The effects of a minimum wage increase in a model with multiple unemployment equilibria

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Abstract

We introduce the heterogeneity of labor in a simple imperfectly competitive aggregate labor market model "à la Manning (1990)" in order to analyze the effects of an exogenous rise of the legal minimum wage on the unemployment equilibrium, the wage dispersion and the general price level. We assume the presence of "knowledge spillovers" in the individual production function leading to increasing returns to scale at the aggregate level. This assumption involves the possibility of multiple equilibria in the model. Then, thanks to a comparative statics exercise, we show that a rise in the legal minimum wage has no impact on the unemployment equilibrium (wherever the economy is stands), increases the general price level and reduces the wage dispersion. We also find that the larger the proportion of unskilled workers paid at the minimum wage in the total employment, the higher the increase in the general price level is. These results are broadly consistent with Card-Krueger's findings (1995).

Keywords: multiple unemployment equilibria, minimum wage, general price level.

JEL Classification: D43, E24, J24, J31.

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Introduction

The problem of minimum wage hikes has been actively discussed among economists. The primary goal of such a government's intervention is to improve the welfare of low paid workers. However, many are those who think that an increase in the minimum wage leads also to employment losses for the workers at the bottom of the wage distribution as young workers or low-skilled workers. In their book, Card and Krueger (1995) argue that the data on the fast-food industry in U.S. states give no support to this idea. Machin and Manning (1994) analyze also the effect of the minimum wage cuts on employment in the U.K. and reach to the same conclusions. With these works, the debate has been revived as well in the academic area as in the political area.

At the theoretical level, the effect of minimum wage on employment has been analyzed especially with models like the efficiency wage model (Rebitzer and Taylor, 1995) or the monopsony model (Card and Krueger, 1995), i.e. models where the firm has a high power on wage determination. As mentioned by Cahuc and al. (2001), only few papers rely on models using wage bargaining with trade union; even though this way of wage determination appears as an important feature of labor markets. Furthermore, for several years, models with multiple equilibria have been used in order to explain the performance of labor markets. For example, Manning (1992) explains the British unemployment experience in the 1980s by the shift from one equilibrium to another.

According to these reports, the purpose of our paper is to examine the effects of minimum wage increase on labor market's performance in a multiple equilibria model where the wage negotiation takes place between a firm and a trade union. The model used in the paper is essentially the imperfectly competitive model of Manning (1990) in which we introduce the heterogeneity of labor and the presence of knowledge spillovers in the individual production technology.

Considering heterogeneous workers is required when we want focus the analysis on the effects of a minimum wage. Therefore, we consider that the labor force employed by the firm consists of low-skilled workers who are paid at the minimum wage and high-skilled workers who are paid at a negotiated wage. This negotiated wage results from a bilateral bargaining between the firm and the trade union which is assumed to be not dominated by skilled workers.

The assumption of knowledge spillovers is a convenient way to produce increasing returns to scale at the aggregate level while having constant returns to scale at the firm level. In the Manning's model, the possibility of multiple equilibria is due to the presence of increasing returns to scale at the firm level, although this assumption has no strong theoretical justification. However, for several years, knowledge has been considered as a fundamental source of increasing returns to scale and a determinant of the persistence of productivity and income differentials across economic agents of production (Romer, 1986). Thus, in our model, the presence of knowledge spillovers allows us to give theoretical support for the presence of increasing returns to scale. These knowledge spillovers work through the average level of high-skilled labor employed in the economy, the high-skilled labor being both the engine and the carrier of knowledge.

The paper is organized as follows. In the first section, we present the model (the price behavior of the firm and the wage determination) and the general symmetric equilibrium. Then, we explain under which conditions the economy exhibits multiple equilibria and we prove the existence of them. In a third section, we run a comparative statics analysis and we show that a minimum wage increase has no effect on the unemployment equilibrium, increases general price level and reduces wage dispersion. Finally, we compare our findings with those of Card and Krueger (1995).

1 The model

We use the Manning's model (1990) broadly based on Layard-Nickell's model (1985, 1986)[1], in which we introduce the heterogeneity of labor.

1.1 The price behavior of the firm and its demand of labor

We assume a monopolistic competition on the good market. The economy is made up of F identical imperfectly competitive firms producing each one an imperfectly substituable good with ohers. The firm i has a production function of the form[2]:

$$y_i = An_{1i}^{\alpha}n_{2i}^{\beta} \qquad with \qquad \alpha+\beta=1 \tag{1}$$

where n_{1i} represents its employment of high-skilled labor, n_{2i} its employment of low-skilled labor and A an efficiency parameter of labor taken as given by the firm i. A is assumed to be a function of the average employment level of high-skilled labor in the economy n_1:

$$A = n_1^{\alpha\sigma} \tag{2}$$

where $\alpha\sigma > 0$ represents the size of the knowledge spillovers and $\sigma > 0$ a measure of the degree of externalities. Thus, the average level of high-skilled labor used in the economy affects positively the efficiency of labor used by the firms. Given the fact that knowledge is the cause of new technologies development and mainly produced and spread by high-skilled labor force, considering that the labor efficiency depends on the average use of high-skilled labor in the economy is a relevant assumption. Being assumed too small to influence aggregate state of the economy, the firm i takes the average employment level of high-skilled labor as given during its optimization program. Given $\alpha+\beta=1$, we have constant returns to scale at the firm level.

[1] Layard, R. and Nickell, S.J. (1985), "The cause of British unemployment", National Institute Economic Review, February, pp. 62-85.
Layard, R. and Nickell, S.J. (1986), "Unemployment in Britain", Economica, Vol. 53, pp. S121-70.

[2] As in Manning (1990), the number of firm is assumed fixed and the capital is excluded for simplicity.

Consist of high-skilled and low-skilled labor, we can write the total employment of the firm i as:

$$n_i = n_{1i} + n_{2i} \tag{3}$$

Let $\gamma_i = n_{2i}/n_i \in]0;1[$ the proportion of low-skilled labor employment in the total employment for the firm i and $\theta_i = \underline{w}/w_{1i} \in]0;1[$ a measure of the wage dispersion between the high-skilled and low-skilled labor where $\underline{w}$ represents the minimum wage which is earned by low-skilled workers and w_{1i} the wage of high-skilled workers which is negociated. The lower θ_i is, the larger the gap between the low-skilled worker's wage and high-skilled worker's wage is. Consequently, the total wage cost of the firm i can be written as a function of the high-skilled worker's wage w_{1i}, the total labor demand and the two parameters defined above :

$$w_i n_i = w_{1i} n_{1i} + \underline{w} n_{2i} = \left[1 + (\theta_i - 1)\gamma_i\right] w_{1i} n_i = C(\gamma_i, \theta_i) w_{1i} n_i \tag{4}$$

with $C(\gamma_i, \theta_i) \in]0;1[$ and where $w_i = C(\gamma_i, \theta_i) w_{1i}$ represents a wage index paid by the firm to its workers. We can also write the production technology of the firm i as a function of n_i and γ_i:

$$y_i = AB(\gamma_i) n_i \qquad where \qquad B(\gamma_i) = (1-\gamma_i)^{\alpha} \gamma_i^{\beta} \tag{5}$$

The demand for the firm i's output is assumed to be given by:

$$y_i^d = \left(Y^d/F\right)(p_i/P)^{-s}, \qquad s > 1 \tag{6}$$

where Y^d is the total aggregate demand, p_i the firm i's price, P the general price level and s the demand elasticity in the good produced by the firm i[3].

The real profit of the firm i can be written as:

$$\pi_i/P = (p_i/P)\, y_i - C(\gamma_i, \theta_i)(w_{1i}/P) n_i \tag{7}$$

Each firm i chooses p_i in order to maximize its real profit. During this optimization beahvior, the firm takes the aggregate state of the economy and the real wage of high-skilled workers as given and, produces exactly the amount of demanded product. Thus, it solves the following program:

$$\begin{aligned} (pi/P)^* = \arg \quad \max \quad & \pi_i/P \\ s.t \qquad\qquad & y_i = AB(\gamma_i) n_i \\ & y_i^d = \left(Y^d/F\right)(p_i/P)^{-s} \\ & y_i = y_i^d \\ & w_{1i}/P \qquad given \end{aligned}$$

The first order condition of this program gives us the following partial equilibrium pricing equation:

$$(p_i/P)^* = \left(\frac{s}{s-1}\right) \frac{C(\gamma_i, \theta_i)}{AB(\gamma_i)} \left(\frac{w_{1i}}{P}\right) \tag{8}$$

[3] This specification of the demand function is derived from CES preferences (see Blanchard and Kiyotaki (1987), and Julien and Sanz (2007)).

This equation show that the price fixed by the firm is an increasing function of labor cost. Then, we use the equations (5), (6) and (8) to obtain an employment equation corresponding to the total labor demand of the firm i:

$$n_i^* = \left[\left(\frac{s}{s-1}\right) C(\gamma_i,\theta_i)\right]^{-s} [B(\gamma_i)]^{s-1} \left(Y^d/F\right) \left(\frac{w_{1i}}{P}\right)^{-s} \tag{9}$$

The total labor demand of the firm has the common features found in the literature, i.e. it is downward sloping in the high-skilled real wage-employment space, decreasing in the labor cost and increasing in the total aggregate demand.

1.2 The wage determination at the firm level

The negotiation is decentralized and rest on the wage ("right-to-manage" model). The low-skilled labor wage corresponds to the legal minimum wage. It is fixed by law and not negotiated between the firm and the trade union. The negotiation concerns just the high-skilled labor wage w_{1i}.

The trade union hasn't insider's behavior and cares about the welfare of both high-skilled and low-skilled employees for which the wage is fixed exogenously. Consequently, we assume that it weights equally the welfare of all employees of the firm and considers it as a whole during the negotiation. We assume futher that it gives as much importance to the employment level as to the wage level of its members[4]. Its utility function is given by:

$$\begin{aligned} V\left(w_i/P, n_i\right) &= \left[\left(\frac{w_i}{P}\right) - w_R\right] n_i \\ &= \left[C(\gamma_i,\theta_i)\left(\frac{w_{1i}}{P}\right) - w_R\right] n_i \end{aligned} \tag{10}$$

where w_R is the real reservation wage which is exogenous at the firm level. This reservation wage represents the alternative utility of workers who become unemployed. The firm's utility is represented by its real profits.

The high-skilled real wage is negotiated in order to solve the following Nash bargaining program:

$$\left(\frac{w_{1i}}{P}\right)^* = \arg \quad \max \quad [\pi_i/P]^{(1-\rho)} \left[\left\{C(\gamma_i,\theta_i)\left(\tfrac{w_{1i}}{P}\right) - w_R\right\} n_i\right]^{\rho}$$

$$\begin{aligned} s.t \qquad & n_i = n_i^* \\ & w_R \quad given \end{aligned}$$

Where $\rho(1-\rho) \in]0;1[$ represents the bargaining power of the union (of the firm). The first order condition of this program give us the high-skilled real wage that results from this negotiation:

$$\left(\frac{w_{1i}}{P}\right)^* = \frac{\chi(\rho,s)}{C(\gamma_i,\theta_i)} w_R \qquad with\ \chi(\rho,s) = \left(\frac{\rho}{s-1}\right) + 1 \geq 1 \tag{11}$$

[4]Here, the union's preferences correspond to the utilitarian model of Oswald (1982).

Where $\chi(\rho, s)$ is the mark-up. This wage equation is traditional and says that the high-skilled real wage is marked-up over the reservation wage with a mark-up which is greater than one and increasing in the bargaining power of union. Now, we turn to the general equilibrium of this economy.

1.3 The general equilibrium

At the general symmetric equilibrium, all the firms and trade unions are identical and take the same decisions. Thus, we have these following equalities: $p_i=P$, $w_{1i}=w_1$, $n_{1i}=n_1$, $n_{2i}=n_2$, $n_i=n$, $y_i=y=Y/F=Y^d/F\ldots$

Since the number of firms in the economy is assumed fixed, the aggregate production can be deduced from the sum of the F identical individual production technologies:

$$Y = Fy = F(1-\gamma)^{\alpha(1+\sigma)}\gamma^{\beta}n^{(1+\alpha\sigma)} \tag{12}$$

At the aggregate level, we have increasing returns to scale of labor whatever the size of knowledge spillovers in the economy. These increasing returns to scale are completely external to the firm[5] but internal to the economy. They correspond to social increasing returns. Generating a positive externality on the firms' production efficiency, the high-skilled labor employment decisions of firms are strategic complements in the model. Thus, the knowledge spillovers reinforce the complementarities already introduced by monopolistic competition and give the possibility of multiple equilibria existence.

Given the definition of the unemployment rate $u = 1 - (Fn/\overline{N})$, where $\overline{N}$ represents the total labor force in the economy, we insert (12) in (9) and we obtain the aggregate price equation (PS for price setting) which relates the aggregate real wage of high-skilled labor to the unemployment rate u and other variables:

$$\left(\frac{w_1}{P}\right)_{PS} = \frac{\left[(1-u)\,\overline{N}/F\right]^{\alpha\sigma}}{\dfrac{s}{s-1}\dfrac{C(\gamma,\theta)}{(1-\gamma)^{\alpha(1+\sigma)}\gamma^{\beta}}} \tag{13}$$

In order to determine the aggregate wage equation, we need to model the reservation wage which is exogenous at the firm level. Like Manning (1990), we use this convenient specification :

$$\begin{aligned} w_R &= u(B/P) + (1-u)(w/P) \\ &= u(B/P) + (1-u)C(\gamma,\theta)(w_1/P) \end{aligned} \tag{14}$$

Where u is the unemployment rate and B/P the real unemployment benefits and w/P the real wage index paid by the firms to the workers. the unemployment benefits are assumed to be the same for all the workers whatever their skill

[5] The firm is assumed to take the decision of other firms as given and to have no influence on aggregate state of the economy.

level and lower than the minimum wage. We introduce (14) in (11) and we obtain the following aggregate wage equation (WS for wage setting) which relates also the real high-skilled labor wage to the unemployment rate and exogenous variables as the real unemployment benefits:

$$\left(\frac{w_1}{P}\right)_{WS} = \frac{\chi\left(\rho,s\right)(B/P)u}{\left[1-\chi\left(\rho,s\right)(1-u)\right]C(\gamma,\theta)} \tag{15}$$

We can see that this expression admits a vertical asymptote in the space $(u,\dfrac{w_1}{P})$ when the unemployment rate gets close to $\underline{u} = \dfrac{\chi\left(\rho,s\right)-1}{\chi\left(\rho,s\right)} < 1$ what implies an inferior bound to the definition interval of the unemployment rate. We see that the larger the mark-up on the reservation wage, the higher this inferior bound of the unemployment rate is. This lower bound represents the unemployent trap of this economy, i.e. the minimum unemployment rate that the economy can achieve given its features on the good and labor market.

2 Multiple equilibria

In order to show the existence of multiple equilibria in this economy, we first analyze the properties and the shape of the aggregate equations. Secondly, we show graphically the different possible cases. Then, we prove the existence of multiple unemployment equilibria.

2.1 Equation's properties

For both equilibrium equations we compute the first and second order derivatives to determine their shape, and theirs limits towards the bounds of the unemployment rate's definition interval to determine their relative position.

2.1.1 The aggregate price equation

The first and second order derivatives of the aggregate price equation give us:

$$\frac{\partial\left(\frac{w_1}{P}\right)_{PS}}{\partial u} = (-\alpha\sigma)\frac{(\overline{N}/F)^{\alpha\sigma}}{\frac{s}{s-1}\frac{C(\gamma,\theta)}{(1-\gamma)^{\alpha(1+\sigma)}\gamma^{\beta}}}(1-u)^{(\alpha\sigma-1)} \tag{16}$$

$$\frac{\partial^2\left(\frac{w_1}{P}\right)_{PS}}{\partial u^2} = (-\alpha\sigma)(1-\alpha\sigma)\frac{(\overline{N}/F)^{\alpha\sigma}}{\frac{s}{s-1}\frac{C(\gamma,\theta)}{(1-\gamma)^{\alpha(1+\sigma)}\gamma^{\beta}}}(1-u)^{(\alpha\sigma-2)} \tag{17}$$

Proposition 1 *The PS curve is always downward sloping in the space* $(u,\dfrac{w_1}{P})$ *whatever the size of the knowledge spillovers in the economy. The PS curve is concave (linear) in the space* $(u,\dfrac{w_1}{P})$ *when* $\alpha\sigma \in \left]0;1\right[$ $(\alpha\sigma = 1)$*, and convex when* $\alpha\sigma > 1$.

Proof. We verify that the expression (16) is always negative when $\alpha\sigma > 0$, and the expression (17) is negative (null, positive) when $\alpha\sigma \in]0;1[$ ($\alpha\sigma = 1$, $\alpha\sigma > 1$). ■

The limits computations give[6]:

$$\lim_{u\to\underline{u}^+}\left(\frac{w_1}{P}\right)_{PS} = \left(\frac{\overline{N}/F}{\chi(\rho,s)}\right)^{\alpha\sigma}\left[\frac{s}{s-1}\frac{C(\gamma,\theta)}{(1-\gamma)^{\alpha(1+\sigma)}\gamma^{\beta}}\right]^{-1} > 0$$

$$\lim_{u\to 1^-}\left(\frac{w_1}{P}\right)_{PS} = 0.$$

These results remain true whatever the size of knowledge spillovers in the economy

2.1.2 The aggregate wage equation

The first and second order derivatives of the aggregate wage equation give us:

$$\frac{\partial\left(\frac{w_1}{P}\right)_{WS}}{\partial u} = \frac{[1-\chi(\rho,s)]}{[1-\chi(\rho,s)(1-u)]\,u}\left(\frac{w_1}{P}\right)_{WS} \tag{18}$$

$$\frac{\partial^2\left(\frac{w_1}{P}\right)_{WS}}{\partial u^2} = \frac{-2\chi(\rho,s)}{[1-\chi(\rho,s)(1-u)]}\frac{\partial\left(\frac{w_1}{P}\right)_{WS}}{\partial u} \tag{19}$$

Proposition 2 *When $\chi(\rho,s)>1$ and $u \in]\underline{u};1[$, the WS curve is always downward sloping and convex in the space $(u,\frac{w_1}{P})$ whatever the size of the knowledge spillovers in the economy.*

Proof. We verify that the expression (18) is always negative and the expression (19) is always positive when $\chi(\rho,s)>1$ and $u \in]\underline{u};1[$. ■

The limits computations give:

$$\lim_{u\to\underline{u}^+}\left(\frac{w_1}{P}\right)_{WS} = +\infty \quad \text{and} \quad \lim_{u\to 1^-}\left(\frac{w_1}{P}\right)_{WS} = \frac{\chi(\rho,s)(B/P)}{C(\gamma,\theta)} > 0.$$

Thus, we have the following interesting result for the relative position of the two curves:

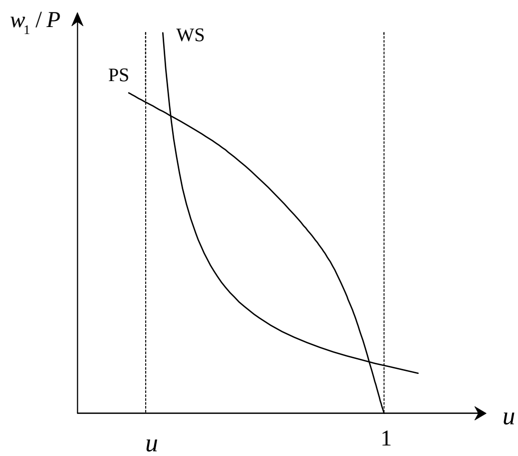
- When $u \to \underline{u}^+$, $\left(\frac{w_1}{P}\right)_{PS} < \left(\frac{w_1}{P}\right)_{WS}$: the PS curve is below the WS curve.
- When $u \to 1^-$, $\left(\frac{w_1}{P}\right)_{PS} < \left(\frac{w_1}{P}\right)_{WS}$: the PS curve is below the WS curve.

[6] $\underline{u}^+$ and 1^- represent the extrem values of the definition interval of $u \in]\underline{u}, 1[$.

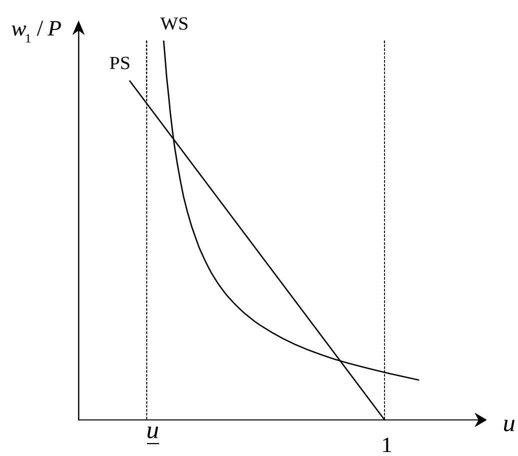
2.2 The different possible cases

Following the previous reports, we can conclude that three different cases can occur in the model according to the size of the knowledge spillovers in the economy. In the three cases, the PS curve always stands below the WS curve as we get close to the bounds of the definition interval of u. These different cases can be graphically represented in this way:

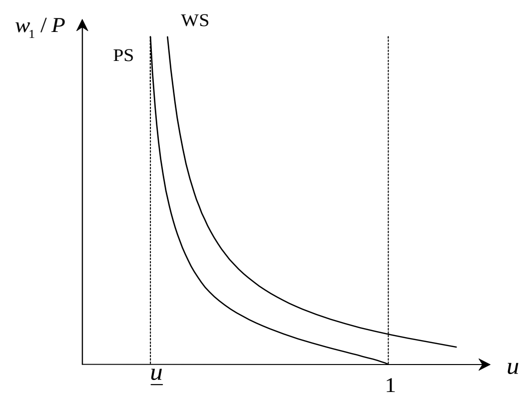
- If $\alpha\sigma \in]0;1[$: Diagram 1



- If $\alpha\sigma = 1$: Diagram 2



- If $\alpha\sigma > 1$: Diagram 3



The diagrams show us that it exists two equilibria if and only if $\alpha\sigma \in \left]0;1\right]$, i.e. if the size of knowledge spillovers is not too large, more specifically if the returns to scale at the aggregate level are not above 2. If the size of knowledge spillovers is too important ($\alpha\sigma > 1$) and the returns to scale too large, no equilibrium exists[7]. In what follows we assume that the condition $\alpha\sigma \in \left]0;1\right]$ is always satisfied and the case with no equilibrium is moved aside from the analysis.

2.3 Multiple unemployment equilibria

As we can see on the diagrams 1 and 2, both equilibria correspond to the intersections between the PS and WS curves.

Let $\Gamma(u) = \left(\dfrac{w_1}{P}\right)_{PS} - \left(\dfrac{w_1}{P}\right)_{WS}$, both equilibria correspond to the roots of this expression.

Proposition 3 *When $\alpha\sigma \in \left]0;1\right]$ the economy exhibits two distinct unemployment equilibria which belong to the interval $u \in \left]\underline{u};1\right[$.*

We demonstrate easily that the expression $\Gamma(u)$ is concave on the interval $u \in \left]\underline{u};1\right[$ when $\alpha\sigma \in \left]0;1\right]$. It reaches its maximum above the abscissa axis and it cuts this axis twice in the interval $u \in \left]\underline{u};1\right[$ (for more detailed calculations see annex A).

According to the diagrams 1 and 2, we can characterize the two equilibria as follow: one named the "low equilibrium" with a high unemployment rate and a low high-skilled real wage $\left(u_L, \left(\dfrac{w_1}{P}\right)_L\right)$ and another named the "high

[7] The case where the two curves are convex and intersect themselves twice is excluded by limits calculations.

equilibrium" with a low unemployment rate and a high high-skilled real wage $\left(u_H, \left(\frac{w_1}{P}\right)_H\right)$.

3 Effects of an exogenous increase in the minimum wage

In this section, we analyze the effects of an exogenous increase in the minimum wage on the unemployment equilibrium and the high-skilled labor real wage thanks to a comparative statics exercise. Then, we find that a policy raising the minimum wage has no effect on the unemployment equilibrium and reduces the purchasing power of the high-skilled workers whatever the nature of the equilibrium. Finally, we compare these findings with those of Card and Krueger (1995) and find some similarities. The problem of equilibrium selection is not discussed.

3.1 Comparative statics analysis

A policy that implements a rise in the nominal minimum wage is expressed in the model by an increase in the parameter θ.

Proposition 4 *Whatever the nature of the equilibrium, an increase in the nominal minimum wage has no effect on the unemployment equilibrium. Conversely, it involves a decrease in the high-skilled real wage equilibrium that is larger when the economy is at the high equilibrium than when it is at the low equilibrium.*

We demonstrate using the Cramer's rule on the equilibrium system composed of the PS and WS curves that $\frac{du_L}{d\theta} = \frac{du_H}{d\theta} = 0$ and $\frac{d\left(\frac{w_1}{P}\right)_H}{d\theta} < \frac{d\left(\frac{w_1}{P}\right)_L}{d\theta} < 0$ (See the annex B for details).

To give an intuition to these results, we may say things in this way: On the one hand, following an increase in the minimum wage, the firms respond to this rise in production cost by a rise in its own prices. Indeed, the partial derivative of (8) according to θ_i shows us that the firm raises its price proportionally about its share of low-skilled employment γ_i[8]. As a consequence, this individual price increase leads to a higher general price level at the general symmetric equilibrium. On the other hand, given the behavior of the trade union, the latter consents to a decrease in the real wage of workers who don't take advantage of this nominal wage rise in order to keep the employment level unchanged. As a result, the negotiations lead to a lower high-skilled real wage. In this way, at the general equilibrium, the total demand of labor as well as the unemployment rate remain unchanged whereas the high-skilled real wage decreases.

[8] The partial derivative of (8) gives us $\frac{\partial\left(\frac{p_i}{P}\right)^*}{\partial\theta_i} = \left(\frac{s}{s-1}\right)\frac{\gamma_i}{AB(\gamma_i)}\left(\frac{w_{1i}}{P}\right) > 0$

This effect on the general price level can be also deduced by considering the shift of the two curves. The partial derivatives of the PS and WS curves according to the parameter θ give us:

$$\frac{\partial \left(\frac{w_1}{P}\right)_{PS}}{\partial \theta} = -\left(\frac{w_1}{P}\right)_{PS} \frac{\gamma}{C(\gamma, \theta)} < 0 \tag{20}$$

$$\frac{\partial \left(\frac{w_1}{P}\right)_{WS}}{\partial \theta} = -\left(\frac{w_1}{P}\right)_{WS} \frac{\gamma}{C(\gamma, \theta)} < 0 \tag{21}$$

Since $\left(\frac{w_1}{P}\right)_{PS} = \left(\frac{w_1}{P}\right)_{WS}$ at the equilibrium , both curves move in the same extent and in the same direction. These shifts are the result of an increase in the general price level. The expressions (20) and (21) show also that the variation of the general price level is increasing in γ, i.e. the proportion of low-skilled labor employment in the total employment. Thus, we can conclude that the increase in the general price level is large enough to cover the new production cost and that an economic policy increasing the minimum wage mainly involves a cost push inflation in the economy.

An other interesting feature of these results is the fact that the implications of this economic policy on unemployment are the same whatever the equilibrium of the economy. Indeed, we have the same comparative statics results at the low and high equilibrium. Consequently, the selection equilibrium analysis is not approached here.

3.2 Comparisons with Card and Krueger's findings

Card and Krueger (1995) analyze the effects of such a policy by comparing the labor market performances of the 50 US states[9] before and after the 1990 and 1991 increases in the federal minimum wage.

They identify different employment outcomes on the concerned group of workers affected by this rise in the minimum wage in different time periods and regions of country. There is compelling evidence that the estimated employment effects aren't significantly different from zero. These rises have no negative employment effect and even sometimes a positive employment effect on the concerned group of workers (Card and Krueger, 1995, pp-389). According this evidence, the rise in the minimum wage does not imply a necessary rise on unemployment as most models suggest.

They analyze also the effects of a higher minimum wage on prices in the fast-food restaurants industry which is the leading employer of low-wage workers. Their results show that *"the price increases of about the magnitude required to cover the higher cost of labor associated with the rise in the minimum wage"* (Card and Krueger, 1995, pp-390).

Another set of their empirical results is about the effect of a higher minimum wage on the distribution of hourly wages. They find that *"these increases in the*

[9] They divide the states into three groups: two where the wages are high and this increase has little or no effect and one where the wages are low and this increase has important effect.

federal minimum wage led to significant increases in wages for workers at the bottom of the wage distribution, and to a reduction in overall wage dispersion" (Card and Krueger, 1995, pp-391).

Our model's results strongly support the two first Card and Krueger's results. As for the effect of a higher minimum wage on the distribution of hourly wages, the similarities with our model are more intuitive. Indeed, we cannot clearly show that the rise in minimum wage increases the wage of workers at the bottom of wage distribution due to the discrete types of labor assumption. Nevertheless, our model allows us to conclude that such a policy leads to a reduction in real wage dispersion since the real wage of high-skilled labor decreases and the real wage of low-skilled labor increases or remains unchanged[10].

The most important discrepancy between these empirical evidences and the common theory concerns the employment effect of a higher minimum wage. Therefore, Card and Krueger (1995) attempt to give a theoretical explanation of their findings by considering alternative models of labor market from the "textbook" model. They consider models where the wage is either taken by the firm (variants of the "textbook" model) or set by the firm (monopsony model[11]), but never models where the wage of workers paid more than the minimum wage results from a negotiation between the firm and a trade union. Machin and Manning (1994), who examine the impacts of a minimum wage decline on employment in U.K., find similar empirical evidences and attempt also to explain theoretically these effects by a monopsony model. Dickens and al. (1999) present a general theoretical model whereby employers have some degree of monopsony and in which minimum wage increases can have positive, neutral or negative effects on employment. Thus, our model can be also considered as an additional theoretical explanation of the effects of minimum wage increase or decrease on the unemployment.

Conclusion

In this paper, we have presented an imperfectly competitive model in which multiple unemployment equilibria can occur when the returns to scale of labor are constant at the firm level and increasing at the aggregate level due to the presence of knowledge spillovers in the economy. Then, we have shown that a minimum wage increase raises the general price level as a result of cost push inflation, decreases the real wage of workers who earn more than the minimum wage, and so reduces the real wage dispersion in the economy. Last but not least, we demonstrate that this economic policy has no effect on unemployment equilibrium if the wage setting system doesn't restore automatically the nominal wage gap that the minimum wage hike has reduced. Since these results come out

[10] The effect of price increase on real minimum wage depends on the extent of this increase, but in all case it cannot be superior to the extent of nominal minimum wage increase.

[11] Machin and Manning (1994): *"A monopsonistic labor market is one in which an employer possesses some market power in setting wages, so that the supply of labor to the firm is a positive function of the wage paid."*

irrespective to the nature of the equilibrium, we don't care about the selection problem in this paper.

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ANNEX

Annex A. Proof of proposition 3

Let $\Gamma(u) = \left(\frac{w_1}{P}\right)_{PS} - \left(\frac{w_1}{P}\right)_{WS}$ defined on $u \in \left]\underline{u}, 1\right[$, with $\Gamma(u) \in C^2$. Then, we must show that $\Gamma(u)$ admits two distinct positive roots in the interval u when $\alpha\sigma \in \left]0; 1\right]$ i.e. that $\Gamma(u)$ cuts twice the abscissa axis in the interval of $u \in \left]\underline{u}; 1\right[$ in the space $(u, \frac{w_1}{P})$.

$\Gamma(u)$ is a concave function on the interval $u \in \left]\underline{u}; 1\right[$ when $\alpha\sigma \in \left]0; 1\right]$. Indeed, we have:

$$\frac{\partial^2\Gamma(u)}{\partial u^2} = \frac{\partial^2 \left(\frac{w_1}{P}\right)_{PS}}{\partial u^2} - \frac{\partial^2 \left(\frac{w_1}{P}\right)_{WS}}{\partial u^2} < 0$$

Furthermore, we have:

$$\lim_{u \to \underline{u}} \Gamma(u) \to -\infty \quad > \quad \lim_{u \to 1^-} \Gamma(u) \to -\frac{\chi\left(\rho, s\right)\left(B/P\right)}{C(\gamma, \theta)}$$

Now, we just verify that a part of the graph of the function $\Gamma(u)$ is above the abscissa axis in the interval of $u \in \left]\underline{u}; 1\right[$ when $\alpha\sigma \in \left]0; 1\right]$, so we show that its maximum is positive and belongs to the interval of $u \in \left]\underline{u}; 1\right[$.

The polynom $\Gamma_u(u) = \frac{\partial \left(\frac{w_1}{P}\right)_{PS}}{\partial u} - \frac{\partial \left(\frac{w_1}{P}\right)_{WS}}{\partial u}$ is null when $\left[\frac{-\alpha\sigma}{(1-u)} - \frac{(1-\chi\left(\rho, s\right))}{\left[1-\chi\left(\rho, s\right)(1-u)\right]u}\right]\frac{w_1}{P}$ is null. This polynom has one positive root $\widetilde{u} = \frac{(1-\alpha\sigma)\left[\chi\left(\rho, s\right)-1\right]-\sqrt{\Delta}}{-2\alpha\sigma\chi\left(\rho, s\right)}$, with $\Delta = (\alpha\sigma-1)^2\left[\chi\left(\rho, s\right)-1\right]^2 + 4\alpha\sigma\chi\left(\rho, s\right)\left[\chi\left(\rho, s\right)-1\right] > 0$, which belongs to the interval $u \in \left]\underline{u}; 1\right[$. In addition, we have $\Gamma(\widetilde{u}) > 0$ when $\alpha\sigma \in \left]0; 1\right]$ and $\chi\left(\rho, s\right) > 1$. Thus, $\Gamma(u)$ admits two distinct positive roots in the interval u when $\alpha\sigma \in \left]0; 1\right]$ and $\chi\left(\rho, s\right) > 1$ and we have two distinct unemployment equilibria.

Annex B. Proof of proposition 4

Let $\left(\frac{w_1}{P}\right)_{PS} = \Psi(u, \theta)$ and $\left(\frac{w_1}{P}\right)_{WS} = \Phi(u, \theta)$, the high equilibrium $(u_H, \left(\frac{w_1}{P}\right)_H)$ is solution of this system:

$$\begin{cases} \Psi(u_H, \theta) - \left(\frac{w_1}{P}\right)_H = 0 \\ \Phi(u_H, \theta) - \left(\frac{w_1}{P}\right)_H = 0 \end{cases}$$

After total differentiation and arrangements, we obtain this system:

$$\begin{cases} \dfrac{\partial \Psi(u_H,\theta)}{\partial u_H}\dfrac{du_H}{d\theta} + \dfrac{\partial \Psi(u_H,\theta)}{\partial \theta} - \dfrac{d\left(\frac{w_1}{P}\right)_H}{d\theta} = 0 \\ \dfrac{\partial \Phi(u_H,\theta)}{\partial u_H}\dfrac{du_H}{d\theta} + \dfrac{\partial \Phi(u_H,\theta)}{\partial \theta} - \dfrac{d\left(\frac{w_1}{P}\right)_H}{d\theta} = 0 \end{cases}$$

The application of Cramer's rule and the properties of the PS & WS curves give us:

$$\frac{du_H}{d\theta} = \frac{\dfrac{\partial \Psi(u_H,\theta)}{\partial \theta} - \dfrac{\partial \Phi(u_H,\theta)}{\partial \theta}}{\dfrac{\partial \Psi(u_H,\theta)}{\partial u_H} - \dfrac{\partial \Phi(u_H,\theta)}{\partial u_H}} = \frac{\left[\Phi(u_H,\theta) - \Psi(u_H,\theta)\right]\frac{\gamma}{C(\gamma,\theta)}}{\dfrac{\partial \Psi(u_H,\theta)}{\partial u_H} - \dfrac{\partial \Phi(u_H,\theta)}{\partial u_H}}$$

$$\frac{d\left(\frac{w_1}{P}\right)_H}{d\theta} = \frac{\dfrac{\partial \Phi(u_H,\theta)}{\partial u_H}\dfrac{\partial \Psi(u_H,\theta)}{\partial \theta} - \dfrac{\partial \Psi(u_H,\theta)}{\partial u_H}\dfrac{\partial \Phi(u_H,\theta)}{\partial \theta}}{\dfrac{\partial \Psi(u_H,\theta)}{\partial u_H} - \dfrac{\partial \Phi(u_H,\theta)}{\partial u_H}} = \Phi(u_H,\theta)\frac{-\gamma}{C(\gamma,\theta)} < 0$$

At the equilibrium u_H, we have $\Phi(u_H,\theta) = \Psi(u_H,\theta)$. Therefore, an increase of minimum wage has no effect on unemployment equilibrium as $\dfrac{du_H}{d\theta} = 0$. The same argument give us the same result at the low equilibrium $(u_L, \left(\dfrac{w_1}{P}\right)_L)$.

Inversely, an increase in minimum wage leads to a decrease of $\left(\frac{w_1}{P}\right)_H$ which is more important when the proportion of low skilled worker γ is high. In the same manner, such policy leads to a decrease of $\left(\frac{w_1}{P}\right)_L$ when the economy is at the low equilibrium, but as $\Phi(u_H,\theta) > \Phi(u_L,\theta)$ this decrease is larger at the high equilibrium than at the low equilibrium.