On Strategic Market Games: From Nash to Walras or to Autarky

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Abstract

This paper provides exchange economies, construed as Shapley–Shubik strategic market games, where no equilibrium features trade, and no convergence takes place towards competitive outcomes. A knife-edge separates the qualitative nature of some results. On one hand, a minor reduction in an agent's endowment can ensure the realization of Walras equilibrium to any desirable approximation. On the other hand, slightly enhanced endowment may suffice to bring about autarky. Satiation in preferences plays a key role in establishing these results. To illustrate their relevance, I consider emissions trading under the Kyoto Protocol.

Keywords: strategic market games, Nash and Walras equilibrium, convergence, satiation, emissions trading.

1 Introduction

Perfect competition continues to hold a central place in economic analysis. During the last three decades, and reflecting a surge in noncooperative game theory, a number of major research programs have arisen concerned with the strategic foundations of perfect competition. Of particular promise and note in this regard is the Shapley–Shubik strategic market game [12]. A simple and tractable instance of this game is the main object of the current paper, the impetus for which is a passage in Shapley and Shubik's (op. cit.) original work:

"we do not assume that the utility of the payment commodity is additively separable... But it is possible that such an assumption might simplify some of our results or proofs or ensure uniqueness or other good behavior on the part of the noncooperative equilibria

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just as it does for the classical competitive equilibria.[1] At least the question merits further study."

Following this suggestion, I accommodate here just two goods and let all utility functions be quasi-linear. Moreover, each agent displays satiation in his use of the nonmonetary good.[2] Regrettably, even this much simplification may yield little of the good behavior hoped for. To wit, identified below are exchange economies where autarky is the only Nash equilibrium in the strategic market game. That particular finding is not novel, as others have concluded likewise (e.g., Busetto and Codognato [2] and Cordella and Gabszewicz [3]). The reasons for the result obtained here differ, however, from this literature. Therefore, some implications are worth bringing out further. A Kyoto-type example suits this purpose. Readers who prefer to shy away from that particular case may instead consider two more tractable examples provided below. These make use of the following standing notation.

I is a finite set of agents, all regarded as producers. Each $i \in I$ is endowed with a_i units of a commodity good—in the nature of a production factor—which generates payoff $u_i(\cdot)$. The other good serves as money and is in 'sufficient' supply in the Shapley–Shubik sense.

Example 1. There are four agents, and each has marginal production payoff $u_i'(x_i) = \max\{10 - x_i, 0\}$. The initial allocation of the commodity good $(a_1, ..., a_4) = (8, 8 + \varepsilon, 12, 12)$.

The results for this case appear after a second example:

Example 2. There are three agents with the same payoff functions as in Example 1, and the initial allocation $= (8, 8, 15)$.

Results of both examples (*On existence*) A competitive equilibrium exists in both examples for any value of ε. If $\varepsilon < 0$, then both economies also possess Nash equilibria (up to first-order optimality conditions) that feature trade in the strategic market game. Such trading equilibria are eliminated when $\varepsilon > 0$ in Example 1; it is also eliminated in Example 2 if the setting is replicated once or more.
(*On convergence*) When $\varepsilon \to 0^-$ in Example 1, then a Nash equilibrium with trade converges to a Walras equilibrium.
(*On destruction*) If $\varepsilon = 1$, every agent in Example 1 will benefit from (unilaterally) destroying 1.5 units of the commodity good before playing the game. □

[1] With an additively separable 'u-money' in sufficient supply, the competitive allocations of the other goods are just those that maximize [the aggregate utility of the nonmoney commodities]; thus a "fixed point" situation reduces to a simpler maximization problem.

[2] This final feature complies with some literature dealing with the market for rights to release greenhouse gases under the Kyoto Protocol, meaning marginal products eventually become nil.

The results on existence and destruction are more or less direct consequences of Theorem 1, provided shortly. The convergence result will only be shown via a Kyoto-type simulation, displayed in Section 3. Section 4 collates some bibliographic remarks.

2 The game and the result

Before spelling out the strategic market game, recall (from Footnote 2) that a **competitive equilibrium** allocation $(x_i)_{i\in I}$ solves

$$\max\left\{\sum_{i\in I} u_i\left(x_i\right) : \sum_{i\in I} x_i = \sum_{i\in I} a_i\right\} \tag{1}$$

while an equilibrium price p can be modeled as a Lagrange multiplier (shadow price) associated with the constraint. This equilibrium exists under quite weak and plausible assumptions (including those specified in the upcoming Assumption). Moreover, if u_i is concave for all $i \in I$, then a sufficient condition for an equilibrium allocation (price) to be unique is that each u_i is strictly concave (continuously differentiable). Finally, an interior equilibrium is characterized by all agents having the same marginal payoff equal to the clearing price.

The version of the **strategic market game** adopted here goes as follows. Each agent $i \in I$ places $q_i \in [0, a_i]$ units of the good and $b_i \geq 0$ units of money on the trading post. Suppose each has enough money so that there are no upper bounds on b_i. Name aggregate *supply* and *bid*

$$Q := \sum_{i\in I} q_i \text{ and } B := \sum_{i\in I} b_i,$$

respectively. The commodity good is traded at the unit price

$$p = \frac{B}{Q} \tag{2}$$

when $Q > 0$; otherwise $p = 0$. Because our interest is in equilibria with trade, we proceed under the hypothesis that $p > 0$. Agent $i \in I$ is paid pq_i for his own supply and takes home $\frac{b_i}{p}$ units of the good. Each agent cares for the sum of his production payoff $u_i(\cdot)$ plus market revenues $pq_i - b_i$. A profile $(q_i, b_i)_{i\in I}$ is declared a **Nash equilibrium** of the game iff (q_i, b_i)

$$\text{maximizes}\{u_i(a_i - q_i + \frac{b_i}{p}) + pq_i - b_i\} \tag{3}$$

for all $i \in I$ with $(q_j, b_j)_{j\in I\setminus\{i\}}$ regarded as given.

Write $\Pi_i(\cdot)$ for the objective function in (3) so that

$$\frac{\partial \Pi_i}{\partial b_i} = u_i'(a_i - q_i + \frac{b_i}{p})\frac{p - b_i \frac{\partial p}{\partial b_i}}{p^2} + \frac{\partial p}{\partial b_i} q_i - 1 \tag{4}$$

and

$$\frac{\partial \Pi_i}{\partial q_i} = u_i'(a_i - q_i + \frac{b_i}{p})\left(-1 - \frac{b_i \frac{\partial p}{\partial q_i}}{p^2}\right) + p + \frac{\partial p}{\partial q_i} q_i - \lambda_i. \tag{5}$$

In (5), λ_i is the shadow price associated with $q_i \leq a_i$. By the Karush–Kuhn–Tucker conditions, a Nash equilibrium profile $(q_i, b_i)_{i \in I}$ must comply with

$$\frac{\partial \Pi_i}{\partial b_i} \leq 0,\ b_i \geq 0 \text{ and } \frac{\partial \Pi_i}{\partial b_i} b_i = 0; \tag{6}$$

$$\frac{\partial \Pi_i}{\partial q_i} \leq 0,\ q_i \geq 0 \text{ and } \frac{\partial \Pi_i}{\partial q_i} q_i = 0; \tag{7}$$

$$\lambda_i \geq 0,\ q_i \leq a_i \text{ and } (q_i - a_i)\lambda_i = 0 \tag{8}$$

for all $i \in I$, with

$$\frac{\partial p}{\partial b_i} = \frac{1}{Q} \text{ and } \frac{\partial p}{\partial q_i} = -\frac{p}{Q}. \tag{9}$$

Definition (On first-order Nash and satiation).
(*i*) A set of choices $(q_i, b_i)_{i \in I}$ that satisfies the first-order optimality conditions is called a *first-order Nash equilibrium* of the game.
(*ii*) A *smallest satiation point* $\hat{x}_i$, is a strictly positive finite number for which $u_i(x_i) < u_i(\hat{x}_i)$ when $x_i \in [0, \hat{x}_i)$ and $u_i(x_i) = u_i(\hat{x}_i)$ when $x_i \geq \hat{x}_i$.
(*iii*) An agent has *excess endowments* if $a_i > \hat{x}_i$ while an agent is *short* if $a_i < \hat{x}_i$.
(*iv*) The economy has *excess endowment in aggregate*, if $\sum_{i \in I} a_i > \sum_{i \in I} \hat{x}_i$.

Remark (On Nash and first-order Nash).
While a genuine Nash equilibrium is of first order, I cannot exclude the possibility that first-order equilibria may exist that are not Nash. My analysis relies on the broader first-order concept. This implies that the results of Theorem 1 and Cases 2 & 3 in Section 3 are more general than when dealing exclusively with genuine Nash equilibria. Moreover, Cases 1, 4 & 5 in Section 3 are not necessarily Nash, as I have only verified that they are of first order.

Assumption (Standing till the end of this section).
(*i*) $u_i(\cdot)$ is nondecreasing, concave and continuously differentiable for each $i \in I$;
(*ii*) there exists a *smallest satiation point* $\hat{x}_i > 0$ for all $i \in I$;
(*iii*) there is at least one agent who is *short* and at least two agents with *excess endowments*; and

(iv) there is *excess endowment in aggregate.*

The first part of Assumption (iii) only serves to provide an (interesting) economy where autarky is Pareto inefficient. The concavity assumption in (i) is never explicitly used but guarantees the existence of a competitive equilibrium.

Theorem 1 (On lack of existence of equilibrium with trade). *A Nash equilibrium with trade does not exist.*

The proof follows after four lemmas.

Lemma 1 (On vanishing margins). *Any feasible allocation implies that $u_j'(a_j - q_j + \frac{b_j}{p}) = 0$ for at least one $j \in I$.*

Proof. Suppose on the contrary that $u_i'(a_i - q_i + \frac{b_i}{p}) > 0$ for all $i \in I$. For the latter to be true, $a_i - q_i + \frac{b_i}{p} < \hat{x}_i$ must hold for all $i \in I$. Final consumption summed over all agents then amounts to

$$\sum_{i \in I}(a_i - q_i + \frac{b_i}{p}) = \sum_{i \in I} a_i < \sum_{i \in I} \hat{x}_i.$$

The equality follows by (2), while the inequality contradicts Assumption (iv) on aggregate excess. Thus, there exists at least one agent $j \in I$ with $u_j'(a_j - q_j + \frac{b_j}{p}) = 0$. □

Lemma 2 (On bids and supply when the margin is nil). *Suppose there exists an equilibrium where at least two agents have offered strictly positive supplies. Then, any agent j who is satiated in equilibrium, i.e., $u_j'(a_j - q_j + \frac{b_j}{p}) = 0$, will bid nothing and must have supplied at least $a_j - \hat{x}_j$.*

Proof. (4) for agent j amounts to

$$u_j'(a_j - q_j + \frac{b_j}{p}) \cdot \frac{p - b_j \frac{\partial p}{\partial b_j}}{p^2} + \frac{\partial p}{\partial b_j} q_j - 1 = \frac{1}{Q} q_j - 1 < 0$$

as $u_j'(\cdot) = 0$ and $q_j < Q$. Hence, for condition (6) to hold for agent j, we must have $b_j = 0$. Concerning his supply, if $a_j \leq \hat{x}_j$, and because $q_j \geq 0$, statement $q_j \geq a_j - \hat{x}_j$ follows trivially. Should $a_j > \hat{x}_j$ and $q_j < a_j - \hat{x}_j$, then by the definition of $\hat{x}_j$ and by Assumption (ii), it must be true that $u_j' = 0$ in equilibrium, so that the right-hand side of (5) reduces to

$$p \cdot \left(1 - \frac{q_j}{Q}\right) - \lambda_j. \tag{10}$$

Because $\hat{x}_j$ by definition is strictly positive, the constraint $q_j \geq 0$ cannot bite when $q_j < a_j - \hat{x}_j$, hence λ_j will be nil. Because by assumption $q_j < Q$, expression (10) becomes strictly positive, this contradicts condition (7). Thus, $q_j < a_j - \hat{x}_j$ is a contradiction yielding $q_j \geq a_j - \hat{x}_j$. $\square$

Lemma 3 (On agents who bid). *Suppose there exists an equilibrium allocation where $u_j'(a_j - q_j + \frac{b_j}{p}) = 0$ for at least one $j \in I$. Then the total final consumption among $i \in I \setminus \{j\}$ is strictly greater than their aggregate smallest satiation points.*

Proof. Because there is one agent j with a vanishing margin, then by Lemma 2, $b_j = 0$ and

$$p = \frac{\sum_{i \in I \setminus \{j\}} b_i}{\sum_{i \in I \setminus \{j\}} q_i + q_j}.$$

Suppose now, and contrary to what is claimed, that

$$\begin{aligned}
0 &\leq \sum_{i \in I \setminus \{j\}} \hat{x}_i - \sum_{i \in I \setminus \{j\}} (a_i - q_i + \frac{b_i}{p}) \\
&= \sum_{i \in I} \hat{x}_i - \hat{x}_j - \sum_{i \in I \setminus \{j\}} a_i + \sum_{i \in I \setminus \{j\}} q_i - \frac{\sum_{i \in I \setminus \{j\}} q_i + q_j}{\sum_{i \in I \setminus \{j\}} b_i} \sum_{i \in I \setminus \{j\}} b_i \\
&< \sum_{i \in I} a_i - \hat{x}_j - \sum_{i \in I \setminus \{j\}} a_i + \sum_{i \in I \setminus \{j\}} q_i - \sum_{i \in I} q_i \ (*) \\
&= \sum_{i \in I \setminus \{j\}} a_i + a_j - \hat{x}_j - \sum_{i \in I \setminus \{j\}} a_i - q_j \\
&= a_j - \hat{x}_j - q_j \leq 0,
\end{aligned}$$

a contradiction. The inequality in (*) follows by Assumption (*iv*), while the last inequality comes from Lemma 2. $\square$

Lemma 4 (On the number of suppliers). *If an equilibrium profile with trade exists, then there are at least two agents with strictly positive supplies.*

Proof. Because there is trade, $Q > 0$. Suppose now, on the contrary, that $q_i = Q$ for exactly one $i \in I$ with $q_j = 0$ for all else. By Assumption (*iii*) an agent $k \neq i$ with excess endowments must then exist, i.e., $a_k - \hat{x}_k > 0$. Because $q_k = 0$, agent k will in equilibrium have $u_k'(\cdot) = 0$. This implies that the right-hand side of (5) for agent k reduces to $p \cdot \left(1 - \frac{q_k}{Q}\right) - \lambda_k = p - \lambda_k = p > 0$ because $\lambda_k = 0$ (by the same argument as in Lemma 2). The last inequality contradicts (7) for agent k. $\square$

Proof of Theorem 1. Suppose that a Nash equilibrium with trade exists, i.e.,

B, Q and p are all > 0. By Lemma 1, there will exist at least one agent with $u_j'(a_j - q_j + \frac{b_j}{p}) = 0$. From Lemma 4, there must be more than one supplier. Thus, Lemma 2 comes into effect and an agent with $u_j' = 0$ must have offered a bid $b_j = 0$ and supplied $q_j \geq a_j - \hat{x}_j$. By Lemma 3, there will be excess endowments in aggregate among all $i \in I \setminus \{j\}$, and by Lemmas 1 and 2, there will once again exist at least one agent $i \in I \setminus \{j\}$, with $u_i'(a_i - q_i + \frac{b_i}{p}) = 0$, $b_i = 0$ and $q_i \geq a_i - \hat{x}_i$; and so on. By the logic of induction, this implies that all agents will choose $b_i = 0$, which contradicts $B > 0$. $\square$

3 The Kyoto case

This section considers the exchange of rights to emit greenhouse gases under the 1997 Kyoto Protocol [14] and brings out some results that are qualitatively speaking akin to the examples in Section 1.

3.1 Parameters

The parameters of the model stem originally from the (intertemporal CGE) model MERGE of Manne and Richels [8] as reported in Godal and Klaassen [6]. In line with the implementation of the Kyoto Protocol, the period under consideration is 2008–2012. While the player set I should ideally be those governments with quantified commitments that have ratified the agreement, the available parameters cover only the relevant regions. These are CANZ (Canada, Australia and New Zealand), EEFSU (Eastern Europe and the former Soviet Union)[3], OECDE (European OECD countries) and Japan. Payoff functions are normalized to $u_i(\hat{x}_i) = 0$ and therefore measure what is sometimes referred to as (negative) abatement costs. The functional form is given by

$$u_i(x_i) = \begin{cases} -\frac{1}{2\beta_i}(\alpha_i - \beta_i x_i)^2 & \text{when} \quad x_i \in [0, \alpha_i/\beta_i) \\ 0 & \text{when} \quad x_i \geq \alpha_i/\beta_i \end{cases}$$

where $\alpha_i, \beta_i > 0$. Hence, the smallest satiation point (also known as 'business-as-usual emissions') are given by $\hat{x}_i = \alpha_i/\beta_i$, while the marginal payoff becomes equal to $\max\{\alpha_i - \beta_i x_i, 0\}$. The values of α_i, β_i, endowments a_i, the computed $\hat{x}_i$, and the marginal (and total) payoff in autarky for the Kyoto-relevant MERGE regions are given in Table I.[4]

[3] The Kyoto countries in this group comprise Bulgaria, the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Romania, the Russian Federation, Slovakia, Slovenia and the Ukraine.

[4] Throughout, units are abbreviated with M-million, B-billion, t-metric ton, C-carbon, USD-1997 US dollars and yr-year.

Table I. Parameter values of the model, smallest satiation points, marginal payoff and payoff in autarky.*

Variable	Payoff functions		Endow.	S.S.P	M.P.	P.P.
Symbol	α_i	β_i	a_i	$\hat{x}_i$	$u'_i(a_i)$	$u_i(a_i)$
Units	US$/tC	US$yr/M(tC)^2	MtC/yr		US$/tC	BUS$/yr
CANZ	693	2.216	215	312.7	216.6	−10.6
EEFSU	1410	1.569	1314	898.7	0.0	0.0
Japan	1727	4.933	258	350.1	454.3	−20.9
OECDE	1883	1.813	860	1038.6	323.8	−28.9
Total			2647	2600		−60.4

* 'S.S.P.' is the smallest satiation point; 'M.P.' is marginal payoff; 'P.P.' is the production payoff.

Table I illustrates that the EEFSU has so much excess endowment that this also holds in aggregate ($2647 > 2600$). It is also clear that with this parameterization of the model, a competitive equilibrium (as in Montgomery [9] i.e., a solution to (1)) exists where all agents have the same vanishing marginal payoff ($= p = 0$).[5] The associated allocation belongs to the interior ($x_i > 0$ for all $i \in I$), but is not unique.[6]

3.2 On existence

Case 1 (*A first-order Nash equilibrium with trade*). If the four regions play a strategic permit market game, then a first-order Nash equilibrium exists as in Table II.

Table II. A first-order Nash equilibrium with trade in the strategic market game.*

Variable	Supply	Bid	Demand	Cons.	Sold	M.P.	M.R.	P.P.	T.P.
Symbol	q_i	b_i	b_i/p			$u'_i(\cdot)$		$u_i(\cdot)$	
Units	MtC/yr	MUS$/yr	MtC/yr			US$/tC	BUS$/yr		
CANZ	0	2546	17.3	232	−17.3	178.3	−2.5	−7.2	−9.7
EEFSU	99.6	0	0	1214	99.6	0.0	14.7	0.0	14.7
Japan	0	6115	41.5	299	−41.5	252.6	−6.1	−6.5	−12.6
OECDE	0	6019	40.8	901	−40.8	249.8	−6.0	−17.2	−23.2
Total	99.6	14679	99.6	2647	0.0	147.4	0.0	−30.8	−30.8

* 'Cons.' is final consumption and amounts to $a_i - q_i + b_i/p$; 'Sold' is $q_i - b_i/p$; 'M.P.' is marginal payoff evaluated at final consumption; 'M.R.' is market revenue; 'P.P.' is the

[5] This seemingly odd result is in line with other studies of the Kyoto agreement without US participation (see, e.g., Springer [13]) which project either a low or vanishing price. The explanation lies with the economic setbacks of the former Soviet states.

[6] The allocation $x_i := \hat{x}_i + \mu_i \sum_{j \in I} (a_j - \hat{x}_j)$ for all $i \in I$ where μ_i is any number satisfying $\mu_i \geq 0$ for all $i \in I$, and $\sum_{i \in I} \mu_i = 1$ will be competitive (for our parameters).

production payoff evaluated at final consumption, and total payoffs 'T.P.' are production payoffs plus market revenue. The equilibrium price is located in the column M.P. and row Total.

The figures in Table II illustrate that the EEFSU gives a positive supply of permits and no bid, while the other regions give positive bids and no supply. This equilibrium therefore satisfies the so-called 'no-wash-sale' condition, signifying that both supply and bids cannot be strictly positive for any agent (see Shapley and Shubik [12, p. 964]). It should be emphasized, however, that in this and subsequent simulations, the no-wash-sale condition is not a binding constraint and only serves to help compute equilibrium. The results in Table II also demonstrate that all agents are better off with trade than in autarky.

Case 2 (*On replication and autarky*). For theoretical interest (not realism), suppose now that the economy is replicated exactly once. Then, the Assumption (including the last part of (*iii*)), becomes satisfied, and autarky becomes the only equilibrium in the game. Clearly, a perfectly competitive equilibrium exists with a price $p = 0$. If the economy is replicated time and again, autarky will prevail.

A not so appealing feature of the parameterization of the model is that countries (governments) are aggregated into regions. This may be reasonable for a group like OECDE, because the members of the European Union have agreed as a group to comply with the Protocol. However, for the EEFSU, no such agreement is commonly known to exist. It therefore appears better to split the EEFSU before modeling trade.

Case 3 (*On disaggregation and autarky*). The MERGE-projected emissions for the EEFSU suggest that this region will have excess endowments in 2008–2012 (Table I). This is supported by actual 2005 emissions available at www.unfccc.org.[7] Moreover, also according to this site, Russia has a little less than two-thirds of the total emissions for this region. To improve the modeling slightly, we split the EEFSU into two agents: Russia and the remainder (called Ukraine+). Write

$$\begin{array}{ll} \alpha_{Russia} := \alpha_{EEFSU}; & \alpha_{Ukraine+} := \alpha_{EEFSU}; \\ \beta_{Russia} := \frac{\beta_{EEFSU}}{2/3}; & \beta_{Ukraine+} := \frac{\beta_{EEFSU}}{1/3}; \\ a_{Russia} := (2/3)\, a_{EEFSU} \text{ and} & a_{Ukraine+} := (1/3)\, a_{EEFSU}. \end{array}$$

With these definitions, the region's endowment remains unchanged, and for any common marginal payoff for the two subregions, total emissions will equal aggregate

[7] More specifically, the data on http://unfccc.int/ghg_emissions_data/ghg_data_from_unfccc/time_series_annex_i/items/3814.php for 2005 show that Russian emissions are about 28–29% below 1990 emissions, while the same figures for the Ukraine are 55–58% (depending on whether so-called Land Use, Land-Use Change and Forestry (LULUCF) activities are included).

EEFSU for the same marginal payoff. When adopting this method of disaggregation (and now replacing the EEFSU with Russia and Ukraine+), then the Assumption is satisfied (again including the last part of (*iii*)), and no Nash equilibrium with trade exists.[8] The results of other simulations (not shown) demonstrate that by excluding either Russia or Ukraine+ from the game, a first-order equilibrium with trade exists. Therefore, this is an instance where adding one agent to an economy leads to the elimination of trade in the strategic market game.

3.3 On convergence

Case 4 (*From Nash to Walras*). Continuing now with Russia and Ukraine+ as separate players. I next remove a sufficient amount of the endowment of one agent in the economy so that Assumption (*iv*) does not come into effect. The chosen agent is Japan, which has the highest willingness to pay for a permit in autarky (any other agent could have been picked in its place). Japan's new (reduced) endowment will then be increased in small steps to see the effect on the trading equilibrium in the game. For ease of interpreting the results, write

$$D := \frac{\sum_{i \in I} a_i - \sum_{i \in I} \hat{x}_i}{\sum_{i \in I} \hat{x}_i}$$

for the degree of excess endowments available in the economy. Thus, $D > 0$ means that there is excess endowments in aggregate, while $D < 0$ signifies that there is no such excess.

Table III. From Nash to Walras when perturbing the endowment of Japan.*

End.	Exc. end.	Nash marg. payoff, $u_i(a_i - q_i + b_i/p)$					Prices	
Japan	D	CANZ	Russia	Ukr.+	Japan	OECDE	Nash	Walras
MtC/yr		US$/tC						
10	−0.0775	108.22	33.01	63.26	271.23	127.44	96.27	109.38
110	−0.0391	59.23	16.73	32.70	101.41	72.93	49.44	55.12
160	−0.0198	30.93	8.33	16.46	43.54	39.44	24.79	27.98
185	−0.0102	16.09	4.22	8.39	20.53	20.98	12.62	14.42
197.5	−0.0054	8.55	2.21	4.41	10.39	11.29	6.63	7.63
203.75	−0.0030	4.76	1.22	2.44	5.64	6.33	3.67	4.24
206.875	−0.0018	2.86	0.73	1.46	3.35	3.81	2.20	2.55
208.4375	−0.0012	1.91	0.49	0.98	2.22	2.55	1.46	1.70
209.21875	−0.0009	1.43	0.37	0.73	1.66	1.92	1.10	1.28

[8] The same conclusion follows for any disaggregation that satisfies the following conditions (J being the set of subregions the EEFSU is split into): $\sum_{j \in J} a_j = a_{EEFSU}$ and $a_j > \hat{x}_j$ for at least two members of J. Note that disaggregating other regions has no effect on the conclusion. Hence, even if all governments decide to allocate permits to firms, and trade is subsequently modeled as a strategic market game between (thousands of) firms, no equilibrium with trade will exist.

* 'Exc.' is short for 'Excess'; 'End.' for 'endowment'; 'marg.' for 'marginal' and 'Ukr.' for 'Ukraine'. Observe that D is free of units.

For comparisons of Nash and Walras, recall that the marginal payoffs in a competitive equilibrium are identical across agents and equal the associated price. Table III demonstrates that the marginal payoffs in the strategic market game differ substantially across agents when the economy is not so well furnished (first row). As the endowment of Japan increases, so that the degree of excess endowment in aggregate (D) approaches zero, the marginal payoffs in Nash get closer and closer to the associated equilibrium price along with the competitive price. The table also shows that in the strategic market game equilibrium, permit sellers (Russia and Ukraine+) have a lower marginal payoff than the Nash equilibrium price, while the opposite is the case for those that come forward as buyers. This feature confirms intuition, and can be shown to hold more generally.

A more interesting variable to examine when it comes to convergence is overall efficiency. Therefore, write W^A, W^C and W^S for the welfare (aggregate production payoff) evaluated in autarky, the competitive equilibrium and in the strategic market game (when there is trade) respectively. More precisely

$$W^A := \sum_{i \in I} u_i\left(a_i\right), \; W^C := \sum_{i \in I} u_i\left(x_i\right) \text{ and } W^S := \sum_{i \in I} u_i\left(a_i - q_i + \frac{b_i}{p}\right).$$

Clearly, each of these welfare functions depend on a_{Japan}. The number

$$E := \frac{W^S - W^A}{W^C - W^A}$$

then serves as an efficiency index for the strategic market game relative to the competitive equilibrium for any given a_{Japan}. Thus, $E = 0$ when there is no trade in the strategic market game, and $E = 1$ when that game is as equally efficient as a competitive equilibrium.

Figure 1 shows how the efficiency index E varies with the degree of excess endowments D, for the same values of a_{Japan} as in Table III (10 million tons of carbon per year is represented by the leftmost point).

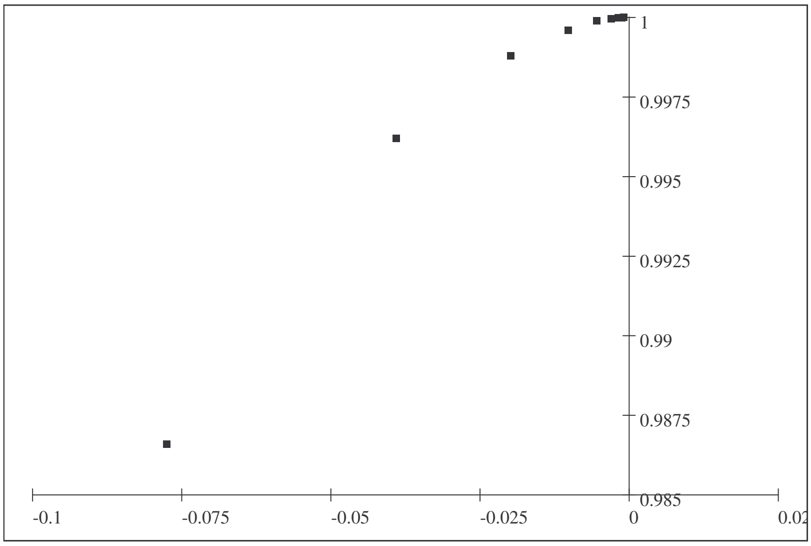

Figure 1. Efficiency of the strategic market game relative to that of competitive equilibrium (E, y-axis) versus the degree of excess endowments (D, x-axis) when perturbing the endowment of Japan.

We can see from Figure 1 that when Japan's endowment is 10 million tons of carbon per year, then the strategic market game generates about 98.7% of the potential gains from trade. When increasing Japan's endowment towards about 210, so that D approaches zero, then the efficiency index becomes close to one, i.e., the welfare in the strategic market game approximates that of Walras. With further increases in a_{Japan}, making $D > 0$, the Assumption (including part (*iv*)) comes into effect and $E = 0$.

3.4 On destruction

Case 5 (*Profitable endowment destruction*). Consider next the parameterization of the economy in Case 3. If Japan destroys 48 units of its endowment before playing the game, then a first-order Nash equilibrium with trade exists with the characteristics given in Table IV ($D = -0.0002$).

Table IV. A first-order Nash equilibrium when Japan destroys 48 units of its endowment.*

Variable	Supply	Bid	Demand	Cons.	Sold	M.P.	M.R.	P.P.	T.P.
Symbol	q_i	b_i	b_i/p			$u_i'(\cdot)$		$u_i(\cdot)$	
Units	MtC/yr	MUS$/yr	MtC/yr			US$/tC	BUS$/yr		
CANZ	0	71.3	97.3	312	−97	0.956	−0.071	−0.0002	−0.0715
Russia	277.1	0	0	599	277	0.244	0.203	0	0.2030
Ukraine +	138.5	0	0	299	138	0.488	0.101	0	0.1015
Japan	0	102.9	140.5	350	−140	1.106	−0.102	−0.0001	−0.1030
OECDE	0	130.3	177.9	1038	−177	1.280	−0.130	−0.0005	−0.1308
Total	415.7	304.4	415.7	2599	0	0.732	0	−0.0008	−0.0008

* 'Cons.' is final consumption and amounts to $a_i - q_i + b_i/p$; 'Sold' is $q_i - b_i/p$; 'M.P.' is marginal payoff evaluated at final consumption; 'M.R.' is market revenue; 'P.P.' is the production payoff evaluated at final consumption, and total payoffs 'T.P.' are production payoffs plus market revenue. The equilibrium price is located in the column M.P. and row Total.

A perfectly competitive equilibrium with this parameterization produces a clearing price $p = 0.561$. Not surprisingly, Table IV demonstrates that Japan is substantially better off compared to autarky (see Table I). Had some other agent instead destroyed the 48 units, the qualitative nature of the results would remain unchanged.

4 Bibliographic remarks

On existence. Dubey and Shubik [5] provided sufficient conditions for the existence of at least one Nash equilibrium in strategic market games. Peck et al. [10] showed that such an equilibrium can entail trade. On at least one account, this paper is in line with Cordella and Gabszewicz [3] and Busetto and Codognato [2] by providing a two-good, quasi-linear exchange economy where autarky is the only Nash equilibrium. On other accounts, they differ. First, because of satiation the economies brought out here do not satisfy the Dubey–Shubik [5] assumptions. Second, in Cordella and Gabszewicz's [3] no-trade economy, a finite replica number exists for which there eventually are Nash equilibria with trade. Here, (Ex. 1, Sec. 1 & Case 2, Sec. 3) the opposite was the case as we started with a trading equilibrium which was eliminated when replicated any number of times. Third, the competitive equilibrium in Cordella and Gabszewicz [3] has the property that the marginal rate of substitution between the two goods is not equalized across all agents. Therefore, such an equilibrium does not belong to the interior of the commodity sets. Presented above are economies where the perfectly competitive allocation is in the *interior* of the commodity sets—hence margins are equalized, but the competitive price is on the *boundary* of the price set.

The main result of this paper also has the implication that the sufficient conditions given by Peck et al. [10] for the existence of an equilibrium with trade cannot generally be extended to cases where the indifference curves are not in the interior of the commodity sets. Thus, the question posed by Busetto and Codognato [2, p. 301] concerning this matter is partially addressed.

On convergence. Positive results on convergence from Nash to Walras in Shapley–Shubik strategic market games are found in Dubey and Geanakoplos [4], Dubey and Shubik [5] and Sahi and Yao [11], among others. Koutsougeras [7] also adds to this topic from a somewhat different perspective. As shown above, and in contrast to Cordella and Gabszewicz [3], no such convergence may result here.

On destruction. Aumann and Peleg [1] identified an exchange economy with the property that an agent could benefit from destroying some of his endowment of a good in which he subsequently would become a *competitive seller*. Brought out above (Ex. 1, Sec. 1 & Case 5, Sec. 3) are instances where the *strategic* market game mechanism could profitably be manipulated via endowment destruction by *every* agent in the economy.

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