[3] To derive the second limit, remark that, as $g(\delta_P)$ is continuous on $[0, 1]$, it is bounded. Thus, the first term of $L'_E(f_1)$ will be negligible with respect to the second term, for sufficiently large f_1.

Now, to obtain the second result of proposition 1, rewrite (1) as:

$$f_1^* = \frac{\delta_A c_A + c_P}{E\left[\delta_P \mid \delta_P > c_P/f_1^*\right]} + \frac{l}{\int_{c_P/f_1^*}^{1} \delta_P dG}.$$

This implies that:

$$f_1^* \geq \delta_A c_A + c_P + \frac{l}{E\left[\delta_P\right]},$$

given that $E\left[\delta_P\right] < E\left[\delta_P \mid \delta_P > c_P/f_1^*\right] < 1$ and $0 < \int_{c_P/f_1}^{1} \delta_P dG < E\left[\delta_P\right]$, for all $f_1 > c_P$.

Finally, to show that f_1^* is increasing in the loss l, first recall that, as $f_1^* > \varphi$, $L'_E\left(f_1^*\right) < 0$. Then, use the implicit function theorem to obtain $\mathrm{d}f_1^*/\mathrm{d}l = -1/L'_E\left(f_1^*\right) > 0$. $\square$

A.2. Proof of Proposition 2.

We show below that, if $\delta_D \leq \alpha E\left[\delta_P\right]$, with $\alpha = \left(c_A + c_P + l\right)/\left(\delta_A c_A + c_P + l\right) > 1$, then the inequality $L_S\left(f_1\right) \leq k_0 + l$ holds true.

First remember that, by definition of f_1^*:

$$L_S\left(f_1^*\right) = L_D\left(f_1^*\right).$$

Thus, by definition of $L_D\left(f_1\right)$:

$$L_S\left(f_1^*\right) = \left(1 - G\left(c_P/f_1^*\right)\right)\left(\delta_D f_1^* + c_D\right).$$

Using:

$$f_1^* = \frac{1}{E\left[\delta_P \mid \delta_P > c_P/f_1^*\right]}\left(\delta_A c_A + c_P + \frac{l}{1 - G\left(c_P/f_1^*\right)}\right).$$

we can obtain upon susbtitution:

$$\begin{aligned} L_S\left(f_1^*\right) &= \frac{\delta_D}{E\left[\delta_P \mid \delta_P > c_P/f_1^*\right]}\left(\left(1 - G\left(c_P/f_1^*\right)\right)\left(\delta_A c_A + c_P\right) + l\right) \\ &\quad + \left(1 - G\left(c_P/f_1^*\right)\right) c_D. \end{aligned}$$

Finally, as $G\left(c_P/f_1^*\right) \geq 0$ and $E\left[\delta_P \mid \delta_P > c_P/f_1^*\right] \geq E\left[\delta_P\right]$, it is clear that:

$$L_S\left(f_1^*\right) \leq \frac{\delta_D}{E\left[\delta_P\right]}\left(\delta_A c_A + c_P + l\right) + c_D.$$

Therefore, a sufficient condition for the inequality to hold true is:

$$\frac{\delta_D}{E\left[\delta_P\right]}\left(\delta_A c_A + c_P + l\right) + c_D \leq c_A + c_D + c_P + l,$$

from which we immediately can derive the sufficient condition stated in Proposition 2. □

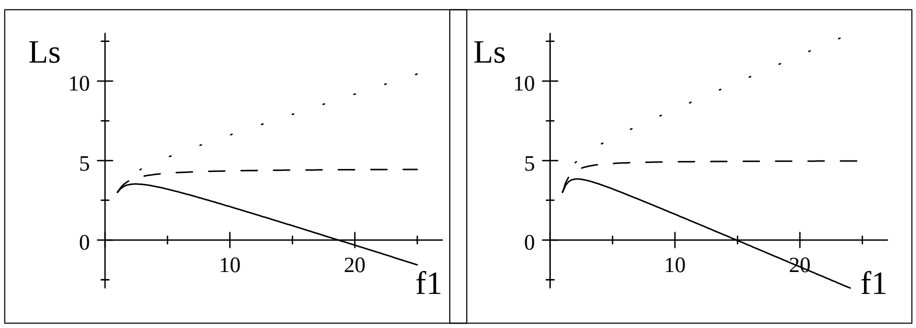
A.3. Derivation of assumption 2 when $G(\delta_P) = (\delta_P)^\lambda$, with $\lambda \geq 1$.

Consider the case where the probability cumulative function is $G(\delta_P) = (\delta_P)^\lambda$. The associated probability density function is $g(\delta_P) = \lambda (\delta_P)^{\lambda-1}$. The associated expected value is $E[\delta_P] = \lambda / (\lambda+1)$.

Integrating, for all $f_1 > c_P$, we can obtain:

$$L_S(f_1) = \left(1 - \left(\frac{c_P}{f_1}\right)^\lambda\right)(\delta_D f_1 + k_1) - \frac{\lambda}{\lambda+1}\left(1 - \left(\frac{c_P}{f_1}\right)^{\lambda+1}\right) f_1 + l.$$

The following figure represents it, in the case where $c_P = 1$, $k_1 = 2$, $l = 3$, and considering in turn $\delta_D = (1/2) E[\delta_P]$ (line) $\delta_D = E[\delta_P]$ (dash) and $\delta_D = (3/2) E[\delta_P]$ (dots). The left and right parts respectively deal with the cases where $\lambda = 1$ and $\lambda = 2$.



$\lambda = 1$ $\lambda = 2$

To derive the limits of $L_S(f_1)$, rearrange the precedent expression as:

$$L_S(f_1) = A + Bf_1 - C(f_1)^{1-\lambda} - D(f_1)^{-\lambda},$$

where:

$$\begin{aligned} A &= k_1 + l > 0, \\ B &= \delta_D - E[\delta_P], \\ C &= \delta_D (c_P)^\lambda > 0, \\ D &= (k_1 - E[\delta_P] c_P)(c_P)^\lambda > 0. \end{aligned}$$

When $\lambda > 1$, as $\lim_{f_1 \to +\infty} (f_1)^{1-\lambda} = \lim_{f_1 \to +\infty} (f_1)^{-\lambda} = 0$, this implies that $\lim_{f_1 \to +\infty} L_S(f_1) = -\infty$, when $B < 0$, $= A$, when $B = 0$, and $= +\infty$, when $B > 0$. In the special case of a uniform distribution (i.e., $\lambda = 1$), the same results hold, except that $\lim_{f_1 \to +\infty} L_S(f_1) = A - C$, when $B = 0$.

To characterize the variations of L_S, differentiate it to get:

$$\begin{aligned} L'_S(f_1) &= B + [(\lambda - 1) C f_1 + D\lambda] (f_1)^{-\lambda-1} \\ L''_S(f_1) &= -[(\lambda - 1) C f_1 + (\lambda + 1) D] \lambda (f_1)^{-\lambda-2} < 0 \end{aligned}$$

From this, if $B \geq 0$, then $L'_S(f_1) > 0$, for all f_1. Otherwise, if $B < 0$, remark first that $\lim_{f_1 \to c_P} L'_S(f_1) = \lambda (\delta_D + k_1/c_P - 1) > 0$ and, as $\lim_{f_1 \to +\infty} (f_1)^{-\lambda-1} = \lim_{f_1 \to +\infty} (f_1)^{-\lambda-2} = 0$, $\lim_{f_1 \to +\infty} L'_S(f_1) = B < 0$. Thus, given that $L'_S(f_1)$ is decreasing (for $L''_S(f_1) < 0$), there exists F such that $L'_S(f_1) > 0$, for $f_1 < F$, and $L'_S(f_1)$, for $f_1 > F$. □

A.4. Proof of property 1.

The expected social costs given D caused an accident is:

$$\begin{aligned} L_S(f_1) &= \int_{c_P/f_1}^{1} ((\delta_D - \delta_P) f_1 + k_1) \, \mathrm{d}G + l, \text{ if } f_1 > c_P, \\ &= l, \text{ otherwise.} \end{aligned}$$

Part (a) is immediate, as $L_S(f_1) - l$ does not depend on l.

In the rest of the proof, we will limit our attention to $f_1 > c_P$.

To show part (b), differentiate to obtain:

$$L'_S(f_1) = \frac{c_P}{(f_1)^2} (\delta_D f_1 + \delta_A c_A + c_D) \, g\left(\frac{c_P}{f_1}\right) + \int_{c_P/f_1}^{1} (\delta_D - \delta_P) \, \mathrm{d}G.$$

From this, we can show that ([4]):

$$\begin{aligned} \lim_{f_1 \to c_P} L'_S(f_1) &= \frac{1}{c_P} (\delta_A c_A + c_D + \delta_D c_P) \, g(1), \\ \lim_{f_1 \to +\infty} L'_E(f_1) &= \delta_D - E(\delta_P). \end{aligned}$$

Under assumption 1, which means that $L'_S(f_1)$ is decreasing, part (c) directly follows from part (b). □

[4] To derive the second limit, remark that, as $g(\delta_P)$ is continuous on $[0, 1]$, it is bounded. Thus, the first term of the derivative will be negligible, for sufficiently large f_1

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