Rules of Action for War and Recursive Optimization: Massé's "Jeu des Réservoirs" and Arrow, Harris, and Marschak's "Optimal Inventory Policy"

Chapter 4 of *Protocols of War: The Mathematical Nexus of Economics, Statistics, and Control Engineering 1940-1960*

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> I hold that the essential distinction between statics and dynamics, if the terms must be used, is not the same as, nor even closely related to that between stocks and flows. In fact the most perfect instances of statical problems are those which deal with "steady flows" of labour capital and goods, of wages, interest and prices in a stationary country: in which each year is just like the past, in which each generation is like that which went before.
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> It may be noted, as an incidental confirmation of this opinion, that our choicest illustrations of the statical or stationary state relate to agricultural and not to mineral prices. Now the annual output of a farm is a true flow: but the annual output of a mine is not a true flow, it comes out of stock: and, if the mineral veins are not practically unlimited, the exhaustion of the stock will disrupt the statical rest. (Alfred Marshall 1898, 46)

This study focuses on two pioneering, but independently derived protocols for dynamic recursive optimization: Pierre Massé's "Application des probabilitiés en chaine á l'hydrologie statistique et au jeu des réservoirs" (formulated in 1940 but not published until 1944) and Kenneth Arrow's, Theodore Harris's, and Jacob Marschak's, "Optimal Inventory Policy" (formulated in 1950 and published in 1951). Both of these studies were prompted by a war-time need for normative guidelines on decision making that would, in a climate of uncertainty, minimize costs while managing flows to and from a stock that is carried over into future stages of decision-making. The mathematical protocols designed to answer the war-time needs were dynamic, stochastic and recursive. Indeed these two studies are two of the earliest formal statements of what Richard Bellman later called "dynamic programming," and they herald a new approach to optimization that now permeates the new classical macroeconomics.

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During the brief war with Germany from September 1939 to June 1940 and under the Vichy regime, Massé, a French hydroelectric engineer stationed in the Pyrenees, faced the problem of determining how much water should be taken from the reservoirs each month in order to minimize the current and future use of coal - the extremely scarce alternative source of electrical power. Massé articulated a mathematical approach to deriving optimal decision rules for operating hydroelectric dams. He brought a stochastic, marginal approach to dynamic production problems, asserting, for example, the efficacy of equating the marginal utility of the flow of water with the marginal expected value of water left in the reserve stock then using a recursive algorithm to determine the marginal expected utility of the water left in the reservoir. Massé's 1944 study influenced postwar developments in the French Marginalist School, optimization studies at the RAND Corporation in the mid-1950s, and his own subsequent work on investment policy and national planning.

In the summer of 1950, the US Office of Naval Research brought Arrow and Marschak to the RAND Corporation, the US Air Force think tank, for a summer research project on logistics and military inventory control. Arrow and Marschak nested the observed business practice of a two-bin inventory policy into a decision-theoretic framework inspired by Abraham Wald's sequential analysis (see chapter 3) for the military during World War II. With the help of the mathematician Theodore Harris, they developed a mathematical model of the two-bin policy in a dynamic, stochastic setting –a multi-stage Markovian decision process where unused inventory could be used in subsequent stages and demand was uncertain. The Arrow, Harris and Marschak paper on "Optimal Inventory Policy," hectographed for the Navy's Logistics Project and published in *Econometrica* in 1951, initiated over a decade of US military-funded research into dynamic, stochastic, inventory policy.

Few historians of science in France and no English-speaking historians of economics have highlighted Massé's marginal, stochastic, recursive approach to optimization, but my purpose here is not make a case for "who said it first." Rather it is to show that during the 1940s and 1950s an applied mathematics evolved in France that was strikingly similar to that which developed a few years later in the USA. In both cases, a multi-stage decision-making protocol emerged from interdisciplinary reasoning for a state client intent upon efficiently allocating, in the absence of a market, very scarce resources to successfully wage war. The cross-disciplinary climate fostered by the curriculum of the Ecole Polytechnic and subsequently when graduates worked together in French nationalized industry was comparable to that which occurred in the American military research groups of the Applied Mathematics Panel during the World War II and the RAND Corporation during the early years of the cold war.

The planning maneuvers that emerged from these institutional environments in France and the USA were built on a continental European erudition of conditional probabilities and shared many features including,

- Reasoning with physical analogies
- Embodying the economic criterion of minimizing costs or maximizing utility over time into a functional equation
- Incorporating uncertainty into mathematical representations of the problem
- Assuming that decisions would be made sequentially in discrete steps and conditional upon the states in other stages
- Nesting the solution process of the original optimization problem in a minimization of computational resources achieved by
    - ☐ Taking the dual approach by solving problems in policy space rather than function (criterion) space
    - ☐ Coding the protocol as a recursive algorithm
    - ☐ Seeking approximate rather than exact solutions
    - ☐ Using constraints imposed by the physical process to ensure convergence
- Declaring the solution in the form of quantified rules of action

Table 1 compares and contrasts the key features of the applied mathematics for a client in the work of Massé for the French l'Union d'Électricité and the work of Arrow, Harris, and Marschak for the US Navy. In 1944, Massé was unaware of similar work for the US military by Abraham Wald on sequential analysis, and even by 1950 Arrow and his colleagues at RAND had not yet seen the studies by Massé. The sections that follow compare details in the studies, but what is striking is that these two independent studies had similar, but very novel mathematical approaches to how a government or corporate manager should regulate a reserve stock in the face of an uncertain future. Both of these exercises in planning through algorithms of optimal control were significant landmarks in the history of operations research and are conceptually similar to Abraham Wald's wartime

Table 1 Comparison of the Recursive Optimization Protocols of Massé 1944 and Arrow, Harris, Marschak 1951

	Massé 1944	Arrow, Harris, Marschak 1951
War Context	WWII, France fighting Germany & Vichy regime, coal very scarce	Cold War, US stockpiling for possible war with Soviet Union
Institutional context for cross-disciplinary thinking	*Ecole Polytechnique* and state-regulated electricity monopoly	US Navy's Logistics Conference at the US Air Force's think tank- RAND Corporation
Physical process	Decision-maker operates dam on a reservoir to supply hydroelectric power to network of plants	Policy-maker manages inventory of raw materials for production process
Economic Criterion	Minimize mathematical expectation of costs of using coal.	Minimized expected loss
Dynamic feature	Water left in reservoir in first month is available for use in next month	Stock not used in first period can be use in second period
Stochastic feature	Uncertain precipitation into the reservoir Uncertain final demand for electricity	Uncertain demand for final good
Known values	Fixed charges, penalties for shortage, costs of having to use coal substitute, probability distributions of precipitation and demand	Purchasing price of supplies including fixed cost for restocking, cost of storage, penalty for shortage, discount rate, probability distribution of demand for final good
Constraints	Reservoir cannot go below empty nor above full level	Stock cannot fall below 0 and should not be greater that *S*
Policy Rule of action	Increase the flow from the reservoirs to turbines if the marginal utility of the flow is greater than marginal expectation of the water left in the reservoir Decrease the flow if marginal utility of flow is less than the marginal expectation of the water in the reservoir.	Two-bin inventory policy (*S,s*) Fix the parameters *S* and *s.* Order only when stock falls to *s* (quantity in second bin) or lower then order to raise total stock to *S*.
Unknown to solve for with recursive protocol	Optimal value of marginal expectation of water in reservoir that minimizes tonnage of coal used	Optimal values of *S* and *s* that minimize loss (costs)
Temporal Structure	Utilities calculated at beginning of the period Month is unit of time, seasonal variation important	Costs calculated at end of period
Effective computation of solution	Recursive operation working from future backwards to solve for rule of regulation. Graphs used to handle constraints	No solution given, numerical methods suggested

work on sequential analysis (discussed in the previous chapter) and Richard Bellman's work on dynamic programming at RAND in the 1950s (discussed in the subsequent chapter).

One of the objectives of this paper is to illustrate how in these two different national settings the institutional context of working for a client competing in an international war led to a similar combination of economics, statistics, and engineering. Therefore, in each case I will begin with describing that institutional context before I go into detail on the specific studies and I will end the respective sections discussing the subsequent legacy of each study.

Normative Economics and les Grandes Écoles

The simple regulation that Massé devised for optimizing the flow of water from the reservoir was equating the marginal utility of the flow of water with the marginal expected value of water left in reserve. In deriving this rule Massé drew heavily on the marginal reasoning of Vilfredo Pareto (1906), but the major challenge he faced was defining and determining the value of the marginal expectation of the water in reserve. It was in meeting this challenge that he took from his *polytechnicien'*s tool bag the Markov chains of Maurice Fréchet, the log-normal distributions identified by Paul Levy and Robert Gibrat, the stochastic gaming of Emile Borel, and the recursive approximation techniques of Blaise Pascal.

Although Pierre Massé is often described as an economist, his formal education was in engineering and mathematics. In the late 1940s and 1950s, there were several engineers, including Bill Phillips, Charles Holt, and Arnold Tustin, who crossed over into economics and pioneered new models and statistical techniques in their new profession (see chapter seven). For the British and American engineers, the attraction of such a crossover came from both a keen desire to engineer a postwar prevention of another worldwide depression and the considerable increase in demand for and status of economists, particularly in the USA, that accompanied the professionalization of the discipline during World War II and the cold war (see, for example, Homan 1946 and Bernstein 2001).

The USA only began to witness the phenomenon of a government employing engineers-turned-economists to practice normative economics (telling the client how to minimize costs or how to maximize damage from nuclear bombs) during the 1940s. There was, however, a much older tradition combining engineering, normative economics and government service in France. In the mid nineteenth century, for example, Jules Dupuit developed a utility theory in the course of his work as *Inspecteur-Général des Ponts et Chaussées.* Indeed, Dupuit's measurement of public utility by the area underneath the demand curve was in keeping with his oath of public welfare that he declared when he joined the state corps of engineers (see Porter 1991, Porter 1995, and Klein 1995).[2]

The combination of normative economics, engineering and mathematics was fostered by a French tradition of employment of a relatively large percent of professionals in the state engineering corps after their formal higher education tracking through the selective *grandes écoles*. The would-be state engineers had to take two to three years of intensive post-*Bacculaureát cours préparatoires* for the entrance exams into the *Ecole Polytechnique*. This expensive barrier to entry, along with the law that engineers employed by the state were recruited solely from the *Ecole Polytechnique* ensured that the students and thus the state engineering corps were from families of the social elite.[3] The "X" was a key symbol on the coat of arms of the *Ecole Polytechnique*, and once admitted, the *polytechnicien*s carried with them the label of the year in which they passed their entrance exam and were promoted into the *Ecole Polytechnique* (for example, the description following Massé's name is X 1916 or X promo 1916).[4] The students at the *Ecole Polytechnique* often completed additional courses at the *Ecoles d'application*, including the *Ecole des Mines*, and the *Ecoles des Ponts et Chaussées,* which is where Massé studied. The courses at the *Ecole Polytechnique* and the *Ecoles d'application* were more theoretical and mathematical than engineering courses in the USA or Britain and they were considered

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[2] Michel Armatte (1996, 1998) has told the stories of two other French engineers before WWII (François Divisia and Robert Gibrat) bringing their training in mathematics and statistics to the discipline of economics.

[3] Private enterprises generally hired engineers trained at lower-tiered, less selective institutions such as the *Ecole Centrale des Arts et Manufactures* or the *Ecoles d'Arts et Métiers*. Several historians of science have written about the nineteenth and twentieth century training of French engineers in the *grandes écoles*, including Shinn 1980, Weiss 1982, Kranakis 1989, Belhoste et. al 1994.

[4] In 1916, Massé was admitted into the *Ecole Polytechnique* and the science program at the *Ecole Normale Supérieure*, (a prestigious training school for teachers). He, however, did not act on either opportunity until the end of his World War I military service in 1918, at which time he chose the *Ecole Polytechnique* (Boiteux 1987).

more demanding than the science courses at French academic universities. The *grandes écoles*, however, did not award doctorates, so some *polytechnicien*s, such as Massé defended their doctoral theses at a university after their engineering training (see Massé 1935).

The culture and curriculum of the *Ecole Polytechnique,* which Napoleon established in 1794 for the training of military engineers, fostered a commitment to state service and normative economic thinking. There was, however, an additional stimulus during the great depression of the 1930s. A high unemployment rate, prolonged stagnation, and a deskilling of engineering jobs that had accompanied an accelerated move toward mass production, encouraged the *polytechnicien*s to look for solutions to the economic crisis from within their own ranks. In an attempt to counter the oversupply of laborers calling themselves engineers, the *polytechnicien*s successfully persuaded the state in 1934 to legislate a definition of the professional title of *ingénieur* (see Grelon 1986). More relevant to our focus, however, hundreds of *polytechnicien*s joined a group to study the depression and possible solutions to the crisis. In 1931 three *polytechnicien*s, Gérard Bardet, André Loizillon, and John Nicoletis, established a group called *X-Crise* with a bulletin called *X-Information*. Through their bulletin, they called upon the *polytechnicien*s to do their duty, as a professional elite, to study the facts of the crisis and to research causes and solutions.[5] By the end of 1933, the group had grown to over 2000 members, some of whom were not *polytechnicien*s, and had established a *Centre polytechnicien d'études économiques* (CPEE). Their monthly bulletin, *X-Crise - Bulletin du Centre Polytechnicien d'Etudes Economiques* contained treatments on the economic crisis from a variety of ideological perspectives and from a variety of disciplines. This is evident from a collection of articles and CPEE lectures reproduced in a book celebrating the 50th anniversary of *X-Crise* (Centre polytechnicien d'études économiques 1982). Marc Bloch, Louis Vallon, Françios Divisia, Paul Valéry, Ernest Mercier, René Roy, and Robert Gibrat were among the historians,

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[5] For example Bardet, wrote in the first issue of X Information in August 1931 (quoted in Centre polytechnicien d'études économiques 1982, 13):

> Devant cette situation, n'avons-nous pas le devoir, en dehors de toute considération de parti, de réfléchir, de comprendre et de prévoir ? On se plaît, à répéter que nous constituons une élite. Il appartient donc au milieu polytechnicien d'examiner en toute indépendance d'esprit les données de ce problème vital. Nous y intéresser, c'est justifier notre réputation et être fidèle aux principes selon lesquels l'Ecole polytechnique a été conçue.

philosophers, economists, statisticians and polytechnians who actively participated in *X-Crise*. A common debate within the movement was whether the economy should be "free" or managed. Both sides of this debate drew on a *polytechnicien*'s faith in rationality in ending the crisis, as is evident in the title of one of the most widely known works to come out of X-Crise written by Georges and Édouard Guillaume (1937), *Économique rationnelle*. Michel Armatte (2000, 130) has documented the keen interest of the *polytechnicien*s in using mathematics to address the economic crisis and he asserts that the *X-Crise* movement strongly influenced the post-war econometric work by Divisia, Roy, and Maurice Allais, as well as the operational research approach of Massé.

From an economist's perspective, it is interesting that the lectures and articles associated with the CPEE were as likely, if not more so, to be in the area of what we know called microeconomics, as they were to be macroeconomics. Massé, while acknowledging the significance of the general theory of John Maynard Keynes, explicitly confined his normative work in the early 1940s to the level of an individual firm. The depression, the war, and the nature of the physical process he studied forced Massé to incorporate uncertainty into his theory of the optimal operation of the firm (see for example, Massé 1946, I: 15-16 and Massé 1984, 68). Indeed, Massé came to assert that the only way to overcome the vagaries of chance was through a rule of regulation of a reserve that fully acknowledged future uncertainty.

The War and Hydroelectric Power

In 1928, Massé began his career in working with hydroelectric power. His initial employment was with a regional company, but he soon became involved in a larger network (l'Union d'Électricité) that served as the foundation for the nationalized Électricité de France created in 1946. For several years Massé worked constructing and regulating the hydroelectric works at several mountain lakes and reservoirs in the Pyrenees, including the Portillon hydro plant, and the Chastang plant on the Dordogne. In his memoirs, Massé admitted that he had a penchant for water (Massé 1984, 35).[6] The changeable rain and snow

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[6] In 1935, he successfully presented his Ph.D. thesis in mathematics at the University of Paris on the subject of dampening turbulence in water currents. Several mathematicians had looked at the propagation of surface waves in the abstract, but these earlier studies had not taken into consideration the inevitable dampening of these oscillations upstream and downstream. In addition to considering this realistic and practical aspect of the gradual dampening of the oscillations, Massé constructed a mathematics that treated upstream effects as

feeding mountain streams, the stock of water in a reservoir, the controlled flow of water from a reservoir, and the translation of water power into electricity were not only variables in Massé's microeconomic mathematical models, they also became his metaphors and levers for macroeconomic analysis of investment and for planning in national economies. Massé saw the regulation of reserves as the key to confronting chance uncertainties, and he used the image of a dammed reservoir with uncertain precipitation feeding into the reservoir to explore rules for regulating the optimal flow from the reservoir. Massé's opening line for his major work on *Reserves and the Regulation of the Future* was "It is by putting in reserve that man frees himself from chance." (Massé 1946, 3)[7]

Despite all his emphasis on water as a variable and a metaphor, it was Massé's wartime concern with coal that sparked his recursive model for determining the optimal flow from a reservoir. The great shortage of coal in first eight months of the war between France and Germany evoked what Massé called "the irony of war and my meeting with chance" (Massé 1984, 67).[8] During the brief war, the primary concern of the managers of the electrical network, which had been greatly centralized in France in 1939, was to minimize the use of scarce coal. In the absence of an internal market in coal and thus prices, the kilowatt hours that had to come from coal was the key variable Massé and other managers had to minimize. The managers of the network faced choices - to take water from the reservoirs for hydroelectric power (thus reducing immediate need for coal and/or avoiding penalties for meeting demand) or use coal instead and save the water for future use. They also faced the choice of which reservoir to draw the water from, for example, it was advisable to first draw the water from the central reservoir nearest the population centers in preference to taking it from the reservoir further away in the Alps. It was also essential to consider seasonal patterns including a high demand for electricity during winter, and the great inflow of water that usually, but not always, came with the melting of the snow in late spring. The biggest challenge to any decision-making protocol for the French electricity

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an image of downstream effects. He determined that the perturbations created by the operation of hydroelectric turbines were like waves that were the average between short, purely undulating oscillations and waves with a trend-like current. Massé compared predictions from his mathematical model with results from several experiments on French rivers.

[7] C'est par la mise en réserve que l'homme se libère du hasard. Libération qu'il faut entendre au sens le plus large, intellectuel, biologique et social. (Massé 1946, I :3)

[8] "Je remonte ainsi à la drôle de guerre et à ma rencontre avec aléa." (Massé 1984, 67)

industry was the end of summer when three out of four years one could expect a wet autumn (see for example, Massé [1959] 1962, 326). Depending on the availability or cost of coal and the evaluation of shortage penalties, one would seek safety and keep the reservoir full, risking a waste of water if the rains came, or seek immediate savings and draw the water for hydroelectric power, risking a shortage in a dry autumn that came once in four years.

Massé drew on the empirical evidence of seasonal patterns, the qualitative decision-making practice he observed from his colleagues, and the training and discourse of *polytechnicien*s of his generation to construct a theoretical framework for making a decision each month as to how much water should be taken from each reservoir in order to minimize the amount of coal used over the course of time. Massé sketched out his procedure for determining rules of action in the *Annuaire hydrologique de la France pour 1940* (not published until 1943). He presented a full treatment of his mathematical protocol to the Société de Statistique de Paris in June 1944 and published a two volume book on using reserves to control the future in 1946 (the first volume was on the deterministic future of the short term and the second one on the stochastic future of the long term). Massé ([1959] 1962) also spelled out his recommended mathematical procedure for the management of hydroelectric reservoirs in the 1962 English translation of *Optimal Investment Decision: Rules for Action and Criteria for Choice*.[9]

In those uncertain months of war with Germany in early 1940, when an efficient operation over time of the reservoirs was essential to minimize France's use of scarce coal, Massé realized that the mathematical solution had to take the form of a conditional policy - a policy for each stage, based on the state for that moment, which depended not only upon uncertain new inflows, but also upon the decision taken in the previous stage.[10] Massé (1946 I: 11) used the image of a fork to describe what strategists and game theorists of the 1950's would later call a decision tree. Thus the appropriate solution to any mathematical

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[9] In this later work, Massé replaced the minimization of coal with the more general goal of minimizing costs or maximizing profit and he discusses other approaches to inventory management and investment decisions including the study of Arrow, Harris and Marschak and George Dantzig's linear programming.

[10] Puisque l'inconnue ne peut pas être sans contradiction, la suite ne varietur des états de la réserve, elle ne peut être qu'une règle conditionnelle, - une stratégie, - définissant à chaque époque, en fonction des transmettre pendant la période élémentaire immédiatement suivante de manière à réaliser un optimum économique en probabilité. (Massé 1984, 70)

problem constructed to represent the decision confronting the manager of the reservoir was a rule of action that could serve as a strategy for each stage based on a description of that stage that included information based on possible choices made in previous stages.

Massé (1944, 207) explained that the *jeu* in his description of this problem, "*le jeu des réservoirs*" captured two meanings- *jeu* meaning the operation of the reservoirs, and, more relevant to his goals, *jeu* meaning speculating on an uncertain future, as one would do in a game of chance. His answer to the game was a rule for regulating the flow from the reservoir: increase the flow of water from the reservoirs to the turbines if the marginal utility of the flow from the reservoir is greater than the marginal expectation of the water kept in the reservoir; decrease the flow of water from the reservoirs to the turbines if the marginal utility of the flow from the reservoir is less than the marginal expectation of the water kept in the reservoir. Where the conditional, probabilistic statistical reasoning and the recursive mathematical algorithm came in was in the major problem of estimating the value (the marginal expectation) of the water kept in reserve. For Massé, the marginal expectation was the derivative of the total expected value of the reserve in respect to the accumulated volume and the probable utility of an extra kilowatt hour in reserve.

Massé's Rule of Exploitation and Principle of Regulation

Massé stated that the ultimate goal was to maximize utility or minimize costs, but he realized that given the physical process he was working with, including the constraints of not being able to go beyond an empty or full reservoir, the most effective way to compute a solution was to solve for a rule that would achieve an optimum rather than to solve for the maximum value of utility. As we will see in the next chapter, Richard Bellman demonstrated that in multi-stage decision problems, seeking the solution in what Bellman called policy space was the dual of solving the problem in function [criterion] space. In his work for the USAF, Bellman realized that the solution the military clients wanted was a policy and often solving for a policy was a far easier route to solving the optimization problem than solving first for the value of the maximum returns or minimum costs. The seasoned wisdom of the decision-maker gave clues to good starting points for approximations in policy space and the constraints on the problem often helped in ensuring that these approximations in policy space led to a relatively rapid convergence to a solution.

The classical approach of working in function space (for example, using the calculus of variations to analyze functions which minimized or maximized a value over time subject to constraints-) often did not lead to an actual solution, but rather only to a declaration on the existence and uniqueness of the solution. Bellman demonstrated that using approximations in policy space were more likely to achieve an actual solution using less computational resources. Indeed the scarcity of computational resources often meant that approximate rather than exact solutions were sought. The client-orientation also meant that the sought-after solution took the form of a simple rule of action that could be applied in a variety of circumstances.

Prewar mathematical approaches to determining the optimum over time were of little use to Massé because they had assumed perfect knowledge - in reality there was uncertainty in terms of the inflow into the reservoir and the final demand for electricity and any decision had to take those uncertainties into account. Massé, did however, draw on the marginal approach to determining optima as highlighted in Vilfredo Pareto's (1906) *Manual of Political Economy*. Massé particularly found useful Pareto's notion of arbitrage in time that compared the values of the same good at different times. One of the key steps in Massé's analysis was to declare a necessary condition for maximizing total utility - a condition for the optimum: *the marginal utility of the flow of water from the reservoir into the turbines should be as much as possible equal to the marginal expectation of the water left in the reservoir.*[11] This strategy for achieving an optimum made sense in the scheme of Pareto if one saw, as Massé did, the marginal expectation of the reserve as similar to a marginal cost of production incurred by an *enterprise d'arbritrage* managing transfers between the known present and the unknown future (Massé 1944, 209).

The marginal utility of a flow of water into the turbines could be measured by the equivalent energy value of coal saved (or after the war, by marginal revenues). A major stumbling block for Massé, however, was how to measure the marginal expectation of the

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[11] C'est-à-dire l'égalité de l'utilité marginale de la làchure et de l'espérance marginale de la réserve résiduelle. Massé 1946, 209. Restated in Masse [1959, 1962, 329): "At the optimum the marginal revenue of power withdrawn from the reserve is equal to the marginal expectation of the power left in reserve."

water in the reservoir.[12] The water left in the reservoir had value for future production of electricity, but because its value was all in the future, it had to be a value that was

- Discounted (taking into consideration that present is preferred to future)[13]
- Probable (future inflows into the reservoir and future demand for electrical power were subject to chance)
- Conditional (the amount in the reservoir at each future stage depended upon decision as to how much to take out in the previous stage)

To specify probability distributions for inflows and demand, Massé drew on the empirical work of his *polytechnicien* colleagues in the hydroelectric industry, Robert Gibrat and Étienne Halphen. In 1932, Gibrat had demonstrated that the logarithms of the flows approximately followed a normal, Gaussian law of errors.[14] Halphen looked at the correlation of the flows over times, and demonstrated that the logarithms of the flows were Gaussian and that they were correlated to each other over time in a simple Markovian chain. In assuming a simple chain, (values in one month depended on value in the previous month and no further past history was necessary), Massé also relied on Maurice Fréchet's (1938) and Paul Levy's (1937) demonstrations of how over a long period of time, one could in certain circumstances assume a statistical regularity independent of initial conditions (the ergodic principle).

To achieve the conditional feature essential to this process of making decisions for and uncertain future, Massé used a recursive algorithm for working backwards from a point in the future to the present and dealt with the problem of the terminal conditions. This is how Massé described it in a later work that used the same technique:

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[12] With regard to interpreting what he meant by "marginal" in the marginal expectation of the water in reserve, Massé ([1959] 1962, 333) explained, "We can conceive of this increment as a thin layer of oil lying on top of the reservoir. According to circumstances … this increment will be kept in reserve, with drawn from the reservoir, or divided between reserve and immediate withdrawal."

[13] In the 1944 study when costs and utility were measured in terms of the kilowatt-hour equivalent of coal, Massé did not incorporate discounting to compute present values.

[14] Michelle Armatte describes Gibrat's novel articulation of the law of proportional effect (the lognormal law) , how this major contribution compared with alternative models of Pareto, Karl Pearson and Francis Edgeworth, and how Gibrat applied this lognormal law to measuring income inequality and to prediction in the hydroelectric industry.

> Now that we have defined the notion of strategy and shed light on its fundamental aspects, we shall take up its use in stochastic processes.
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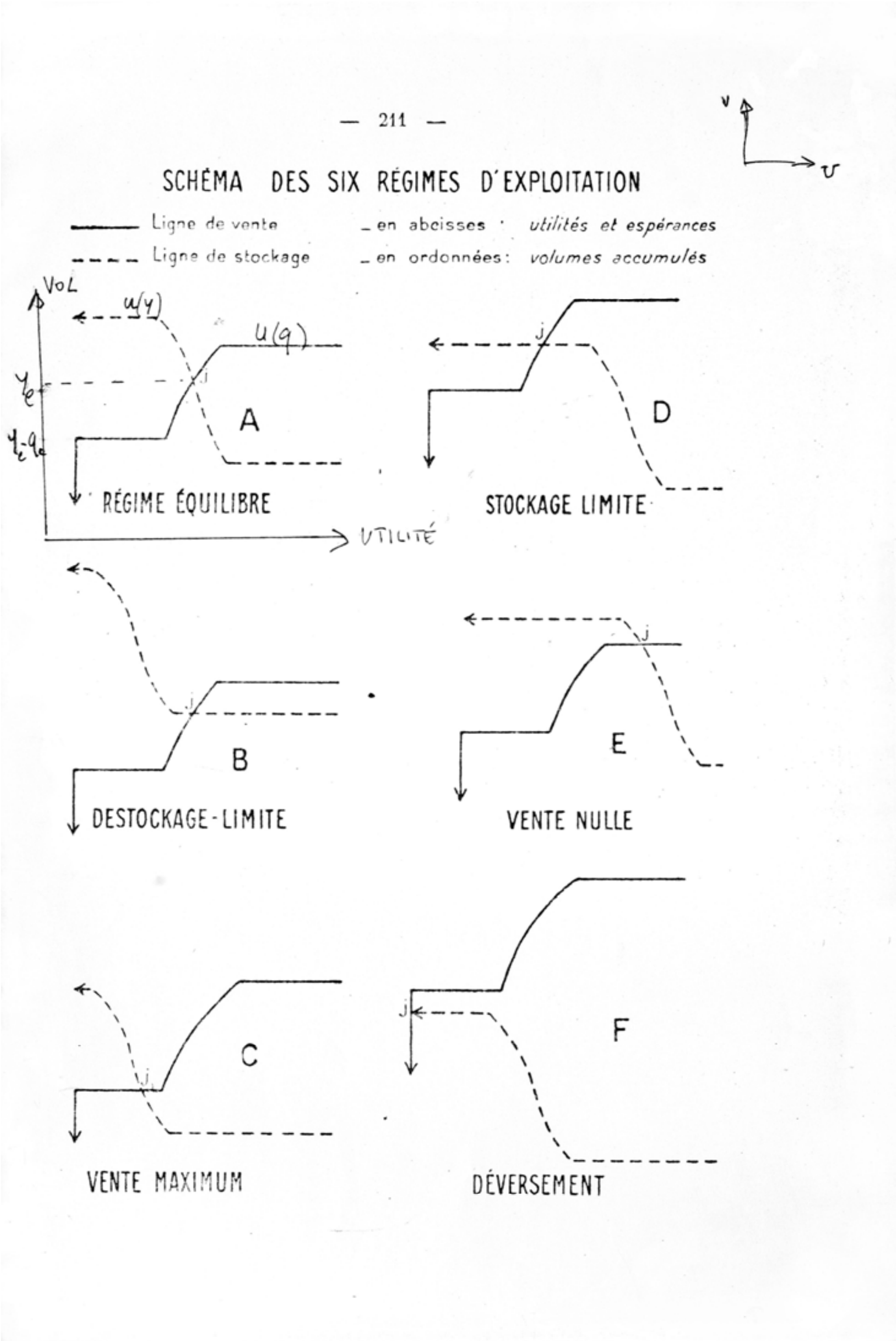
> It use implies a number of assumptions, which may be reduced to the following: it assumes that a description of future states can be attempted in terms of probabilities and that under these conditions it is possible to pose the problem of the optimum in terms of mathematical expectations….
>
> [My] approach proceeds by retracing the course of time; it tries to approximate via small adjustments a strategy that will be constantly optimal for a given "terminal convention." Each round in the iterative process determines simultaneously the optimal expectation and the optimal strategy, which becomes progressively independent of the terminal convention. (Masse [1959] 1962, 266, 274)

One problem with such a recursive approach is how to insure rapid convergence to a solution. To partly ensure this and to make his technique applicable to the physical process under study, Massé used the constraints of the reservoir system to give boundaries to the solution process. The reservoir could not go beyond full or empty. As the water level in the reservoir dropped and the demand for power increased, the manager was faced with five successive stages:

- Discharge (full reservoir, excess wasted in spillover, no thermal power used)
- Maximum reserve
- Equilibrated system
- Maximum use of thermal power
- Shortage (reservoir empty, thermal power used, but still not enough)

Massé used graphs for each stage to illustrate the achievement or lack thereof of an optimum value (see Figure 1). The graphs enabled the user to specify some of the values

Figure 1. Massé's (1944, 211) schema of actions that would result from six of the possible intersections of the marginal utility of the flow (solid line) and the marginal expected utility of the water remaining in the reservoir (broken line). In each case the value of the marginal utility or marginal expectation is measured on the horizontal axis, and the vertical axis measures volume of the reserve at end of period or value of random demand at beginning of period minus power withdrawn from the reservoir during the period.



for the equation of the sum of the marginal expectations (equation No. 6, Massé 1944, 210, see Table 2 for Massé's definitions):

$$s_{n+1}(X) = \int_I s_n(Y) k_n(x) dx + \int_E u_n(q) k_n(x) dx \,,$$

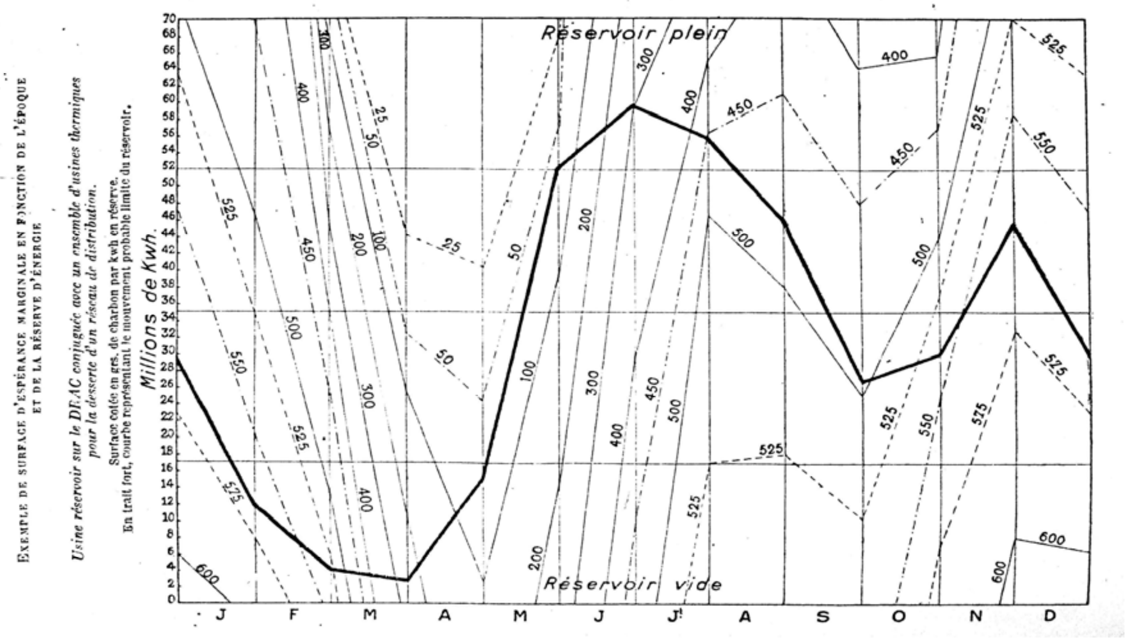
Table 2: Massé's (1944, 208) definitions of symbols

Symbol	Definition
x	volume entrant au cours d'une période;
q	volume transmis (ou lâchure) au cours d'une période ;
X	volume en réserve au début de la période ;
Y	volume en réserve à la fin de la période;
M	maximum capacité du réservoir ;
$k_n(x)$	loi de probabilité de x au cours de la n^0 période ;
$U_n(x)$	utilité totale de la lâchure q au cours de la n^0 période
$S_n(Y)$	espérance totale de la réserve Y à la fin de la n^0 période;
$u_n(q) = \frac{dU_n(q)}{dq}$	utilité marginale de la lâchure q;
$s_n(Y) = \frac{dS_n(Y)}{dY}$	espérance marginale de la réserve Y.

Massé also used the graph reproduced in Figure 2 to illustrate the values of the marginal expectation of the reservoir that one could expect given the typical seasonal pattern of the volume of the reservoir.

Massé's recursive process of optimization was a specific case of what Richard Bellman would in 1950 call "dynamic programming". As we will see in the next chapter, the key to Bellman's dynamic programming was model construction based on the principle of optimality: "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision." (Bellman 1954, 285 but repeated word for word in many of Bellman's publications). Bellman formalized the protocol and generalized the

Figure 2. Massé's (1944, 213) diagram of the relationship between the value of the marginal expectation of the reservoir in relation to the seasonal limits of the reservoir. The months of the year are marked on the horizontal axis and vertical axis measures millions kilowatt hours. The bold solid line indicates the typical seasonal limits of the energy stored in the reservoir- coming close to its maximum capacity in June and to near empty in March. The diagonal lines indicate the probable limits (broken lines) and level of marginal expectation in grades of carbon per kilowatt hour in the reservoir.



principle, but as Kenneth Arrow pointed out in his 1957 Presidential address to the Econometric society, the studies of Massé and, a few years later, of Arrow, Harris, and Marschak were precursors to the technique of dynamic programming.

> It would be easy to show that much of the reasoning used in capital theory has in fact made use of the principle of optimality. The explicit recognition of this principle has stemmed from the work of P. Massé (though the formulation is somewhat different), and of A. Wald on sequential analysis of statistical data, which can be regarded as a specific case of the general principal. An application to the theory of inventory holdings was given by Arrow, Harris and Marschak; see also Dvoretsky, Kiefer and Wolfowitz. R. Bellman was the first to see the generality of the procedure, to which he has given the name of dynamic programming. (Arrow, 1957)

Economics in the EDF and CNRS

Massé conceived of his protocol for recursively deriving a rule of action in the uncertain months of war between France and Germany. On June 22, 1940, Prime Minister Henri Philippe Pétain surrendered to Germany. The surrender agreement conceded northern and western France and the entire Atlantic Coast to the German occupation. Pétain's government, with their new capital in Vichy, retained administrative jurisdiction over the remaining two-fifths of the country.[15] Under the Vichy agreement *Liberté, egalité, fraternité* was replaced with *travail, famille, patrie* (work, family, fatherland). In his memoirs, Massé wrote of his efforts with the French resistance during this time. It also appears that he and others under the Vichy regime took refuge in their "*travail*", which could serve as a singular source of pride and achievement in a circumscribed life. Massé (1946, I: 6) was explicit that his analysis of decision-making fleshed out during the first half of the 1940s was invariant to any surrounding social structure and without preoccupation on the economic system in which it was used.[16]

For Massé (1946, I: 22), the notions of "optimum" and "control" were interwoven. In the forward to his book on investment choices, Massé explained:

> This book is an introduction to the search for the optimum, understood as the optimum in both quantity and quality. It is an attempt to develop rules for action and criteria for choice. There will therefore be no cause for surprise if we systematically follow a normative approach at the expense of the descriptive. (Massé [1959] 1962, ix)

This emphasis on the normative and Massé's ingenious combination of thinking at the margin, assuming conditional probabilities, and approximating optimal policies with recursive computational algroithms strongly influenced his later theoretical treatment of

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[15] Pétain's government also agreed to identify and hand over all Jews to the Germans, to prevent emigration from France, and to pay for German occupation costs. In April 1942, Pierre Laval took on effective administrative control of the Vichy territory. On November 11, 1942, Germany occupied the whole of France and disbanded the small Vichy military force. France was liberated from German occupation on September 10, 1944.

[16] Les méthodes que je vais essayer de développer ont un caractère invariant par rapport aux transformations de la structure sociale. Elles s'appliquent à n'importe quelle définition de l'entité détentrice des réserves et de la mesure corrélative des utilités et des coûts. Leur objet unique est de dégager, sans préoccupation de système, les règles d'action optimum correspondant à ces définitions. Cela ne signifie point que je sois indifférent à l'évolution de la structure sociale, mais simplement que je traite ici une autre question. (Massé 1946, I: 6)

economic investment, his career in the hydroelectricity industry, and the work of other French economists. Massé extended his rule of regulation, derived for the *le jeu des réservoirs*, to investment in general: “At the optimum point, the marginal expectation of the actual cost (the cost of period t) is equal to and opposite in sign to the marginal expectation of future discounted costs (those posterior to period t). (Masse [1959] 1962, 274)”. He also preached and practiced the necessity of forming expectations based on conditional probabilities in his key role as Commissaire Géneral du Plan for France from 1959 to 1966 (see for example, Massé 1962 and 1965 and Boiteux 1987).

In 1946, Massé became Director of Equipment of Électricité de France. In 1948 he was promoted to Assistant Director General and Director of Economic Studies of the EDF and in 1957 he became he became president of l’Électricité de Strasbourg. From 1966 until 1969, Massé was Directeur Général Adjoint at Électricité de France. Through the work of Massé and his colleagues, particularly Marcel Boiteux, the EDF was in the early 1960s the only public utility enterprise in the world to use marginal cost pricing as a basis for charges and for investment policy (Nelson, 1963, 474).[17] Two histories of the EDF (Picard et. al. 1985 and Lévy-Leboyer & Morsel 1994) document the major influence of marginal economic thinking in general and Massé in particular on the course of the EDF. As Martin Chick described it,

> What is striking about the post-1945 history of the EDF is the central role played by economists in devising criteria and methods for the pricing of output and for the proportionate use of thermal and hydro sources of energy. That such allocative issues attracted the attentions of economists is not surprising. What is more unusual is the extent to which their ideas were implemented. (Chick, 1998, 130)

Jacques Drèze (1964) has documented the importance of the economic work in the EDF, along with the work of the *polytechnicien* Maurice Allais (X 1931) on transportation, in establishing a French marginalist school. More recently, in a biographical essay, Nobel laureate Edmond Malinvaud (X 1942) described the context in which he, Massé and other French engineering economists worked:

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[17] James Nelson (1964) edited a collection of English translations of several key works by Massé, Boiteux, and other colleagues in the EDF on marginal cost pricing. See also Chick 2002.

> In my environment during the 1950s and 1960s, economics was viewed mainly as a normative science, to be used for public policies and public management. My older equivalents, [e.g. Massé] who had entered professional activity before the war, had developed an acute sense of the need for conducting economic affairs more rationally. This was the case of engineers serving in public utilities or in the ministries controlling transports, mining, and the like. (Malinvaud 2001, 5)[18]

During the 1950s the French Centre National de la Recherche Scientific (CNRS) provided major forums for the exchange of European and American approaches to mathematical, statistical, and engineering economics. In 1951, the CNRS began sponsorship of a research group that met under the direction of René Roy, Inspecteur Général des Ponts et Chaussées. A paper by Boiteux (1951) on marginal cost pricing appeared in the first issue of the group's *Cahiers du séminaire d'éconmétrie*. A sense of the international scope of Roy's seminar can be glimpsed from a 1953 seminar that brought together presentations by Roy, Massé, Tjalling Koopmans, and H. S. Houthakker. A CNRS conference on dynamic models in Paris brought together Roy, Allais, Massé, Frechet, Frisch, William Baumol, R. M.Goodwin, Lawrence Klein, and H. Theil, among others. Roy, Massé, Fréchet, Gibrat, Maurice Allais, Malinvaud, Divisia, as well as Arrow, Marschak, Ragnar Frisch, Herman Wold, Leonard Savage, Paul Samuelson, Milton Friedman, and George Shackle were among the participants at the five-day Colloque international d'économétrie sponsored by the CNRS in Paris May 1952. It was in these CNRS forums that Massé and Arrow and Marschak were finally able to meet face to face and discuss the recursive optimization protocols they had independently developed for optimal management of stocks and flows.

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[18] Malinvaud asserts that this normative approach that drew heavily on both mathematics and statistics, was not readily taken up in French academia outside the Grandes Écoles. Malinvaud, did however, feel comfortable in European workshops with Ragnar Frisch, Jan Tinbergen, Richard Stone and others who worked on planning and shared an ethic of public service: "Although technical issues were being discussed, most of us agreed on the ultimate aim of our work, namely to improve economic management in our democratic societies. Thinking in terms of economic planning was viewed at the time as appropriate. (Malinvaud 2001, 6)." In his autobiographical essay, Malinvaud (2001, 6) acknowledged that the would-be planners were too optimistic and paid insufficient attention to private incentives and the increases in complexity that accompanied increased globalization. Malinvaud, however, lamented the subsequent downgrading of "old practical rules for cost-benefit analysis" and the triumph of political considerations over economic rationality that accompanied the mid-1980's trend towards privatization and the disdain for government intervention.

War and the Business Practice of the Two-Bin Inventory Policy

Immediately after World War II, the US military extensively funded basic research into resource allocation, which they considered on par with the two other military decision processes of strategy and tactics.[19] Control of inventories large in scope and scale was one of the major resource allocation problems confronting US military forces. They held these inventories in preparation for M-day (the unnamed day when troops and equipment would be fully mobilized for engagement in a newly declared war) and D-day (the unnamed day when a major operation was to commence to force a settlement). The Proceedings of Aircraft Procurement Conference held in Washington DC in July 1939, illustrate the climate surrounding planning for an M-day that eventually ushered American military forces into WWII. At that conference, General H. H. Arnold spoke to airplane manufacturers about the problems of increasing inventories before M-day and ensuring a high productive capacity after M-day:

> We have partially solved one problem by saying we are going to have a reserve of 2,000. That is the reason we have been standing out for a reserve of airplanes in this present bill. We need 3,000 for operation and 2,000 for reserve, merely to take up the lag until you fellows get into production. Normally, we will be able to get an advance on M-day. But it does not eliminate at all the necessity for building up productive capacity in the United States (Proceedings of Aircraft Procurement, 1939, 14)

General Arnold was in essence specifying the details of a two-bin inventory policy for airplanes: put 3,000 in the first bin, and 2,000 in the second bin. The quantity of 2,000 in the second bin is determined by how many aircraft you expect to use up between the time of the new order and the time that order is delivered.[20] Private businesses, and to a lesser extent some military units, commonly used the two-bin policy as a practical solution to the inventory problems of when and how much to reorder in a situation where there was a time

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[19] (see, for example, Smith 1968, 3)

[20] In the notation of the two-bin policy problem, the value 5,000 airplanes ordered is S and the 2,000 planes in the "reserve bin" is s. Business writers (see, for example Wilson 1934) enhanced the widespread business practice of the two-bin policy with their templates for graphs and data tables that would make the practice more "scientific". These scientific approaches, however, while dynamic, assumed certainty of demand. Arrow, Harris, and Marschak demonstrated how to get the optimal values for S and s when demand for the final good was a random variable.

lag between ordering and restocking and there were costs associated with storage, reordering, and running out of your stock before satisfying demand.

In the summer of 1950, the Office of Naval Research convened a logistics research conference at the Air Force's RAND Corporation to examine inventory control. Two economists participating in the small research group, Kenneth Arrow and Jacob Marschak, honed in on the business practice of the two-bin policy. With the help of the mathematician Theodore Harris, they developed a mathematical model of the two-bin policy in a dynamic, stochastic setting –a multi-stage decision process where unused inventory could be used in subsequent stages and demand was uncertain. In their study, hectographed for the Navy's Logistics Project and published in *Econometrica* in 1951, Arrow, Harris and Marschak modeled the optimal maximum stock *(S)* and best reordering point *(s)* as functions of the distribution of the random variable of demand, the cost of reodering, and the penalty for a shortage.

Arrow, Harris, and Marschak's study on "Optimal Inventory Policy" was an important bridge between Abraham Wald's sequential analysis (see chapter 3) and Richard Bellman's dynamic programming (see chapter 5). In the summer of 1948, Arrow, along with Meyer Girshick and David Blackwell, put the SRG's sequential analysis protocol into a formal "decision-theoretic framework," as Arrow (2002, 1) called it. They formulated a loss function specified a recursive process that at each stage recalculated the values of a few parameters based on the solution at the previous stage such that the loss function was minimized. Two summers later at RAND, Arrow applied a similar recursive optimization framework to work on the inventory management problem for the Navy. Wald's successful sequential protocol also inspired Arrow, Harris and Marschak to make "rules of action" central to their inventory control protocol. In place of the rule inspired by Wald's log probability ratio that yielded at each stage the action of accept, reject, or go through another stage of observation, Arrow et. al. used the rules of the two-bin policy: order *S* (the total for both full bins) when the stock falls to *s* or lower (the reserve in the second bin). This rule was prescribed in business literature and widely observed in practice. Arrow, Harris and Marschak used their recursive optimization approach to solve for the optimal values of the parameter *S* and *s*. Bellman, inspired by Wald's work and that of Arrow, Girshick, and

Blackwell (1949) and Arrow, Harris and Marschak (1951) was soon generalizing the recursive optimization protocol by specifying a recursive functional equation that embodied the economic criterion and solving for not only the numerical values of the parameters of various rules of action through approximations in what he called "policy space," but also solving for an optimal rule of action itself.

In their *Introduction to Operations Research*, West Churchman, Russell Ackoff, and Leonard Arnoff (1957, 195) asserted, "More O.R. has been directed toward inventory control than toward any other problem area in business and industry." Their OR text, as with almost all others of that time period, included more than one chapter on inventory control. The research for most of the major studies into optimal inventory control was financed by the military. To understand why a government championing the free market side in an ideological cold war should finance major research projects in production planning, we have to look at the important role resource allocation played in World War II military engagements and cold war stockpiling.

By the end of World War II, the US government was consuming half of industrial production, and the high percent of the US adult population involved in industrial mobilization and military logistical support was considered a major factor in allied success.[21] Between 75% to 90% of the day-to-day activity of US military personnel during World War II and the cold war was devoted to logistics. The US Army ([1948] 1993) declared World War II, "a war of logistics".[22]

Immediately after World War II, the US government was determined not to sink back to prewar levels of unpreparedness – for example, between 1918 and 1933, the US

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[21] George Dantzig (2000) described his work on resource allocation in the Combat Analysis Branch of the Statistical Control Division of the United States Airforces during World War II:

> At the end of the war, it was essentially doing the same thing as planning a whole country. Done on an enormous scale….Everything was planned in greatest detail: all the nuts and bolts, the procurement of airplanes, the detailed manufacture of everything. There were hundreds of thousands of different kinds of material goods and perhaps fifty thousand specialties of people. My office collected data about the air combat such as the number of sorties flown, the tons of bombs dropped, attrition rates. I also became a skilled expert on doing planning by hand techniques.

[22] The quotation from the final report of the Army Service Forces on *Logistics in World War II* continues: "Never before had war been waged on such varied, widespread fronts. Never had one involved so many men, so much materiél, nor such great distances. Never had combat operations so directly affected whole industrial systems and populations."(U.S Army ([1948] 1993, 32).

The title of James Huston [1966] history of logistics is the "sinews of war", but he and other military personnel have also described logistics as "military economics" or "the economics of warfare".

produced had only 35 tanks compared with 88,430 during World War II [Gropman 1997, 54]). The resolve to be better prepared entailed maintaining large cold-war inventories and planning strategies for logistical support.

In 1953, Whitin estimated that the Navy had more than three million different items for which inventory records had to be kept. In the Navy Aviation Supply office alone there were 250,000 items with different stock numbers compared with the total inventory of General Motors that comprised 120,000 items. As Arrow (2002, 2) described the situation in the late 1940s and early 1950s, "The Navy had a clear interest in minimizing inventory costs." The Navy was willing to put considerable funds into research that could enable them to manage their inventory so that they could be war-ready with a minimum of expense. A myriad of economists, who generally professed a price theory that denied the existence of fluctuating inventories, applied their optimization tools to determine decision rules that would yield the lowest expected value for total future costs involving inventories in production planning.

For the logisticians, planning within the military was not only compatible with but also essential for the smooth running of the private "free" markets. It was through logistics that the military mainly affected the economy that the economy affected the military, and the military acknowledged the need to efficiently allocate their resources in the absence of internal markets.[23] Also, researchers funded by the military were encouraged to use examples from and address the needs of private industry. For example, the members of the Carnegie Institute of Technology project on Planning and Control of Industrial Organization struck a deal with the Pittsburgh Plate Glass Corporation:

> it would supply access to problems and to factory and warehouse data; and the academic researchers, using Navy money, would try to solve the problems…it

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[23] For example, in a report for the George Washington University Logistics Research Project, Rear Admiral Henry E. Eccles (1961, 3) explained:

> The purpose of the military system of the United States is to provide forces which can be effectively used in combat to support the political position and objectives of the nation. However, since economic viability is one of the major political purposes of the nation, the armed forces must be created and employed in such a way as to protect rather than jeopardize the economic health of the nation. With this limitation in mind the criteria by which the military system should be judged is necessarily twofold.
>
> A. Combat effectiveness;
>
> B. Economic efficiency.

> was hardly fair to use government money to augment paint profits by solving their particular organizational and administrative problems, so the HMMS [Holt, Modigliani, Muth, Simon] teams turned to writing a book to present their research results to a wider audience." (Holt 2002, 1997)

At the other end of the spectrum, the US military funded abstract mathematical analysis of inventory control that few military officers or industrialists could understand, such as the *Econometrica* articles by Arrow, Arrow, Harris, and Marschak (1951) and Dvoretzky, Keifer, and Wolfowitz (1953). The Air Force, however, also published summaries of this abstract research. For example, in their 1955 RAND paper, Berman and Clark distilled the findings of their fellow RAND colleagues (e.g. Arrow et. al. and Bellman) and explained, "the purpose of this paper is to present what is believed to be a realistic optimal inventory policy for a military organization. Our prime consideration has been practicability of application rather than elegance or generality" (Berman and Clark 1955, 1). Similarly G. Hadley and T. M. Whitin (1964) and Geisler (1962) prepared a RAND memoranda for the US Air Force surveying alternative approaches to inventory theory. The first column of Table 3 adapts their list of the methods the military had used since 1940 (see Hadley and Whitin 1964, 9) and the second column gives examples of each approach.

A description of the broadness of the spectrum of US military-funded inventory control research from abstract to concrete would be incomplete without mentioning the RAND Corporation's design of Monopologs, a simulation game to train Air Force officers and new RAND employees in logistics thinking. When RAND researchers realized that it would be impossible to construct a complex mathematical model of the entire Air Force supply system, they tried a simplified Monte Carlo base-depot model that emphasized the randomness of military demand for aircraft spares. That study revealed key interactions and led RAND researchers to design a game that would simulate the history of a spare part and enable Air Force personnel to see the consequences of their inventory-management decisions (see Figure 3 and Figure 4).[24]

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[24] The base-depot model developed in 1956 is described in Karr 1957. The first description of Monopologs is Hambruger 1956. In the updated description by Jean Renshaw and Annette Heuston (1960) the reader can get

Table 3 Methods of Determining Operating Policies for Inventory Systems Studied by the US Military

Method	Author, Date, & Brief Description
"Scientific " Formulas based on Heuristic-intuitive practices	Harris 1915, Eisenhart 1948 Static (one-period analysis) and stochastic (random demand): Economical lot size varies directly with square root of expected sales and square root of procurement costs and varies inversely with square root of carrying charges. Wilson 1934 Dynamic (stock not used in one period is passed on to next period) and deterministic (known constant demand): Two-bin policy with *s* (ordering point or safety stock) in second bin equal to least number of units on shelves when restocking order is made that will prevent item from running out before order is filled and with *S* being ordering amount.
Analytical: Optimize with respect to parameters in a specific operating rule	Arrow, Harris, and Marschak 1951 Mathematically determined optimal values of (*S, s*) for two-bin operating rule in the dynamic, stochastic situation
Simulation	Hamburger 1956 Monopologs Simulates history of a spare part and the consequences of management choices when demand is random
Analytical- determine set of operating rules that minimizes costs or maximizes return	Bellman, Glicksberg, and Gross 1955 First use of functional equation to determine the structure of the optimal inventory policy Scarf 1960 Demonstrated two-bin (*S,s*) policy was optimal under broad assumption of loss functions

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the rules and tools for playing Monopologs as well as a pattern for cutting-out a demand-generation spinner based on the assumption of a Poisson distribution of demand for the spare aircraft part.

Figure 3. Schema of the environment of Monopologs, the RAND Corporation’s simulation game to train Air Force personnel in inventory control. It consists of the depot (the wholesale distributor) and the bases (the consumers). (Renshaw and Heuston 1960, 8.)

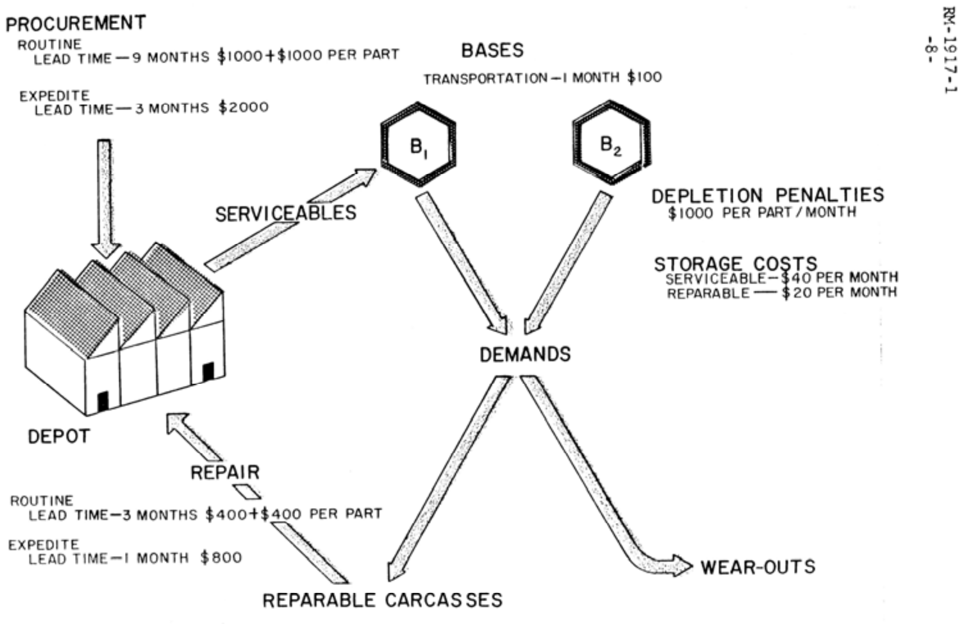
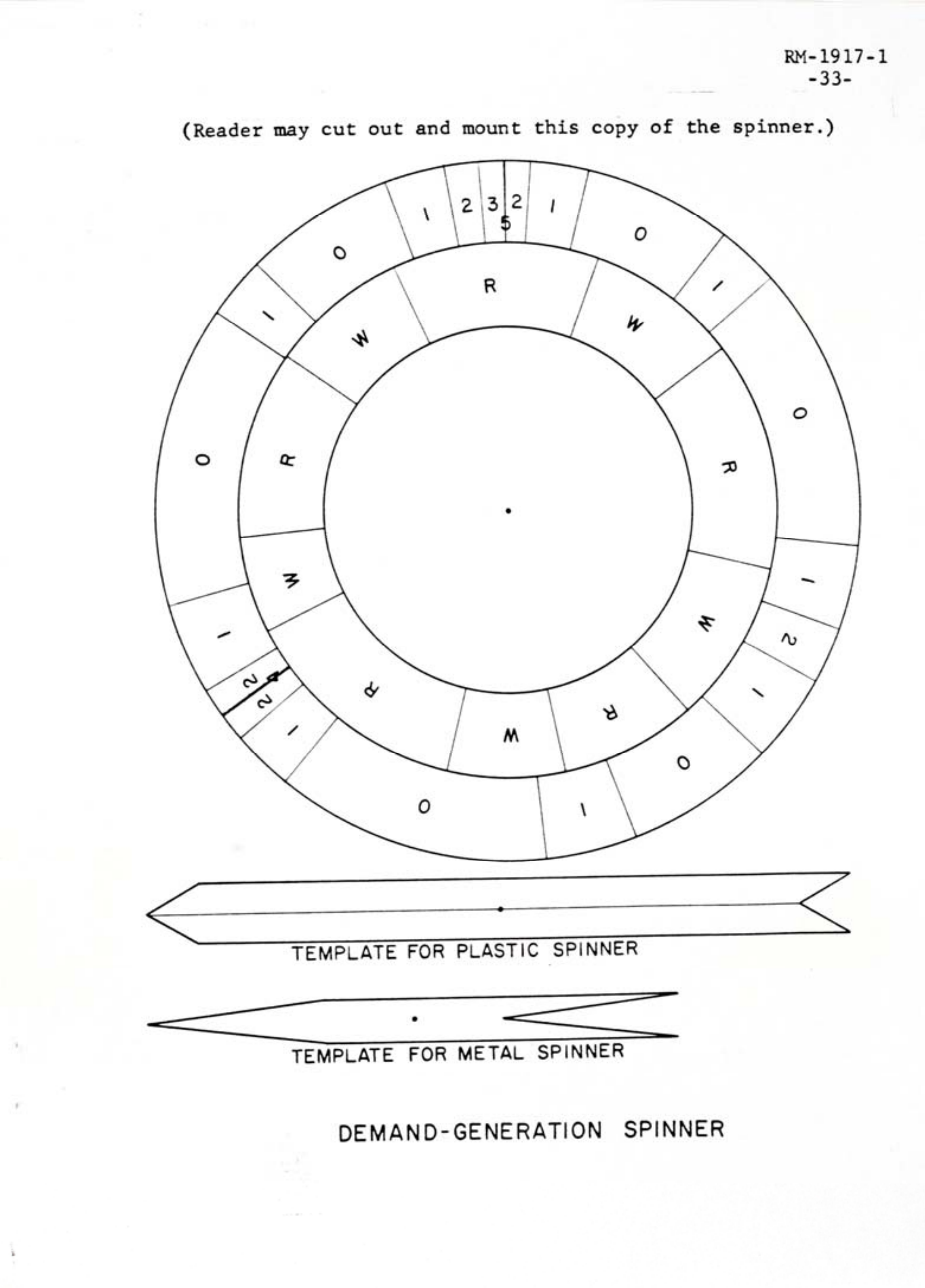


FIG. 1—MONOPOLOGS SYSTEM

Figure 4. The RAND Corporation's random demand generator template for their Monopologs game. The sizes of the sections, which represent demand, are proportional to a Poisson distribution of random demand fluctuations. Air Force personnel, playing the game in order to master complicated inventory control, were expected to cut out the pattern and mount the dial face on a plywood surface with a metal or plastic dial. (Renshaw and Heuston 1960, 33)



RM-1917-1
-33-

(Reader may cut out and mount this copy of the spinner.)

DEMAND-GENERATION SPINNER

Both the Air Force, through the RAND Corporation and the Office of Naval Research (ONR) acted as clearinghouses for inventory control theory including the publication of bibliographies on the subject. In the years immediately following the war, the Office of Naval Research was the primary US agency supporting academic science (see Sapolsky's 1990 history of the ONR). Bruce Old, who left the ONR to work as a consultant for Arthur D. Little, wrote that immediately after the war, "the Navy found itself the sole government agency with the power to move into the void created by the phasing out of the OSRD at the end of the War" (Old 1961, 35). Old acknowledged that when the war ended, many scientists wished "to forget the Navy and return to former pursuits". As Fred Rigby (1976), the first head of the Logistics Branch of the ONR described it, "We actually had to go looking for people to take our money – our contracts, that is – in those days" (Rigby 1976, 407). The ONR, however, eventually found many economists, including Arrow, Samuel Karlin, Herbert Scarf, A. Dvoretzky, J. Keifer, Jacob Wolfowitz, Thomson Whitin, Oskar Morgenstern, Charles C. Holt, Franco Modigliani, John Muth and Herbert Simon, willing to take their money to work on the problems of logistics, and in particular inventory control.[25]

The key role of the military funding for management science can be gleaned from the preface to *Planning Production, Inventories, and Work Force.* Holt, Modigliani, Muth and Simon (1960, v.) stated "The whole industrial community owes the O.N.R. a vote of thanks for its farsighted support of research on decision problems." In 1957 Holt wrote of a new "communications link between the theoretically oriented people in mathematics, statistics, and economics who for many years have worked on decision problems, and the 'practical' managers in the business work who face important decision problems in large numbers. (Holt 1957a,10). Holt credited post-war government sponsorship of research in forging this link and in encouraging businesses to use electronic computers and mathematical algorithms not just for routine data handling, but also to solve their operations research problems (Holt 1957a, 12).

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[25] The mid-1950s state of the art of research on inventory control and Navy interest in that subject can be glimpsed from "An Inventory Control Bibliography" in the *Naval Research Logistics Quarterly* (Lewis et. al. 1955). Most of the entries had been funded either by the ONR or by the RAND Corporation, which was the think tank underwritten in the 1950s by the US Air Force. For histories on the RAND Corporation, the role of economists in RAND, and its influence on economic theory see Jardini 1998 and Mirowski 2002.

In the 1950s, the Office of Naval Research (ONR) financed major projects on production planning and inventory control at George Washington University, Carnegie Institute of Technology, Cornell University, and Princeton University. The ONR published the *Naval Research Logistics Quarterly,* and the ONR-funded Logistics Research Project at George Washington University published the *Logistics Papers*. In the summer of 1950, the Office of Naval Research organized a research group at RAND (subsequent conferences, sponsored by the ONR were held at George Washington University). It was at the 1950 RAND workshop that Kenneth Arrow and Jacob Marschak married the dynamic, stochastic mathematics of decision science with the observed business practice of a two-bin policy for inventory control. Thus began a decade of intensive US military funding for studies in inventory control.

Arrow, Harris and Marschak's Dynamic Model with Uncertainty

During the Great Depression several economists, including Roos 1930, Shaw 1940, and Meltzer 1941, had examined the role of inventories of final goods in production fluctuations and macroeconomic crises. The military, however, contracted Arrow, Harris and Marschak, to develop a microeconomic policy to guide the firm in the management of inventories of raw materials, spare parts, and final goods that would be used in military engagements. The authors thus turned toward business and military practice and literature rather than economic theory for their foundation:

> By regarding the order size as the only controlled condition, and the demand as the only random noncontrolled condition, we do take account of most of the major questions that have actually arisen in the practice of business and nonprofit organizations.
>
> Before formulating the problem, a study was made of the existing business literature on inventory control, using freely the comprehensive bibliographies that were compilied by T. H. Whitlin [1950] of Princeton University, and by Louise B. Haack [1950] of the George Washington University, for projects of the Office of Naval Research at those universities. Arrow, Harris and Marschak 1951, 252

The business and military literature included R. H. Wilson's "scientific" routine for a two-bin inventory policy explained in a 1936 issue of *Harvard Business Review*, business formulas for numerical minimization of costs in L. P. Alford and John R. Bank's (eds) 1944 *Production Handbook*, and estimations of distributions of demand per unit period that J. B. Kruskal and J. J. Wolf (1950) had derived for the Navy. In addition to business practice and literature, Arrow, Harris and Marschak also drew on statistical theory, particularly Abraham Wald's (1947 and 1950) approach to statistical decision functions that he initiated with his World War II work on sequential analysis and W. Feller's (1950) exposition of Markov processes.

Arrow, Harris, and Marschak derived the optimal inventory policies for a dynamic model with known and constant demand, a static model with uncertain demand, and a dynamic model in which demand was a random variable with a known probability distribution. It was the latter model that was so novel and required recursive optimization. As was the case with Massé, the authors started that optimization process with formulas for rules of action (see symbol definitions in Table 4):

> Choose two numbers S and s, $S > s > 0$, and let them define the following rule of action:
>
> $$\text{If } y_t > s, \quad q_t = 0 \quad (\text{and hence } z_t = y_t);$$
>
> (4.4)
>
> $$\text{if } y_t \leq s, \quad q_t = S - y_t \; (\text{and hence } z_t = S).$$
>
> S and s are often called, respectively, the maximum stock and the reordering point (Arrow, Harris, and Marschak 1951, 260).

Figure 5 reproduces their illustration of a typical temporal sequence that stock would take if the two-bin rule was used. The recursion comes in solving the loss function for the present value of all costs, including the cost of making an order and the penalty for stock depletion.

> When maximizing expected utility, the policymaker takes into account the present values of losses, not their values at the time when they are incurred. In commercial practice, α is equal to $1/(1 + \rho)$, where ρ is an appropriate market rate of interest….
>
> If we now define the function
>
> $$L(y) = l_0(y) + \alpha l_1(y) + \alpha l_2(y) + \cdots,$$
>
> we see from definition (4.7) that $L(y_t)$ is the present value at time t of the total expected loss incurred during the period $(t, t + 1)$ and all subsequent periods

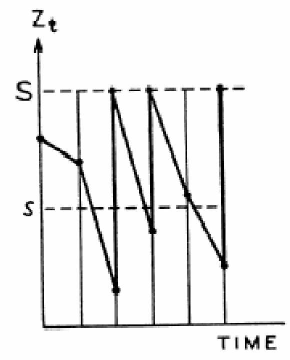
> when y_t is given. By definition, $L(y)$ involves the parameters S and s; and the policymaker fixes these parameters so as to minimize $L(y_0)$….
>
> [Notice that from the way we have defined the rule of action, $L(y)$ is constant for $0 \leq y \leq s$ so that $L(0)$is unambiguously defined.] Putting $y_0 = y$ we obtain from (4.10)and (4.10') the equations
>
> $$(4.11) \qquad L(y) = l(y) + \alpha \int_{0^-}^{S} L(S-x)\,dF(x) + \alpha L(0)[1-F(y)] \quad \text{if } y \leq s,$$
>
> $$(4.12) \qquad L(y) = l(y) + \alpha \int_{0^-}^{y} L(y-x)\,dF(x) + \alpha L(0)[1-F(y)] \quad \text{if } y \leq s.$$
>
> Our problem is to find the function $L(y)$ that satisfies (4.11), (4.12) and to minimize $L(y_0)$ with respect to S, s……
>
> (5.11)
>
> $$L(y) = l(y) + \alpha L(0)[1-F(y-s)] + \int_{0}^{y-s} \{l(y-x) + \alpha L(0)[1-F(y-x-s)]\}\, dH_\alpha(x), \quad y > s$$
>
> …..
>
> $$(5.14) \qquad L(0) = \frac{K + l(S) + \int_{0}^{S-s} l(S-x)\, dH_\alpha(x)}{(1-\alpha)[1+H_\alpha(S-s)]}$$
>
> Knowing $L(y)$ from (5.11) and (5.14), the next step is to find, for a given initial stock y_0 ,the values of s and S which minimize $L(y_0)$. We shall consider only the minimization of $L(0)$, although the procedure could be worked out to minimize $L(y_0)$ for any initial stock y_0….
>
> Presumably the minimization of (5.14) would be accomplished in practice by numerical methods. (Arrow, Harris, and Marschak 1951, 262, 263, 265, 266)

Although the authors demonstrated that it was possible to solve for the optimal values of S and s, given the demand distribution and the minimized loss function, they did not actually solve a sample problem or even explain the numerical methods by which one could do so. In generalizing recursive optimization through what he called “dynamic programming” Richard Bellman would take on those tasks as well as using a single functional equation to express the economic criterion that Arrow et. al. only expressed in words (“Our problem is to find the function $L(y)$ that satisfies (4.11), (4.12) and to minimize $L(y_0)$ with respect to S, s.”, Arrow, Harris, and Marschak 1951, 263). Bellman recognized that the end result for the military had to be rules of action that could be quantified and that the best way to solve the initial dynamic objective function of

Table 4 Symbol Definitions for Arrow, Harris, & Marschak 1951

Symbol	Definitions
S	initial stock level to provide for the demand that will occur during the period
S^*	optimal stock level
x	demand during the period
ξ	unit(s) of product
a, a_0, A, B	constant
$F(x)$	cumulative distribution of demand
$f(x)$	density function – derivative of $F(x)$ with respect to x .
π	depletion penalty
F_t	a Poisson distribution of demand for the period $(t, t + 1)$
μ_1	mean of F_t
$L(S)$	expected net loss
θ_0	fixed constant
x_t	the demand over the interval $(t, t + 1)$
y_t	the stock available at instant t, not including any replenishment
z_t	the stock available at instant t, including the replenishment
q_t	the amount ordered at time t
τ	time between the ordering and the receiving of goods (pipeline time)
s	reordering point
K	constant, cost of handling an order
c	marginal cost of carrying stock during a unit of time
$l(y_t)$	a certain loss with a fixed value of y_t
$l_t, l_t(y_0)$	a loss with a random variable y_t

Figure 5. An illustration of the sort of curve that might be obtained for stock level (Z_t) as a function of time if the two-bin rule is adopted. (Arrow et.al. 1951, 260, Figure 3)



minimizing losses or maximizing military worth was to put it into a functional, recursive equation amenable to approximating the parameter values for the rules of action (approximation in policy space- see next chapter).

A Decade of US Military Interest in Recursive Optimization for Inventory Control

Through the ONR and the Rand Corporation, the US Navy and Air Force, respectively, invested in mathematical and economic studies on inventory control. Table 5 highlights key studies financed by the US military from 1950 to 1960. Aryeh Dvretzky, J. Keifer, and Jacob Wolfowitz (1952a, 1952b, 1953) extended that analysis of Arrow, Harris, and Marschak to unknown forms of demand distributions and to other policies. Even after their work the question remained was the two-bin policy the optimal policy among many policies? Arrow, Harris, and Marschak, had taken a two-bin policy as given, and solved for the optimal values of S and s, but it was not until a decade after the initial study, that Herbert Scarf (1960), mathematically demonstrated the that the two-bin policy was optimal under certain assumptions of costs functions and time lags.

The above-mentioned studies were abstract and mathematical and published in academic publications such as *Econometrica.* It was left to authors such as Edward Berman, Andrew Clark, G. Hadley, and Thomson Whitlin, to interpret the academic prose and findings for the military. There was, however, a major project at the Graduate School for Industrial Administration at Carnegie Institute of Technology in Pittsburgh that combined mathematical innovation and practical applications. Bill Cooper had initiated this project with funding from the U.S. Air Force in the early 1950s. The ONR soon took over the financing for the project on “Planning and Control of Industrial Operations.” It was in that context that the graduate student and junior faculty member John Muth researched the optimal properties of the forecasting model of exponentially weighted moving averages (EWMA or, as economists called it, adaptive expectations, Muth 1960) and named and formulated “rational expectations” (Muth 1961). Structural time series analysts such as Andrew Harvey perceive Muth 1960 as a classic in their field (see Chapters 2 and 7), and new classical macroeconomists, such as Robert Lucas, likewise see Muth 1961 as the prototype for their subsequent work on rational expectations (see Chapters 5 and 7).

Table 5 Selected US Government-funded Studies in Optimal Inventory Control, 1950-1961

Author/date	Military Contract or Influence	Key Contribution
Arrow, Harris, Marschak 1951	Office of Naval Research (ONR) Logistics Conference at RAND, Summer 1950	Determined optimal value of maximum stock *S* and the best reordering point *s* as functions of the demand distribution, the cost of making an order, and the penalty of stock depletion in a fixed two-bin policy in a dynamic, stochastic situation using a Markovian model.
Dvoretzky, Keifer, Wolfowitz 1952a, 1952b, 1953	ONR Project at Cornell University	Determined the distribution of functions of demand as production process continued demonstrating that loss functions and distributions could be used to find optimal ordering policy for non two-bin polices. Existence and uniqueness theorems for optimal solution of two- bin policy.
Whitin 1953	ONR – Economics Research Project at Princeton University, Directed by Oskar Morgenstern, ONR Research Project N6onr-27009	Surveyed Inventory practices in US Navy, scientific methods for intrafirm inventory control, and economic theories on inventories and business cycles
Bellman, Glicksberg, Gross 1955	Air Force RAND Corporation	First use of functional equation and successive approximations in policy space to determine structure of optimal inventory policy and obtain complete solutions of general classes of problems of ordering in the face of uncertain demand including cases involving stockpiles of different items with correlated demand functions
Berman and Clark 1955	Air Force RAND Corporation	Summarized, compared and interpreted the mathematical models of Arrow et. al., Bellman et. al.for a broad military audience hoping to improve the practice of inventory control.
Hamburger 1956	Air Force RAND Corporation	Monopologs- a game to simulate Air Force supply system and the consequences of inventory-management decisions in the face of uncertain demand for aircraft spares
Karr 1957	Air Force RAND Corporation	Monte-Carlo base-depot model to derive requisitioning recommendations

Author/date	Military Contract or Influence	Key Contribution
Scarf 1960	ONR contract with Stanford University	Demonstrated that the two-bin (*S,s*) policy was optimal if holding and shortage costs were linear and in more general cost functions and with time lag in delivery
Holt, Modigliani, Muth, Simon 1961	ONR Project on Planning and Control of Industrial Operations at Carnegie Institute of Technology's Graduate School of Industrial Administration, Contract Nonr-76001, Project Nr 047001	Enhanced effective computation and practical application of optimal inventory policy by approximating all costs by quadratic functions so expected values could replace probability distributions of all random variables. Reduced amount of past data that had to be stored by forecasting sales with an exponentially weighted moving average. Generalized the EWMA model to take into account long-run trends and seasonal effects (Holt-Winters method of forecasting) and determined under what statistical conditions the EWMA was an optimal forecast.

The ONR-funded Carnegie crew, which included Charles C. Holt, Franco Modigliani, Muth and Herbert Simon (with contributions from Charles Bonini and Peter Winters), interviewed managers in 15 companies and assumed that the audience for their final book *Planning Production, Inventories and Workforce* (Holt et. al. 1960) were managers in private businesses as well as the military forces (Holt 2002, 96). Their text was and is widely used in the operations research community, which refers to as the HMMS text. Linear decision rules, quadratic cost functions, and forecasting sales with EWMA, made the inventory and production control models computable and accessible to the business community. The HMMS team demonstrated that the realistic assumption of approximate quadratic costs eliminated the need for dealing with entire distributions of demand: "when costs are quadratic the only datum about future sales that enters into the optimal decisions rule is the expected value" (Holt et. al.1960, 123).[26] As Murray Geisler (1962, 10), the head of the Logistics Department of the RAND Corporation explained:

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[26] Pedro Duarte (2005) has researched the economizing incentives that led many in the operations research field to adopt quadratic costs and criterion functions in the 1950s and how that OR practice fed into the use of quadratic loss functions in US Monetary Policy in the 1960s.

> If in a dynamic programming problem such as that under consideration [optimal inventory control] all the costs can be considered as positive, definite, non-homogenous, quadratic forms in the decision variables, then so far as the decision for the first period is concerned, all random variables can be replaced by their expected value and the problem may be solved as if they were known with certainty. Thus, all that is involved in this case is to take the appropriate partial derivatives and set them equal to solve for the optimal values of the decision variables. Further, this formulation does not depend on the independence of demands in different periods.

The Carnegie Institute team also demonstrated that EWMA forecasts of sales were quick, cheap, easy, and relatively accurate. We can see from Figure 6, the size and relatively "primitive" nature of computers used for inventory control in the late 1950s and early 1960s. The EWMA forecasting model saved on expensive computer storage and processing because one only needed to save and use the single previous forecast and adjust that in light of new demand. The HMMS team touted the accuracy of their forecasting method with several graphical demonstrations of how close the actual forecast was the one-month prediction (see Figure 7). Their simple EWMA equation (Holt et. al. 1960, 260) for predicting sales in this time period based on the prediction of sales in the previous time plus a fraction of the difference between actual and predicted sales in the previous period was:[27]

$$\bar{S}_t = \bar{S}_{t-1} + w_e(S_{t-1} - \bar{S}_{t-1})$$

Holt began his 1957 ONR Research Memorandum on forecasting with

> An exponentially weighted moving average is a means of smoothing random fluctuations that has the following desirable properties: 1) declining weight is put on older data, 2) it is extremely easy to compute, 3) minimum data is required. A new value of the average is obtained merely by computing a weighted average of two variables, the value of the average from the last period and the current value of the variable…The flexibility of the method combined with its economy of computation and data requirements make it especially

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[27] If you substitute in the equation for $\bar{S}_{t-1}$ and then $\bar{S}_{t-2}$ and so forth. For each substitution backward you must multiply the coefficient (1-w_e) by itself an additional time. Given that w_e is a positive number less than one, the weights decline in a geometric progression, thus the name exponentially weighted moving averages. One can also see from this equation that the forecaster need only to store information from the previous period because that captures information from all past stages. See chapter 2 for more explanation.

Figure 6. Photo taken on June 26, 1957 of the Logistics Computer in the Navy Research Project Laboratory on the campus of George Washington University. The research associates, Norman Mason and Charles Hesaltine, are processing the answers to logistics control problems the computer had put on tape. The small plaque on the bottom right of the computer reads: "Developed and Manufactured for the Office of Naval Research by Engineering Research Associates". Source: George Washington University Relations Photo Collection Acc# 133, Folder 821.

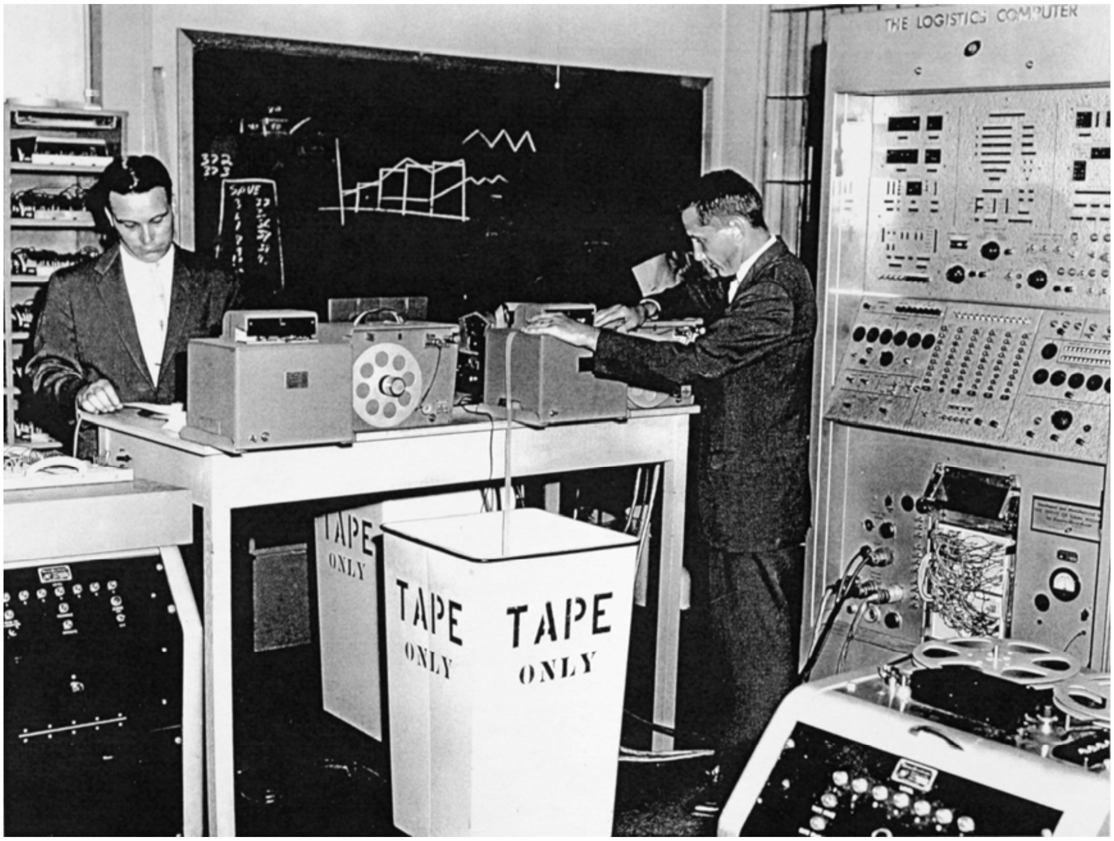
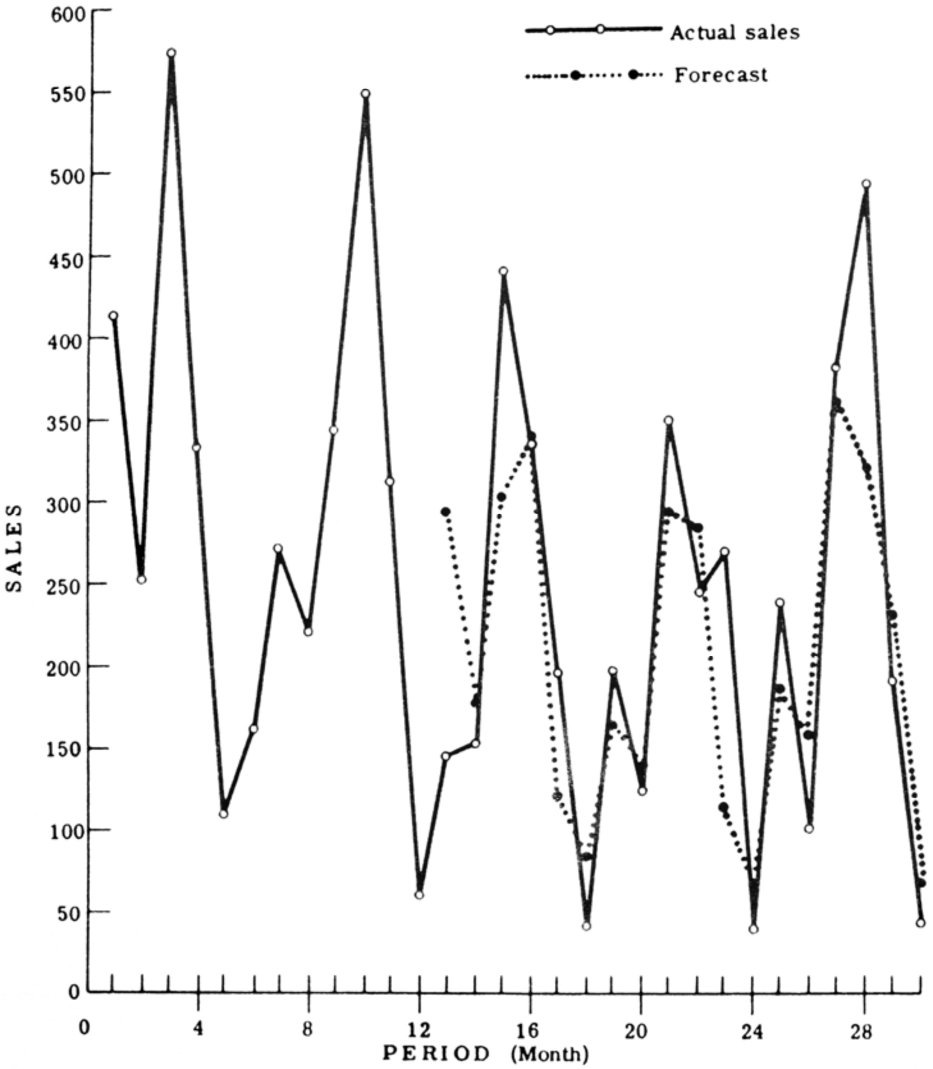


Figure 7. A diagram indicating the accuracy of exponentially weighted moving averages for forecasts intended for production and inventory control. The forecasts for sales were made one month in advance from an exponentially weighted moving average of past sales. (Holt et. al. 1957, 268, Figure 14.2)



> suitable for industrial situations in which a large number of forecasts are needed for sales of individual products. (Holt 1957b, 1)

Holt and Winters extended the applicability of EWMA forecasts by working out how to take into account trends and seasonal components. Holt asserted in 2002, “the Holt-Winters method is now offered as an option in almost all forecasting software packages, and it is more widely used by American business than any other forecasting formula.” (Holt 2002, 98) Holt (99) also pointed out that two members of the HMMS team, Simon and Modigliani, went on to win Nobel prizes in economics and a third member, Muth, inspired another Nobel prize with his work on rational expectations.

Conclusion

In both the French and American cases, the war-time client was paying for a normative rule for efficient multi-stage allocation of resources in the absence of an internal market and in the presence of uncertainty. As with Massé, Arrow, Harris, and Marschak took up the challenge of formulating a decision rule when the stock lasted over several periods and the demand was random. For the manager of the system of hydroelectric dams during the war, the game of the reservoirs was to determine how much water should be taken from the reservoirs each month in order to minimize the current and future use of coal - the extremely scarce alternative source of electrical power. The challenges to determining the optimal flow of water included an uncertain future inflow of water from precipitation, uncertain future final demand for electrical power, and the fact that one’s optimal decision this month was conditional upon the decisions the managers would make in future months. It was an exercise in optimal inventory control for an enterprise engaged in arbitrage through time. The reservoir, with uncertain inflows, diminishing marginal utility, opportunity costs, penalties for shortages, and physical boundaries of empty and full that limited mathematical functions, was an abiding metaphor for Massé as he rose in the ranks to Directeur Général Adjoint of Électricité de France and eventually Commissaire Général du Plan for France in the government of Charles de Gaulle. The recursive technique for conditional decision-making was also a career theme – Massé used it in many settings to determine rules for action.

In several ways, the formal training at the *Ecole Polytechnic,* which was run by the French Ministry of Defense, and the exchange of ideas in the close knit circles of *polytechnicien*s in institutions such as the large utility companies created an interdisciplinary climate similar to that of the US National Defense Research Council during World War II and the RAND corporation that was established as an US Air Force think tank in 1946. Massé, like Abraham Wald in the NDRC and Richard Bellman at RAND, remained sensitive to the physical process he was engineering as he brought together the poly-techniques of marginal economic analysis, sequential statistical inference, and recursive mathematical approximation.

Five major threads are woven into this investigation of studies of optimal inventory control from 1948 to 1960:

1. Wartime exigencies forced an applied mathematics that modeled multistage decision-making in a climate of uncertainty. The requirements of war kept the focus on modeling and improving operations and observed practices. World War II and cold-war agencies required cost-effective, user-friendly, solutions to these mathematical problems. Therefore, the solution process had had to take a final form of a quantified rule of action that non-mathematicians could use and had to rely on approximation to minimize computing costs.

2. Wartime programming was normative resource allocation through time. For the US military, resource allocation, along with strategy and tactics, was one of the three key branches of mathematical decision theory. In an attempt to turn the US military and supporting industries into cost-minimizing rational producers, the Air Force and Navy financed most of the US studies on optimal inventory control in the 1940s and 1950s. As a result, in the first two decades of the cold war more business operations research was directed to inventory control than to any other subject.

3. The breakdown of both the flow-from-the-reservoir stock and the flow-into-the-inventory stock decision processes into sequential stages enabled Massé and the RAND economists to meet computational challenges, through backwards recursion, and modeling challenges posed by the interaction of stocks and flows.

4. During the cold war, the economists' work on analytical modeling for inventory control drew more on the prewar business management literature than it did on the prewar economic literature on the role of inventories in business cycles. Much of the mathematical analysis of inventory control models confirmed the optimality of the intuitive/empirical two-bin approach to restocking decisions that many business managers and a few military agencies had been following for decades. Ironically Massé, the engineer, relied more heavily on economic theory and concepts than did the economists Arrow and Marschak.
5. Although the analysis of the flows connected to the stocks of reservoirs and inventories attracted more interest among operations researchers rather than economic theorists in the 1940's and 1950's, there were three important spin-offs that eventually had major impacts on economic theory: a) modeling costs with a quadratic function; b) modeling expectations with exponential smoothing; c) modeling the decision process with an objective functional equation amenable to policy solutions through recursive approximation.
6. Normative microeconomics developed in production planning for the military client was an important context for modeling expectations. Downstream from the French and US government's attempts to turn themselves into rational producers we find academic economists adapting the same models and algorithms to demonstrate that innately rational consumers render government policy impotent. So this story starts with the normative microeconomics of optimal reservoir regulation and inventory control and ends with the positive macroeconomics of rational expectations (explored more fully in Chapter 5). In other words, we start with the government, in the case of the US Air Force, giving some economists lots of money to develop models and concepts that end up being used by other economists to demonstrate why the government should not intervene in the economy.

In his memoirs, Massé (1984, 70) makes clear that it was the urgent necessities arising from the dark days of war that led to his illumination that a *conditional rule*- a strategy- was the only mathematical solution possible in the problem of how much water you take from the reservoir in order to minimize the use of scarce coal. The rule had to be conditioned on the decisions taken in another stage. Within a decade of Massé's first publication of his rule and protocol for the *jeu des reservoirs*, US military funding for research to fight a cold war would ensure that multi-stage decision-making with Markovian modeling of economic criteria and recursive approximation to determine optimal policy rules of action were at the forefront of applied mathematics, operations research and eventually macroeconomic theory.

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