To Sponsor or Not to Sponsor: Sponsored Search Auctions with Organic Links and Firm Dependent Click-Through Rates

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Abstract

In 2009 sponsored search advertisements generated over $11 billion in revenue for search engines in the US market. Most of these advertisements were sold using an auction mechanism. Several existing papers analyze the auction mechanism currently used under the assumption that customers are only accessed through sponsored links. We extend this literature to incorporate two important market features. In particular, we consider the impact of a second access channel, organic search listings which appear beneath the sponsored links, and we allow for the possibility that both relevance of the advertising firm as well as its position in the sponsored link listings impact the click-through-rate. Our results demonstrate that the existence of an outside alternative leads to less aggressive bidding behavior. The outside alternative also creates an important role for the minimum cost-per-click established by the search engine in maximizing auction revenue. In contrast to equilibrium results in the existing literature, the firm with the highest value per click does not necessarily win the first spot in the sponsored search listings. Moreover, under certain conditions, firms adopt a mixed strategy with regard to participation in the keyword auction, but have a pure bidding stratgey when they do enter the auction.

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1 Introduction

Many firms now have a presence in both traditional and electronic markets. In electronic markets firms typically rely on consumers accessing the firm through a commercial website where customers can find product information, order products, and pay directly. Key issues for firms with a website include visibility to the online audience of potential customers and the ability to convert online investments into revenue. Firms can use several strategies for this purpose. Search engine optimization (*SEO*) can increase the visibility of a website by improving its position among organic links on popular search engines. This strategy requires building a dense network of links and trackbacks through, for example, active participation in social networks or better internal organization of the website (*e.g.* cross linking, URL normalization). Yet, *SEO* has inherent limits because website designers are dependent on search engines regarding both the algorithm used to rank websites and the frequency of search engine (*SE*) updates. As a consequence, the final position of a website in the organic links generated by the *SE* could appear somewhat random, and the effects of any *SEO* strategy should be evaluated in the medium run only.

Websites also may use online advertising as an active strategy to improve their visibility.[1] Since the end of the 1990s, the online advertising market has rapidly developed both in terms of technological possibilities (*e.g.* tracking opportunities) and business models (pay-per-click, pay-per-print, pay-per-sale).[2] The most prominent segment of the online advertising market is sponsored search. Sponsored search enables firms to display sponsored ads alongside organic results produced by the *SE*. In 2009 sponsored search advertisements generated over \$11 billion in revenue for search engines in the US market and €6.7 billion in the European market (Source IAB). Most of these advertisements were sold through keyword auctions. The keyword segment is the largest segment of the sponsored search market in terms of revenues (more than 45%) and is the segment with the highest growth rate (+10% compared to 2008). One important factor explaining the success of sponsored search is the fact that it provides a balanced compromise between several concerns. First, sponsored links are displayed together with organic links. From a user's perspective, they appear to be

[1] See Evans(2007) for a complete survey on online advertising.

[2] See Taylor(2009) for an economic rationale of the current payment schemes in the online advertising industry.

less intrusive than other types of ads (*e.g.* pop-up windows or e-mail advertising). From the advertiser's perspective, they provide the ability to better target different customers based on their search query. This results in more qualified traffic viewing the sponsored ads. Finally, sponsored search is largely based on a pay-per-click principle which is less costly for advertisers because they only incur a charge if the sponsored ad is sufficiently interesting to induce a consumer to click on the advertised link.

A growing body of economics and marketing literature is focused on keyword advertising and the response of firms and consumers to this advertising channel. A first strand of this literature addresses keyword advertising from the point of view of search engines (SE). Based on auctions models, the seminal papers of Varian (2007) and of Edelman *et al.* (2007) examine the specificity of keyword auctions as compared to traditional auctions and look for an optimal auction design to maximize SE revenue. Several subsequent studies have extended this analysis by considering more specific issues in the design of online auctions. Most of these extensions analyze keyword bidding strategies under the assumption that customers are only accessed through sponsored links. Our paper contributes to this literature by incorporating several specific attributes of keyword auction markets. We consider the impact of a second access channel, organic search listings which appear beneath the sponsored links. We allow for the possibility that both the relevance of the advertising firm as well as its position in the sponsored and organic link listings impact the click-through-rate for each listing. Finally, we allow for the possibility that a customer may click on more than one of the links presented in the search listings.

Our model enables us to examine how organic and sponsored links impact the keyword advertising strategies of individual firms and the reservation bids established by the search engines when websites differ according to their popularity or relevance (the probability that a searching customer will click on a given firms sponsored or organic link). We demonstrate that sponsored links induce two opposite effects; a 'crowding out' effect (sponsored links reduce traffic to organic links) and a 'market expansion effect' (the use of sponsored links increases the overall click-through rate) relative to a market with no sponsored links. When the crowding out effect for a particular firm is large, then that firm has little incentive to participate in the keyword auction because creating a sponsored link substantially reduces the firms ability to attract customers through its organic link. As a result, the existence of organic links leads to less aggressive bidding, and, in contrast to equilibrium results in much of the existing literature, the firm which is most relevant or has the highest value per click does not necessarily win the first spot in the sponsored search listings. The interplay between crowding out and market expansion effects also creates an important role for the reservation price (minimum cost-per-click) established by the search engine that has not been considered in previous literature. In particular, the SE can minimize the effect of less competitive bidding by increasing the minimum cost per click. Under certain conditions, the SE establishes a reservation price causes a less popular firm to use a sponsored link to increase its traffic while a more popular firm submits a relatively low bid or chooses not to participate to the keyword auction at all. In such equilibria the more popular firm relies on organic links to attract customers. We also demonstrate that for some parameterizations of our model, firms may not participate systematically in the keyword auction but play mixed strategies instead. In contrast to previous papers, the mixed strategies apply not to the bids submitted by each website but to the decision regarding whether or not to participate in the keyword auction.

Our approach is most similar to Work by Xu et. al. (2009) and Katona and Sarvary (2009) who also consider the role of organic listing in the sponsored search market. One key distinction is that in our model the firm's decision to participate in the keyword auction is endogenous and depends upon the minimum cost per click (*cpc*) established by the SE. In both Xu et. al. and Katona and Sarvary the SE has a fixed number of sponsored listings to sell, and these listings are allocated to the highest bidders. The minimum *cpc* is set to zero so there is no reason for firms to refrain from participation in the keyword auction. Xu et *al.* (2009) investigate a framework in which two firms compete at two levels, both the keyword (advertising) market and the products market.[3] Firms sell a homogeneous product but are endowed with different marginal production costs.[4] There are two types of consumers; shoppers that sample all firms, and non-shoppers that only sample the firm listed first in the search results. They find that the 'disadvantaged' firm (higher production cost) always has an incentive to be ranked first while the firm with the lower production cost has an incentive to bid aggressively only when the advantage from being ranked first significant. The effect

[3] Goldfarb and Tucker (2007) provide an empirical study of this two-level competition.

[4] See also Athey and Ellison (2008) where the differentiation between firms is linked to product characteristics.

of keyword advertising on the price of the product is ambiguous. Katona and Sarvary (2009) consider a first-price auction and show that a less popular site can be ranked before a more popular firm in the list of sponsored links. They also extend these results by considering a dynamic setting to account for customer loyalty over time.[5]

A related strand of the literature emphasizes the role of sponsored links in reducing consumer search costs (see Chen et al., 2009). Taylor (2009) presents a theoretical model in which consumers may choose between organic and sponsored links. With two competing search engines, he shows how the quality of organic results may cannibalize the revenues of the search engine. Yet, in his paper, the behavior of websites in the auction process is not explicitly considered. White (2009) focuses on the interplay between organic and sponsored results when sponsored links are sold at a fixed price and the SE can determine the quality of the ads it accepts. Higher quality ads reduce consumer search costs but may lead to increased competition in the final product market which ultimately reduces SE profit.

The last strand in this literature analyses sponsored search empirically to better understand user response to keyword search results and SE strategies. Ghose and Yang (2009) examine how keywords impact consumer behavior and find that retailer-specific and brand-specific information in paid ads increases the efficiency of online advertising; the former increases the click-through-rate (the number of clicks over the number of times the ad is displayed) and the latter increases the conversion rates into sales. Agarwal *et al.* (2006) find that while the click-through-rate decreases with position, the conversion rate first increases and then decreases with position for longer keywords. They conclude that the top positions in sponsored search advertisements are not necessarily the revenue or profit maximizing positions for advertisers. Complementary to these studies, Rutz and Bucklin (forth.) investigate the interactions between several types of keywords (generic *versus* branded keywords), and find that generic keywords may induce positive spillovers on the efficiency (measured by click-through rate) of branded keywords. Similarly, Jeziorski and Segal (2009) show the prevalence of externalities across ads meaning that the click-through-rate on a given ad in a given position depends on which ads are shown in other positions.

Section 2 presents the model. Section 3 analyzes equilibrium bidding strategies. Section

[5] Chen *et al.* (forth.) also consider a dynamic auction process in which advertisers and search engines may change their strategy according to the performance observed in previous stages. See also Agarwal et al. (2006).

4 discusses the results and concludes.

2 The Model

2.1 Consumers' behavior on the search engine

We consider a duopoly market with search advertising in which first firm, Firm 1 has a higher probability of being relevant to consumers than the second firm, Firm 2. In particular, a randomly chosen consumer who conducts a search on a given keyword which produces only organic links will find the search result listing of firm 1 relevant with probability β_1, and the listing of firm 2 relevant with probability β_2 where $1 \geq \beta_1 > \beta_2 \geq 0$. Because Firm 1 is more relevant, in the absence of any search advertising, the results of the search engine algorithm will always list Firm 1 first and Firm 2 second. To allow for the possibility that sponsored search may expand the market, we assume that if sponsored links are present, then the probability a consumer will find Firm i's sponsored link relevant is $\delta_i \geq \beta_i$. To account for the fact that some consumers may not be willing to look through all sponsored links, we assume that a fraction γ of consumers will consider all of the sponsored links, but a fraction $(1-\gamma)$ will only consider the first sponsored link and will then move on to the organic links if the first sponsored link is not relevant These customers only consider the organic link of the firm listed second in the sponsored links (*i.e.*, they do not consider the organic link of the firm whose sponsored link they previously rejected). If this organic link is relevant, they click on it. Finally, to allow for the possibility that some consumers are averse to utilizing sponsored links, we define α as the probability that a consumer first considers the sponsored links if any sponsored links appear, and $1-\alpha$ as the probability a consumer bypasses the sponsored links and goes directly to the organic links. Given the above search behavior, the (sponsored link-averse) consumer will click on the first relevant organic link she encounters. We are now able to determine the click-through-rate for each firm as a function of the advertising strategies adopted by each firm.

Each firm has the option to pay a fee to the search engine (SE) in order to have a sponsored link appear at the top of the search results. If only one of the two firms sponsors a link, then a link to that firm will appear as the first listing on the search results page. This sponsored link is followed by the organic results which always list Firm 1 first and Firm

2 second. If both firms choose to advertise a sponsored link, then the SE must determine a rule determining which of the two firms is listed first in the sponsored links area of the search results. The strategy of the search engine is modeled in section 3.

2.2 Click-through-rates

Consider the expected click through rates when neither firm sponsors a link. The probability that a consumer clicks on Firm 1's link is β_1. We assume a consumer who clicks on Firm 1's link also considers clicking on the link to Firm 2 with probability $(t-1)/t$, where $t \geq 1$. If $t = 1$, it means that a consumer only clicks on one link (at most) ; after visiting Firm 1 or Firm 2's website, she will never visit further links on the result page. So the probability that a consumer clicks on the link to Firm 2 is $((1-\beta_1) + \beta_1((t-1)/t))\beta_2 = (1-\beta_1/t)\beta_2$.

The probability calculations are somewhat more complex if one of the two firms chooses to advertise. For example, if Firm 2 sponsors a link and Firm 1 does not, then Firm 2's sponsored link appears at the top of the search results followed by the organic links to Firm 1 and then Firm 2. The probability that a consumer clicks on Firm 2's sponsored link is $\alpha\delta_2$, on Firm 2's organic link is $(1-\alpha)(1-\beta_1/t)\beta_2$, and on Firm 1's organic link is $\alpha(1-\delta_2/t)\beta_1 + (1-\alpha)\beta_1 = (1-\alpha\delta_2/t)\beta_1$. The differences in click-through rates when neither firm sponsors a link versus when Firm 2 sponsors a link highlight both a market expansion effect and a crowding out effect that result from sponsoring a link. With no sponsored links, the total capture rate of consumers is $\beta_1+\beta_2-\beta_1\beta_2/t$. When Firm 2 sponsors a link, this increases to $\beta_1+\beta_2-\beta_1\beta_2/t+\alpha(\delta_2-\beta_2)(1-\beta_1/t)$ which is a net increase of $\alpha(\delta_2-\beta_2)(1-\beta_1/t)$. There is also a crowding-out effect in which some consumers who would have clicked on organic links in the absence of advertising, switch to the sponsored link instead. Firm 1's organic click through rate decreases from β_1 to $(1-\alpha\delta_2/t)\beta_1$, and firm 2's organic click-through rate decreases from $(1-\beta_1/t)\beta_2$ to $(1-\alpha)(1-\beta_1/t)\beta_2$. The overall crowding-out effect is $\alpha(\beta_2+(\delta_2-\beta_2)\beta_1/t)$.

If both firms sponsor a link, then the click-through rates depend upon which firm is listed first in the sponsored links. If Firm 1's sponsored link appears first, then the click through rate for firm 1 is $\alpha\delta_1 + (1-\alpha)\beta_1$, and the click through rate for firm 2 is $\alpha\gamma(1-\delta_1/t)\delta_2 + (1-\alpha)(1-\beta_1/t)\beta_2$. Click-through rates under each possible advertising combination are presented in the following table.

Click Through Rates by Firm and Link Type			
	Firms with a Sponsored Link		
Firm/Link	Neither	Only Firm 1	Only Firm 2
1 Organic	β_1	$(1-\alpha)\beta_1$	$(1-\alpha\delta_2/t)\beta_1$
1 Sponsored	0	$\alpha\delta_1$	0
2 Organic	$(1-\beta_1/t)\beta_2$	$\left(1-\frac{\alpha\delta_1+(1-\alpha)\beta_1}{t}\right)\beta_2$	$(1-\alpha)\left(1-\frac{\beta_1}{t}\right)\beta_2$
2 Sponsored	0	0	$\alpha\delta_2$
	Both, Firm 1 Appears First	Both, Firm 2 Appears First	
1 Organic	$(1-\alpha)\beta_1$	$\begin{pmatrix}\alpha(1-\delta_2/t)(1-\gamma)\\+(1-\alpha)\end{pmatrix}\beta_1$	
1 Sponsored	$\alpha\delta_1$	$\alpha(1-\delta_2/t)\gamma\delta_1$	
2 Organic	$\begin{pmatrix}\alpha\left(1-\frac{\delta_1}{t}\right)(1-\gamma)\\+(1-\alpha)\left(1-\frac{\beta_1}{t}\right)\end{pmatrix}\beta_2$	$(1-\alpha)(1-\beta_1/t)\beta_2$	
2 Sponsored	$\alpha(1-\delta_1/t)\gamma\delta_2$	$\alpha\delta_2$	

2.3 Firms' revenues

Firms are interested in maximizing profit generated by the search channel. We assume that revenues are directly correlated with click through rates. In particular, we let v_i denote the expected value to firm i from a customer that clicks on a link to firm i.[6] In addition, let p_i denote the cost per click paid by the firm i to the search engine if firm i is the only firm with a sponsored link, and let $p_{i,k}$ denote the cost per click paid by firm i when both firms sponsor links and firm i is listed in position k. Firms only incur the cost-per-click if a customer reaches the firm through the sponsored link. To determine firm profit, we must distinguish between customers who reach the firm via the sponsored link and the organic link. Let π_i^{st} denote the profit of firm i when only one of the firms advertises a sponsored link and Firm 1 adopts a strategy s and Firm 2 adopts a strategy t, and $s,t \in \{N, A\}$, where A is a strategy of advertising a sponsored link and N is a strategy of not advertising. Finally, let π_i^{AAk} denote the profit for firm i when both firms have sponsored links and firm i's sponsored link appears in position k. Using the above table, the expected cost of attracting a customer can be calculated and subtracted from the click through rate to determine the expected profit

[6] If the probablity that a click on a link to firm i is converted to a sale is ρ_i (*i.e.*, the conversion rate is ρ_i), and the average value of a sale at firm i is s_i, then $v_i = \rho_i s_i$. For simplicity, we assume that ρ_i is the same whether the customer was encountered through a sponsored link as an organic link. However, the model does allow for differences in the probability a customer clicks on a sponsored versus an organic link.

under each possible strategy profile. If neither firm advertises, then

$$\begin{aligned}\pi_1^{NN} &= \beta_1 v_1 \\ \pi_2^{NN} &= \beta_2 \left(1 - \frac{\beta_1}{t}\right) v_2.\end{aligned}$$

If only Firm 1 advertises, then

$$\begin{aligned}\pi_1^{AN} &= \alpha\delta_1 (v_1 - p_1) + (1-\alpha)\beta_1 v_1 \\ \pi_2^{AN} &= \left(1 - \alpha\frac{\delta_1}{t} - (1-\alpha)\frac{\beta_1}{t}\right)\beta_2 v_2.\end{aligned}$$

If only Firm 2 advertises, then

$$\begin{aligned}\pi_1^{NA} &= \left(1 - \alpha\frac{\delta_2}{t}\right)\beta_1 v_1 \\ \pi_2^{NA} &= \alpha\delta_2 (v_2 - p_2) + (1-\alpha)\left(1 - \frac{\beta_1}{t}\right)\beta_2 v_2.\end{aligned}$$

If both firms advertise, then each firm's click-through rate depends upon the placement of its sponsored listing. If firm 1 is listed first, then

$$\begin{aligned}\pi_1^{AA1} &= \alpha\delta_1 (v_1 - p_{1,1}) + (1-\alpha)\beta_1 v_1 \\ \pi_2^{AA1} &= \alpha\left(1 - \frac{\delta_1}{t}\right)\gamma\delta_2 (v_2 - p_{2,2}) + \left(\alpha(1-\delta_1)(1-\gamma) + (1-\alpha)\left(1 - \frac{\beta_1}{t}\right)\right)\beta_2 v_2.\end{aligned}$$

If firm 2 is listed first when both firms advertise, then

$$\begin{aligned}\pi_1^{AA2} &= \alpha\left(1 - \frac{\delta_2}{t}\right)\gamma\delta_1 (v_1 - p_{1,2}) + (\alpha(1-\delta_2)(1-\gamma) + (1-\alpha))\beta_1 v_1 \\ \pi_2^{AA2} &= \alpha\delta_2 (v_2 - p_{2,1}) + (1-\alpha)\left(1 - \frac{\beta_1}{t}\right)\beta_2 v_2.\end{aligned}$$

In the next section, we determine equilibrium bidding strategies of firms 1 and 2. They have first to decide whether they want to participate to the bidding process and then how much they bid.

3 Generalized Second Price Auction

3.1 Cost per click and keyword auction mechanism

Consider a generalized second price auction in which the firm's location in the sponsored links generated by the search engine is determined by a combination of the firm's bid and its relevance (this corresponds to the mechanism currently used by Google). We let c denote the minimum cost per click established by the search engine.[7] This defines the minimum bid to participate to the auction. We assume that the cost per click is c for a specific firm i if both firms advertise and i is listed second or if i is the only firm that advertises

In choosing the location of each bidder, the search engine considers the expected revenue generated by the firm, where the expected revenue per searching customer from listing firm 1 first is

$$\alpha\left(\delta_1 p_{1,1} + \left(1 - \frac{\delta_1}{t}\right)\gamma\delta_2 p_{2,2}\right)$$

and the expected revenue from listing firm 2 first is

$$\alpha\left(\delta_2 p_{2,1} + \left(1 - \frac{\delta_2}{t}\right)\gamma\delta_1 p_{1,2}\right)$$

where $p_{1,j} \leq b_1$, $p_{2,j} \leq b_2$, and $p_{i,2} = c$ for $i = 1, 2$. Recall that firms are only considered for listing in the sponsored links if they submit a bid $b_i \geq c$. Assuming bids exceed c, the search engine will list firm 1 first if

$$\alpha\left(\delta_1 p_{1,1} + \left(1 - \frac{\delta_1}{t}\right)\gamma\delta_2 p_{2,2}\right) \geq \alpha\left(\delta_2 p_{2,1} + \left(1 - \frac{\delta_2}{t}\right)\gamma\delta_1 p_{1,2}\right)$$

or

$$p_{1,1} \geq \frac{\delta_2}{\delta_1} p_{2,1} + \frac{\gamma\left(\delta_1\left(1 - \frac{\delta_2}{t}\right)p_{1,2} - \delta_2\left(1 - \frac{\delta_1}{t}\right)p_{2,2}\right)}{\delta_1}$$

and will list firm 2 first otherwise. Noting that either firm pays a *cpc* of c if is is listed second, if bids are such that firm 1 is listed first, then firm 1 pays a cost-per-click (*cpc*) of

$$\begin{aligned} p_{1,1} &= \frac{\delta_2}{\delta_1} b_2 + \frac{\gamma\left(\delta_1\left(1 - \delta_2/t\right)p_{1,2} - \delta_2\left(1 - \delta_1/t\right)p_{2,2}\right)}{\delta_1} \\ &= \frac{\delta_2}{\delta_1} b_2 + \frac{\gamma c\left(\delta_1 - \delta_2\right)}{\delta_1} < b_2, \end{aligned}$$

where the inequality follows from the assumption that $b_2 > c$, and firm 2 pays a *cpc* of c. If bids are such that firm 2 is listed first, then firm 1 pays a *cpc* of c and

$$\begin{aligned} p_{2,1} &= \frac{\delta_1}{\delta_2} b_1 - \frac{\gamma\left(\delta_1\left(1 - \delta_2/t\right)p_{1,2} - \delta_2\left(1 - \delta_1/t\right)p_{2,2}\right)}{\delta_2} \\ &= \frac{\delta_1}{\delta_2} b_1 - \frac{\gamma c\left(\delta_1 - \delta_2\right)}{\delta_2} > b_1. \end{aligned}$$

[7] For example, estimates provided by the Google AdWords keyword tool suggest that Googles sets a minimum cost per click of \$.05 for any keyword.

Given the assumption that $\delta_1 > \delta_2$, firm 1 may be listed first even if it bids less than firm 2 (i.e. if firm 1 bids between $p_{1,1}$ and b_2) and firm 2 must bid strictly more than firm 1 in order to be listed first. The premium that firm 2 must pay in order to be listed first $(p_{2,1} - b_1)$ is increasing in δ_1 and decreasing in δ_2. In the limiting case in which $\delta_1 \to \delta_2$, the search engine will simply rank the firms according to their bids. In addition, the premium is decreasing in γ and c. This result is intuitive. Recall that γ represents the probability that a consumer considers all of the sponsored links (while with probability $1 - \gamma$ a consumer only considers the first sponsored link and then moves on to the organic links). As γ increases, the cost to the search engine of listing the less relevant Firm 2 first (in terms of revenue that could have been gained by listing the sponsored link to the more relevant Firm 1 first instead) decreases because a consumer who determines that Firm 2's sponsored link is not relevant is more likely to consider (and potentially click on) the sponsored link to firm 1, which generates revenue of c for the SE. Similarly, the revenue generated by a consumer who rejects the sponsored link to Firm 2 and then clicks on the sponsored link to Firm 1 is increasing in c.

3.2 Equilibrium bidding strategies

The profit functions under each scenario can be used to create a payoff matrix for the game in which the firms simultaneously determine their advertising strategies. There will be an equilibrium in which neither firm chooses to advertise if $\pi_1^{NN} > \pi_1^{AN}$, and $\pi_2^{NN} > \pi_2^{NA}$. These restrictions require

$$\begin{aligned} \beta_1 v_1 &\geq \alpha\delta_1 (v_1 - c) + (1-\alpha)\beta_1 v_1, \text{ and} \\ \left(1 - \frac{\beta_1}{t}\right)\beta_2 v_2 &\geq \alpha\delta_2 (v_2 - c) + (1-\alpha)\left(1 - \frac{\beta_1}{t}\right)\beta_2 v_2. \end{aligned}$$

which imply that firm 1 will prefer not to advertise conditional on firm 2 not advertising if $c \geq v_1 (1 - \beta_1/\delta_1) \equiv c_1$, and firm 2 will prefer not to advertise conditional on firm 1 not advertising if $c \geq v_2 \left(1 - \left(1 - \frac{\beta_1}{t}\right)\beta_2/\delta_2\right) \equiv c_2$. An equilibrium in which neither firm advertises exists if $c \geq \max\{c_1, c_2\}$. If the search engine establishes a minimum cost-per-click $c < \max\{c_1, c_2\}$, then at least one firm will advertise with strictly positive probability.

Firm 1 prefers to advertise and be listed first over advertising and being listed second if

$$\alpha\delta_1 (v_1 - p_{1,1}) + (1-\alpha)\beta_1 v_1 > \alpha\left(1 - \frac{\delta_2}{t}\right)\gamma\delta_1 (v_1 - c) + (\alpha(1-\delta_2)(1-\gamma) + (1-\alpha))\beta_1 v_1$$

which implies

$$p_{1,1} < \left(1-\gamma\left(1-\frac{\delta_2}{t}\right)-\beta_1\left(1-\gamma\right)\left(1-\delta_2\right)/\delta_1\right)v_1+c\gamma\left(1-\frac{\delta_2}{t}\right)\equiv\tilde{p}_1.$$

Firm 1 prefers to advertise and be listed first over not advertising given firm 2 does advertise if

$$\alpha\delta_1\left(v_1-p_{1,1}\right)+\left(1-\alpha\right)\beta_1 v_1 > \left(1-\alpha\frac{\delta_2}{t}\right)\beta_1 v_1$$

or

$$p_{1,1} < v_1\left(1-\left(1-\frac{\delta_2}{t}\right)\beta_1/\delta_1\right)\equiv\hat{p}_1.$$

Firm 1 prefers to advertise and be listed second over not advertising given firm 2 advertises if

$$\alpha\left(1-\frac{\delta_2}{t}\right)\gamma\delta_1\left(v_1-p_{1,2}\right)+\left(\alpha\left(1-\delta_2\right)\left(1-\gamma\right)+\left(1-\alpha\right)\right)\beta_1 v_1 > \left(1-\alpha\frac{\delta_2}{t}\right)\beta_1 v_1$$

or

$$c < v_1\left(1-\frac{\beta_1\left(\left(t-1\right)+t\gamma\left(1-\delta_2\right)\right)}{\gamma\delta_1\left(t-\delta_2\right)}\right)\equiv\tilde{p}_{1,2}.$$

If $c > \tilde{p}_{1,2}$, then firm 1 prefers not advertising over advertising and being listed second given firm 2 advertises. Note that $\tilde{p}_1 = \hat{p}_1$ when $c = \tilde{p}_{1,2}$. If $c > \tilde{p}_{1,2}$, then $\tilde{p}_1 > \hat{p}_1$, and if $c < \tilde{p}_{1,2}$, then $\tilde{p}_1 < \hat{p}_1$.

Lemma 1 $\hat{p}_1 > c_1 > \tilde{p}_{1,2}$ *for all* $t > 1$, *and* $v_1\left(1-\beta_1/\delta_1\right) = \tilde{p}_{1,2}$ *for* $t = 1$.

Proof. This follows directly from $\hat{p}_1 - c_1 = \frac{\beta_1}{t\delta_1}\delta_2 v_1 > 0$ and from $c_1 - \tilde{p}_{1,2} = \frac{1}{\gamma}\frac{\beta_1}{\delta_1}\frac{v_1}{t-\delta_2}$ $\left(1-\gamma\delta_2\right)\left(t-1\right)$. ■

Firm 2 prefers to advertise and be listed first over advertising and being listed second if

$$\begin{aligned}&\alpha\delta_2\left(v_2-p_{2,1}\right)+\left(1-\alpha\right)\left(1-\frac{\beta_1}{t}\right)\beta_2 v_2\\ > \;&\alpha\left(1-\frac{\delta_1}{t}\right)\gamma\delta_2\left(v_2-c\right)+\left(\alpha\left(1-\delta_1\right)\left(1-\gamma\right)+\left(1-\alpha\right)\left(1-\frac{\beta_1}{t}\right)\right)\beta_2 v_2\end{aligned}$$

or

$$p_{2,1} < v_2\left(1-\gamma\left(1-\delta_1/t\right)-\frac{\left(1-\gamma-\delta_1+\gamma\delta_1\right)\beta_2}{\delta_2}\right)+c\gamma\left(1-\delta_1/t\right)=\tilde{p}_2.$$

Firm 2 prefers to advertise and be listed first over not advertising given firm 1 advertises if

$$\alpha\delta_2\left(v_2-p_{2,1}\right)+\left(1-\alpha\right)\left(1-\frac{\beta_1}{t}\right)\beta_2 v_2 > \left(1-\alpha\frac{\delta_1}{t}-\left(1-\alpha\right)\frac{\beta_1}{t}\right)\beta_2 v_2$$

or

$$p_{2,1} < v_2 \left(1 - \frac{(1 - \delta_1/t)\,\beta_2}{\delta_2}\right) \equiv \hat{p}_2.$$

Note that $\hat{p}_2 = c_2$. Finally, firm 2 prefers to advertise and be listed second over not advertising given firm 1 advertises if

$$\begin{aligned} &\alpha\left(1 - \frac{\delta_1}{t}\right)\gamma\delta_2\,(v_2 - c) + \left(\alpha\,(1 - \delta_1)\,(1 - \gamma) + (1 - \alpha)\left(1 - \frac{\beta_1}{t}\right)\right)\beta_2 v_2 \\ > &\left(1 - \alpha\frac{\delta_1}{t} - (1 - \alpha)\,\frac{\beta_1}{t}\right)\beta_2 v_2 \end{aligned}$$

or

$$c < v_2 \left(1 - \frac{\beta_2\,(\delta_1\,(t - 1) + t\gamma\,(1 - \delta_1))}{\gamma\delta_2\,(t - \delta_1)}\right) \equiv \tilde{p}_{2,2}.$$

Note that $\tilde{p}_2 > \hat{p}_2$ when $c > \tilde{p}_{2,2}$.

Lemma 2 *$\hat{p}_2 > c_2 > \tilde{p}_{2,2}$ for all $t \geq 1$.*

Proof. See Appendix. ■

Proposition 3 *If $c < \min\{\tilde{p}_{1,2}, \tilde{p}_{2,2}\}$, then in equilibrium Firm 1 bids $\tilde{p}_1$ and Firm 2 bids $\tilde{p}_2$. If $\tilde{p}_1 \geq \frac{\delta_2}{\delta_1}\tilde{p}_2 + \frac{\gamma c(\delta_1 - \delta_2)}{\delta_1}$, then firm 1 is listed first and pays a cpc of $\frac{\delta_2}{\delta_1}\tilde{p}_2 + \frac{\gamma c(\delta_1 - \delta_2)}{\delta_1}$, and firm 2 is listed second and pays a cpc of c. If $\tilde{p}_2 > \frac{\delta_1}{\delta_2}\tilde{p}_1 - \frac{\gamma c(\delta_1 - \delta_2)}{\delta_2}$, then firm 2 is listed first and pays a cpc of $\frac{\delta_1}{\delta_2}\tilde{p}_1 - \frac{\gamma c(\delta_1 - \delta_2)}{\delta_2}$ and firm 1 is listed second and pays a cpc of c.*

Proposition 4 *If $c > \max\{c_1, c_2\}$, then neither firm bids on sponsored links.*

Proposition 5 *If $\tilde{p}_{1,2} \geq c > \tilde{p}_{2,2}$, then in equilibrium firm 1 always bids $\tilde{p}_1$. If $\hat{p}_2 > \frac{\delta_1}{\delta_2}\tilde{p}_1 - \frac{\gamma c(\delta_1 - \delta_2)}{\delta_2}$, then firm 2 bids $\hat{p}_2$ and is listed first and pays a cpc of $\frac{\delta_1}{\delta_2}\tilde{p}_1 - \frac{\gamma c(\delta_1 - \delta_2)}{\delta_2}$ and firm 1 is listed second and pays a cpc of c. If $\hat{p}_2 \leq \frac{\delta_1}{\delta_2}\tilde{p}_1 - \frac{\gamma c(\delta_1 - \delta_2)}{\delta_2}$, then firm 2 does not bid, and firm 1 advertises and pays a cpc of c.*

Proof. Firm 1 always bids because $c \leq \tilde{p}_{1,2}$ implies firm 1 is better off advertising and being listed second than not advertising. Firm 1 bids the maximum amount $\tilde{p}_1$ that it is willing to pay to be listed first. Note that $\frac{\delta_1}{\delta_2}\tilde{p}_1 - \frac{\gamma c(\delta_1 - \delta_2)}{\delta_2} > c$ if and only if $\tilde{p}_1 \geq c\,(\gamma + (1 - \gamma)\,\delta_2/\delta_1)$ which always holds because $\tilde{p}_1 \geq c$ for $c \leq \tilde{p}_{1,2}$. Therefore, if $\hat{p}_2 > \frac{\delta_1}{\delta_2}\tilde{p}_1 - \frac{\gamma c(\delta_1 - \delta_2)}{\delta_2}$, then $\hat{p}_2 > \frac{\delta_1}{\delta_2}\tilde{p}_1 - \frac{\gamma c(\delta_1 - \delta_2)}{\delta_2} > \frac{\delta_1}{\delta_2}c - \frac{\gamma c(\delta_1 - \delta_2)}{\delta_2} = c\left(\frac{\delta_1(1-\gamma)+\gamma\delta_2}{\delta_2}\right) > c$ where the second inequality follows from $\tilde{p}_1 > c$, and the final inequality follows from $\delta_1\,(1 - \gamma) + \gamma\delta_2 >$

δ_2 because $\delta_1 > \delta_2$ by assumption. Therefore, if $\hat{p}_2 > \frac{\delta_1}{\delta_2}\tilde{p}_1 - \frac{\gamma c(\delta_1-\delta_2)}{\delta_2}$, then bidding $\hat{p}_2$ is optimal for firm 2 because it will be listed first at this bid. However, because $c > \tilde{p}_{2,2}$ implies firm 2 is better off not advertising than advertising and being listed second and paying a *cpc* of c, firm 2 does not bid if $\hat{p}_2 \leq \frac{\delta_1}{\delta_2}\tilde{p}_1 - \frac{\gamma c(\delta_1-\delta_2)}{\delta_2}$.

Proposition 6 *If $\tilde{p}_{2,2} > c > \tilde{p}_{1,2}$, then in equilibrium firm 2 always bids $\tilde{p}_2$. If $\hat{p}_1 > \max\left\{c, \frac{\delta_2}{\delta_1}\tilde{p}_2 + \frac{\gamma c(\delta_1-\delta_2)}{\delta_1}\right\}$, then firm 1 bids $\hat{p}_1$ and is listed first and pays a cpc of $\frac{\delta_2}{\delta_1}\tilde{p}_2 + \frac{\gamma c(\delta_1-\delta_2)}{\delta_1}$, and firm 2 is listed second and pays a cpc of c. If $\hat{p}_1 \leq \max\left\{c, \frac{\delta_2}{\delta_1}\tilde{p}_2 + \frac{\gamma c(\delta_1-\delta_2)}{\delta_1}\right\}$, then firm 1 does not bid, and firm 2 advertises and pays a cpc of c.*

Proof. The proof is similar to the proof of proposition 5. However, because $c < \tilde{p}_{2,2}$ does not ensure that $\frac{\delta_2}{\delta_1}\tilde{p}_2+\frac{\gamma c(\delta_1-\delta_2)}{\delta_1} > c$, the additional condition that $\hat{p}_1 > \max\left\{c, \frac{\delta_2}{\delta_1}\tilde{p}_2 + \frac{\gamma c(\delta_1-\delta_2)}{\delta_1}\right\}$ is needed to ensure that bidding on a sponsored link is optimal for firm 1 when the bidding would result in firm 1 being listed first.

Proposition 7 *Suppose* $\min\{c_1, c_2\} > c > \max\{\tilde{p}_{1,2}, \tilde{p}_{2,2}\}$*. If $\hat{p}_2 > \frac{\delta_1}{\delta_2}\hat{p}_1 - \frac{\gamma c(\delta_1-\delta_2)}{\delta_2}$, then firm 2 bids $\hat{p}_2$, firm 1 does not bid, and firm 2 pays a cpc of c. If $\hat{p}_2 \leq \frac{\delta_1}{\delta_2}\hat{p}_1 - \frac{\gamma c(\delta_1-\delta_2)}{\delta_2}$, then firm 1 bids $\hat{p}_1$, firm 2 does not bid, and firm 1 pays a cpc of c.*

Proof. Because $c > \max\{\tilde{p}_{1,2}, \tilde{p}_{2,2}\}$, firm i prefers advertising and being listed first over not advertising, but prefers not advertising over advertising and being listed second (because the *cpc* from being listed second exceeds $\tilde{p}_{i,2}$). Also, $c > \max\{\tilde{p}_{1,2}, \tilde{p}_{2,2}\}$ implies $\hat{p}_i > \tilde{p}_i$, so $\hat{p}_i$ is the maximum firm i is willing to pay if it is listed first. Finally, lemmas 1 and 2 imply $\hat{p}_i > c$, so each firm is willing to pay the *cpc* c if it is listed first, but will not bid if it will be listed second.

Proposition 8 *Suppose $c_1 > c > c_2$ and $c > \max\{\tilde{p}_{1,2}, \tilde{p}_{2,2}\}$. If $\hat{p}_2 < \frac{\delta_1}{\delta_2}\hat{p}_1 - \frac{\gamma c(\delta_1-\delta_2)}{\delta_2}$, then firm 1 bids $\hat{p}_1$, firm 2 does not bid, and firm* 1 *pays a cpc of c. If $\hat{p}_2 \geq \frac{\delta_1}{\delta_2}\hat{p}_1 - \frac{\gamma c(\delta_1-\delta_2)}{\delta_2}$, then there is a mixed strategy equilibrium in which firms decide to bid randomly and submit a bid of $\hat{p}_i$ when they do bid.*

Proof. The proof that firm 1 bids and firm 2 does not if $\hat{p}_2 < \frac{\delta_1}{\delta_2}\hat{p}_1 - \frac{\gamma c(\delta_1-\delta_2)}{\delta_2}$ follows the proofs in the previous propositions because $c < c_1$ implies $\hat{p}_1 > c$. If $\hat{p}_2 \geq \frac{\delta_1}{\delta_2}\hat{p}_1 - \frac{\gamma c(\delta_1-\delta_2)}{\delta_2}$, then firm 2 is willing to pay the premium required to be listed first. But if firm 2 is listed first, then firm 1 prefers not to advertise. However, if firm 1 advertises with probability 0, then firm 2 will choose not to advertise, but then firm 1 prefers to advertise because $c < c_1$. Thus, there is no equilibrium in pure strategies. Note that in an equilibrium with mixed strategies, if both firms bid $\hat{p}_i$, then firm 2 is listed first. Letting σ_i denote the probability that firm i advertises, the equilibrium in mixed strategies satisfies

$$\sigma_2 \pi_1^{AA2} + (1 - \sigma_2)\, \pi_1^{AN} = \sigma_2 \pi_1^{NA} + (1 - \sigma_2)\, \pi_1^{NN}$$

and

$$\sigma_1 \pi_2^{AA1} + (1 - \sigma_1)\, \pi_2^{NA} = \sigma_1 \pi_2^{AN} + (1 - \sigma_1)\, \pi_2^{NN}.$$

Proposition 9 *Suppose $c_2 > c > c_1$ and $c > \max\{\tilde{p}_{1,2}, \tilde{p}_{2,2}\}$. If $\hat{p}_2 > \frac{\delta_1}{\delta_2}\hat{p}_1 - \frac{\gamma c(\delta_1 - \delta_2)}{\delta_2}$, then firm 2 bids $\hat{p}_2$, firm 1 does not bid, and firm 2 pays a cpc of c. If $\hat{p}_2 \leq \frac{\delta_1}{\delta_2}\hat{p}_1 - \frac{\gamma c(\delta_1 - \delta_2)}{\delta_2}$, then there is a mixed strategy equilibrium in which firms decide to bid randomly and submit a bid of $\hat{p}_i$ when they do bid.*

Proof. The proof is similar to the proof in proposition 8.

The above propositions generate several insights into the role of the various parameters on the bids submitted by each firm. Note that in equilibrium each firm will either bid $\tilde{p}_i$ or $\hat{p}_i$. Comparative statics results presented in the appendix demonstrate that optimal bids are strictly decreasing in t, non-increasing in β_1, and β_2, and strictly increasing in δ_1 and δ_2.

As t (a measure of consumers' propensity to visit multiple links) increases, the premium required to be listed first decreases because a consumer who clicks on the first sponsored link is more likely to click on the second sponsored link as well. For the same reason, as t increases the benefit to a given firm of being listed first decreases. This causes bidding for sponsored links to become less competitive and the optimal bids decrease.

As β_1 (a measure of the natural relevance of firm 1) increases, firm 1 has less incentive to advertise because the value of its organic link increases in β_1. As a result, the bids $\tilde{p}_1$ and $\hat{p}_1$ that firm 1 might submit are both decreasing in β_1. However, Firm 2 has a greater incentive to advertise because if Firm 2 does not advertise, as β_1 increases, there is a lower probability that Firm 2 encounters a customer. Interestingly, this does not impact the bid that Firm 2 submits. As the above propositions demonstrate, if Firm 2 submits a bid for a particular keyword, that bid will be either $\tilde{p}_2$, or $\hat{p}_2$, and neither of these values depend upon β_1. This follows from two facts. First, if Firm 1 also advertises, then an increase in β_1 is irrelevant to Firm 2's bid because for the fraction α of customers who consider the sponsored links, the parameter δ_1, not β_1, impacts the probability that Firm 2 encounters a given customer, and the behavior of the fraction $1-\alpha$ who do not consider sponsored links is not influenced by Firm 2's position in the sponsored listings. Second, if Firm 1 does not advertise, then Firm 2 is only interested in sponsoring a link if $c \leq c_2$. However, because $\hat{p}_2 > c_2$, it follows that Firm 2 will bid $\hat{p}_2$, and $\hat{p}_2$ does not depend upon β_1. Similarly, an increase in β_2 causes firm 2 to bid less competitively, but has no impact on the bids of firm 1 because any customer who considers the organic link to firm 2 already will have considered the organic link to firm 1. Increases in δ_1 and δ_2 both increase competition for sponsored links.

The impact of changes in v_1 and γ on optimal bidding strategies is less clear. The optimal bids $\tilde{p}_1$ and $\hat{p}_1$ of firm 1 are increasing in v_1 if and only if β_1 is sufficiently small. In this case, the market expansion effect dominates the crowding out effect so that more competitive bidding by firm 1 for a sponsored link is optimal when v_1 increases. An increase in v_2, on the other hand, always leads to more competitive bidding by firm 2. As the fraction γ of consumers who consider all sponsored links increases, bidding for sponsored links by firm i becomes more competitive if and only if β_i is sufficiently large and $c < \tilde{p}_{i,2}$, so that $\tilde{p}_i > \hat{p}_i$.

3.3 Optimal Search Engine Strategy

The Search Engine (SE) optimally chooses the minimum cost per click c in order to maximize expected search engine revenue from the generalized second price auction. As discussed in the previous section, the order in which the firms are listed in the sponsored links when both firms submit bids is not determined solely by which firm submits the highest bid - the ordering also depends upon the probabilities δ_i that consumers click on the sponsored links. As demonstrated in subsection 3.2, the search engine's choice of the minimum cost per click c determines the bidding strategies of the two firms. As propositions 3 through 9 demonstrate, the choice of c is critical to determining which proposition applies and what the resulting bidding strategies for each firm will be. However, it is not obvious how changes in c will impact SE profit. For the bidding firms, an increase in c makes the alternative of relying on organic links for which the firms incur no cost-per-click relatively more attractive which suggests bidding for sponsored links would become less competitive. However, because the firm listed second pays a *cpc* of c, if both firms bid on sponsored links, then an increase in c reduces the incremental cost the firm listed second must pay in order to be listed first. At the margin, this makes a bid increase attractive to the firm listed second, and increases competition for the first spot in the sponsored links. From proposition 3 and the definitions of $\tilde{p}_1$ and $\tilde{p}_2$, it is apparent that the second effect dominates when c is relatively low ($c < \min\{\tilde{p}_{1,2}, \tilde{p}_{2,2}\}$), so an increase in c leads to more competitive bidding (higher bids) However, once c exceeds $\tilde{p}_{i,2}$, firm i is no longer willing to pay for a sponsored link that is listed second because firm i's return from not advertising is greater than its return from advertising and being listed second. In this case, firm i only bids if the maximum amount $\hat{p}_i$ that it is willing to pay to be listed first is sufficient to ensure that firm i is listed first in the sponsored links. The bid $\hat{p}_i$ does not depend upon c because for firm i the alternative of relying only on organic links for which there is no *cpc* dominates being listed second if $c > \tilde{p}_{i,2}$. In addition, as the following lemma demonstrates, the actual payments made by each firm in an equilibrium in which both firms adopt pure advertising strategies are increasing in c.

Lemma 10 *When both firms advertise, the cost per click paid by Firm* 1, $\frac{\delta_2}{\delta_1}\tilde{p}_2 + \frac{\gamma c(\delta_1-\delta_2)}{\delta_1}$, *and the cpc paid by firm* 2, $\frac{\delta_1}{\delta_2}\tilde{p}_1 - \frac{\gamma c(\delta_1-\delta_2)}{\delta_2}$, *are increasing in c.*

Proof. Substituting the expressions for $\tilde{p}_i$ yields $\frac{d}{dc}\left(\frac{\delta_2}{\delta_1}\tilde{p}_2 + \frac{\gamma c(\delta_1-\delta_2)}{\delta_1}\right) = \frac{\gamma(t-\delta_2)}{t} > 0$ and $\frac{d}{dc}\left(\frac{\delta_1}{\delta_2}\tilde{p}_1 - \frac{\gamma c(\delta_1-\delta_2)}{\delta_2}\right) = \frac{\gamma(t-\delta_1)}{t} > 0$.

This lemma combined with propositions 3, 5, and 6 implies that in any equilibrium in which both firms bid with probability 1, the *cpc* paid by each firm is increasing in the minimum cost per click set by the search engine. This follows from the fact that in any such equilibrium the payment made by the firm listed first in the sponsored listings is one of the two expressions in lemma 10, and the *cpc* for the firm listed second is c. This lemma implies that if the SE maximizes profit under conditions satisfying proposition 3, then $c = \min\{\tilde{p}_{1,2}, \tilde{p}_{2,2}\}$; if it maximizes profit under conditions satisfying proposition 5, then $c = \tilde{p}_{1,2}$; and if it maximizes profit under conditions satisfying proposition 7, then $c = \min\{c_1, c_2\}$. In each of these cases, raising c increases the actual payment made by all participants in the auction without changing the firms that participate. A similar statement cannot be made about propositions 6, 9 or 8 because in the conditions satisfying any of these propositions, and increase in c may cause one of the two firms to drop out of the auction. Thus, the SE must compare profit with both firms bidding under a lower c with profit achieved with only one firm sponsoring a link at a price equal to the upper bound for c in the proposition. Unfortunately, the general conditions under which the SE prefers to set c so that both firms bid, only one firm bids, or firms adopt a mixed strategy are quite complex and do not provide straightforward intuition. Therefore, to gain further insight into the optimal SE strategy, we consider several examples.

3.3.1 Numerical examples

In the first example, suppose $\gamma = t = 1$,(consumers consider all of the sponsored links, but when they click on a sponsored or organic link, they never click on other links later) and $v_1 = kv_2$ where $k > 1$. Under these assumptions, $\tilde{p}_{1,2} = c_1$, and $\tilde{p}_{2,2} = c_2$, so the conditions of propositions 6 through 9 cannot apply. In addition $\tilde{p}_{1,2} > \tilde{p}_{2,2}$ if and only if $k > \frac{\delta_1(\delta_2-\beta_2)}{\delta_2(\delta_1-\beta_1)}$. Suppose this condition holds. If $c = \tilde{p}_{2,2}$, then proposition 3 applies, so firms will bid $\tilde{p}_i$, and firm 1 is listed first and pays $\frac{v}{\delta_2}(\delta_2 - \beta_2 + \beta_2\delta_2)$, firm 2 is listed second and pays $\left(1 - \frac{\beta_2}{\delta_2}\right) v$ and search engine profit is

$$\frac{v}{\delta_2}\left(\delta_2\left(\delta_1 + \delta_2 - \delta_1\delta_2\right) - \beta_2\left(\delta_1 + \delta_2 - 2\delta_1\delta_2\right)\right) > 0.$$

Note that this profit is increasing in both δ_1 and δ_2.

If the SE increases c, then proposition 5 applies. Because $\hat{p}_2 < \frac{\delta_1}{\delta_2}\tilde{p}_1 - \frac{\gamma c(\delta_1-\delta_2)}{\delta_2}$ firm 2 will not bid on a sponsored link, and $c = \tilde{p}_{1,2}$ is optimal for the SE. In this case, SE profit is $\delta_1\tilde{p}_{1,2} = (\delta_1 - \beta_1)\, kv$. Subtracting profit when $c = \tilde{p}_{1,2}$ from profit when $c = \tilde{p}_{2,2}$ yields

$$\frac{v}{\delta_2}\left(\delta_2\left(\delta_1 + \delta_2 - \delta_1\delta_2\right) - \beta_2\left(\delta_1 + \delta_2 - 2\delta_1\delta_2\right) - \delta_2\left(\delta_1 - \beta_1\right) k\right)$$

which is positive if

$$k < \frac{\delta_1\left(\delta_2 - \beta_2\right) + \left(\delta_2\left(\delta_2 - \delta_1\delta_2\right) - \beta_2\left(\delta_2 - 2\delta_1\delta_2\right)\right)}{\delta_2\left(\delta_1 - \beta_1\right)} \equiv \tilde{k}.$$

Note that $\delta_2 > \beta_2$ implies $\delta_2\left(\delta_2 - \delta_1\delta_2\right) - \beta_2\left(\delta_2 - 2\delta_1\delta_2\right) > 0$, so $\tilde{k} > \frac{\delta_1(\delta_2-\beta_2)}{\delta_2(\delta_1-\beta_1)}$. Thus, if $\tilde{k} > k > \frac{\delta_1(\delta_2-\beta_2)}{\delta_2(\delta_1-\beta_1)}$, then it is optimal for the SE to set $c = \tilde{p}_{2,2}$ and induce both firm 1 and firm 2 to bid on sponsored links. However, if $k > \tilde{k}$, then it is optimal for the SE to establish a higher minimum *cpc* of $c = \tilde{p}_{1,2}$ even though doing so will result in only firm 1 bidding on a sponsored link. This example demonstrates that setting c sufficiently small to induce both firms to advertise is optimal if v_1 is not too much larger than v_2. However, if v_1 is substantially larger than v_2 (*i.e.*, if k is sufficiently large), then the SE should set a high cost per click that excludes firm 2 from the sponsored link competition.

Another simple example considers the case in which $v_1 = v_2 = v$, and $\beta_2 = 0$ so the less relevant firm 2 is completely excluded from the organic listings. In this case $\hat{p}_2 = c_2 = \tilde{p}_{2,2} = v$. It can be shown that if $\beta_1 < (\delta_1 - \delta_2) / (1 - \delta_2)$, then the SE sets $c = \tilde{p}_{1,2}$, both firms bid on sponsored links, and firm 1 is listed first. In this case, even though firm 2 is willing to pay v to be listed first in the sponsored links, a bid of v is not sufficient for firm 2 to be listed first because firm 1 submits a bid close to v and has a higher click through rate, so the SE lists firm 1 first in the sponsored links. For slightly larger values of β_1, the SE still sets $c = \tilde{p}_{1,2}$, both firms bid, and firm 2 is listed first. Finally, as $\beta_1 \to \delta_1$, the SE sets $c = v$ and extracts all possible surplus from firm 2, which is willing to pay v. This final outcome is also more likely as t increases, because as t increases firm 1 is more willing to rely on attracting customers through its organic link.

As a third example, suppose that $v_1 = v_2$, and $\beta_1 > \frac{\beta_2(1-\delta_1)+(\delta_1-\delta_2)}{1-\delta_2}$. Note that this implies $\tilde{p}_{1,2} > \tilde{p}_{2,2}$. Because $\tilde{p}_{1,2} > \tilde{p}_{2,2}$, if the SE sets sets $c = \tilde{p}_{2,2}$, then proposition 3 applies. Furthermore, because $\tilde{p}_1$ and $\tilde{p}_2$ are both increasing in c, if the SE chooses a value of c such that proposition 3 applies, it will set $c = \tilde{p}_{2,2}$.

If $c = \tilde{p}_{2,2}$, and $v_1 = v_2 = v$, then $\tilde{p}_2 \geq \frac{\delta_1}{\delta_2}\tilde{p}_1 - \frac{\gamma c(\delta_1-\delta_2)}{\delta_2}$ if and only if

$$\frac{v}{\delta_2}(1-\gamma)(\beta_1 - \beta_2 - \delta_1 + \delta_2 - \beta_1\delta_2 + \beta_2\delta_1) > 0$$

which holds if and only if $\beta_1 > \frac{\beta_2(1-\delta_1)+(\delta_1-\delta_2)}{1-\delta_2}$ which is true by assumption. Because $\tilde{p}_2 \geq \frac{\delta_1}{\delta_2}\tilde{p}_1 - \frac{\gamma c(\delta_1-\delta_2)}{\delta_2}$ when $c = \tilde{p}_{2,2}$, if the SE sets $c = \tilde{p}_{2,2}$, then firm 2 bids $\tilde{p}_2$ and wins and is listed first with a *cpc* of $\frac{\delta_1}{\delta_2}\tilde{p}_1 - \frac{\gamma c(\delta_1-\delta_2)}{\delta_2}$, and firm 1 is listed second with a *cpc* of c. So setting $c = \tilde{p}_{2,2}$ generates search engine profit of

$$\alpha\left(\delta_2\left(\frac{\delta_1}{\delta_2}\tilde{p}_1 - \frac{\gamma c(\delta_1-\delta_2)}{\delta_2}\right) + \left(1-\frac{\delta_2}{t}\right)\gamma\delta_1\tilde{p}_{1,2}\right) = \alpha\left(\delta_1\tilde{p}_1 + \gamma\delta_2 c\left(1-\delta_1/t\right)\right)$$

$$= \alpha\left(\begin{array}{c} \delta_1\left(\frac{1}{t}\frac{v}{\delta_2}\left(\begin{array}{c} t\delta_1 - t\beta_1 + \beta_2\delta_1 + t\gamma\beta_1 - t\gamma\beta_2 - t\gamma\delta_1 \\ +t\gamma\delta_2 + t\beta_1\delta_2 - t\beta_2\delta_1 - t\gamma\beta_1\delta_2 + t\gamma\beta_2\delta_1 \end{array}\right)\right) \\ +\gamma\delta_2 v\left(1 - \frac{\beta_2(\delta_1(t-1)+t\gamma(1-\delta_1))}{\gamma\delta_2(t-\delta_1)}\right)(1-\delta_1/t) \end{array}\right)$$

Alternatively, the SE could set $c = \tilde{p}_{1,2}$. In this case, proposition proposition 5 implies that firm 2 will only bid on a sponsored link if $\hat{p}_2 > \frac{\delta_1}{\delta_2}\tilde{p}_1 - \frac{\gamma c(\delta_1-\delta_2)}{\delta_2}$. It can be shown that if $c = \tilde{p}_{1,2,}$ then $\hat{p}_2 > \frac{\delta_1}{\delta_2}\tilde{p}_1 - \frac{\gamma c(\delta_1 - \delta_2)}{\delta_2}$ if $\beta_1 > \frac{\beta_2(1-\delta_1)+(\delta_1-\delta_2)}{1-\delta_2}$. Thus, if the SE sets $c = \tilde{p}_{1,2}$, then proposition 5 implies firm 2 bids $\hat{p}_2$ and is listed first and pays a *cpc* of $\frac{\delta_1}{\delta_2}\tilde{p}_1 - \frac{\gamma c(\delta_1-\delta_2)}{\delta_2}$, and firm 1 bids $\tilde{p}_1$ and is listed second and pays a *cpc* of $\tilde{p}_{1,2}$. In this case SE profits are

$$\alpha\left(\delta_2\hat{p}_2 + \left(1 - \frac{\delta_2}{t}\right)\gamma\delta_1 v\left(1 - \frac{\beta_1\left((t-1) + t\gamma\left(1-\delta_2\right)\right)}{\gamma\delta_1\left(t-\delta_2\right)}\right)\right)$$

Finally, it can be shown that subtracting the SE profit when $c = \tilde{p}_{1,2}$ from the SE profit when $c = \tilde{p}_{2,2}$ yields a strictly negative value if $\beta_1 > \frac{\beta_2(1-\delta_1)+\delta_1-\delta_2}{1-\delta_2}$. Thus, the SE maximizes expected profit by setting low value of c to induce greater competition between the two firms in their bidding for a sponsored link. This result is intuitive because if β_1 is large, then firm 1 has relatively little incentive to bid on a sponsored link particularly if the *cpc* for customers who arrive through the sponsored link is large. The best the SE can do is set c to extract the maximum possible surplus from having firm 1 listed second, and this occurs when $c = \tilde{p}_{1,2}$. Furthermore, if β_1 is sufficiently large, then $c_2 > c_1$. Using this and propositions 7 and 8 it can be shown that further increases in c result in only firm 2 sponsoring a link and reduce SE profit.

This example shows that when firms have an alternative of relying on organic links to attract customers, it is possible that the optimal bidding strategies will result in the less relevant firm (firm 2) being listed first in the sponsored listings. Furthermore, in this particular example, the SE sets c sufficiently high so that firm 2 is the only firm that sponsors a link in equilibrium. This result relies on β_1 being sufficiently large (or, alternatively, on δ_1 being sufficiently small or δ_2 being sufficiently close to δ_1), so that for firm 1, its organic link attracts enough customers relative to the alternative of a sponsored link.

4 Managerial implications and concluding remarks

This paper investigates strategic behavior of firms on a search engine. We develop a comprehensive framework to account for the existence of asymmetry between these firms. First, some firms may be more relevant for customers than others (i.e. they have a higher probability to be clicked on their organic link). Second, firms may differ in the value generated by each consumer that visits their websites (i.e. some firms can extract more revenues from a consumer than others). Our model also integrates several empirical features regarding consumers behavior on search engine. First, our model closely mimics consumer behavior (successive iterations) when considering a result page. Second, it also explicitly integrates the possibility for some consumers to bypass sponsored links or to consider only the first sponsored linked as documented in the literature on browsing behavior.

Our framework allows us to determine what parameters are important for bidding strategies and how they affect the decision to bid and the amount of bidding. Our results highlight three kinds of equilibrium outcomes. In the first equilibrium outcome both firms bid on sponsored links. Under such an equilibrium, either firm can be listed first in the sponsored links. In general, the more relevant firm, firm 1, is more likely to be listed first as the difference $\delta_1 - \delta_2$ in the relevance of each firm in the sponsored links increases, because this implies consumers are more likely to click on firm 1's sponsored link, so the SE prefers to list firm 1 first. In addition, as β_1 decreases (β_2 increases), firm 1's bid becomes more competitive relative to firm 2, which also makes it more likely that firm 1 will be listed first.

The second type of equilibrium outcome involves the search engine setting the minimum *cpc* sufficiently high that only one of the two firms bids in the keyword auction. This equilibrium occurs which there is a significantly large difference in the willingness to pay between the two firms. For example, if firm 1 has a relatively high click through rate β_1 on its organic links and firm 2 does not, then it may be optimal for the SE to set c high enough so that firm 1 chooses not to bid in the keyword auction. Similarly, if the value v_2 to firm 2 of attracting customer is much higher than v_1, then the SE will optimally set a high value of c which extracts surplus from firm 2 while excluding firm 1. The analysis also demonstrates how the SE's ability to extract surplus in this manner is constrained by each firm's ability to attract customers through its organic links.

The final type of equilibrium involved the firms adopting mixed strategies to determine whether to participate in the auction process. This equilibrium requires that the SE set c sufficiently large that one of the two firms, say firm 1, is better off not advertising if firm 2 also chooses not to advertise, but firm 2 is better off advertising and paying c if firm 1 does not advertise, and firm 1 is better off advertising and being listed first if firm 2 does advertise.

The results and specific examples presented above demonstrate that organic listings can have a significant impact on equilibrium in sponsored search auction – both in terms of the optimal bidding strategies and decision to participate in the auction by firms, and the optimal reservation price established by the SE. In particular, the results are consistent with outcomes in auction sponsored search auctions in which less relevant firms are often listed ahead of more relevant firms in the sponsored links. The analysis also predicts that under certain conditions, highly relevant firms will not appear at all in the sponsored links.

The model can be extended in several directions. Initial analysis extending the model to include more than two firms suggests that it is possible that a Vickrey auction will generate more revenue for the SE than a generalized second price auction under certain conditions. This merits further investigation. Another possible extension would endogenize parameters γ and δ_i as parts of the search engine strategy. Indeed, the SE has the possibility to improve the visibility of the first sponsored link (in contrast to the other sponsored and organic links). For instance, it may choose to display only one sponsored link at the top of a result page, all the other sponsored links being displayed at the left-hand-side of the page which is a less favorable location. In that case, the advantage given to the first sponsored links is significantly increased. Secondly, one may empirically observe that for valuable or popular keywords, the ranking and the identity of sponsored links may change for two identical and successive requests. The ads displayed in the sponsored links section are the results of a random selection process of ranking. Yet, this creates some opportunity for the SE to "manipulate" the probability of clicking on a sponsored link (δ_i in our model) so as to extract more revenues from keywords advertising.

5 Appendix

5.1 Comparative statics on optimal bids:

$\tilde{p}_1 = v_1\left(1-\gamma\left(1-\frac{\delta_2}{t}\right)-\beta_1\left(1-\gamma\right)\left(1-\delta_2\right)/\delta_1\right)+c\gamma\left(1-\frac{\delta_2}{t}\right)$

$\frac{d\tilde{p}_1}{dt} = \frac{1}{t^2}\gamma\delta_2\left(c-v_1\right) < 0$

$\frac{d\tilde{p}_1}{d\beta_1} = -\frac{1}{\delta_1}v_1\left(\gamma-1\right)\left(\delta_2-1\right) < 0$

$\frac{d\tilde{p}_1}{d\gamma} = \frac{1}{t\delta_1}\left(ct\delta_1 - c\delta_1\delta_2 + t\beta_1 v_1 - t\delta_1 v_1 + \delta_1\delta_2 v_1 - t\beta_1\delta_2 v_1\right) > 0$ if and only if $\beta_1 > \frac{\delta_1(t-\delta_2)(v_1-c)}{tv_1(1-\delta_2)}$.

Sign here is unclear. If $c = v_1$, then clearly this condition on β_1 is always satisfied and $\tilde{p}_1$ is increasing in γ. If $c = 0$, then $0 < \frac{\delta_1(t-\delta_2)(v_1-c)}{tv_1(1-\delta_2)} < 1$, so the condition on β_1 may or may not be satisfied.

$\frac{d\tilde{p}_1}{d\delta_1} = \frac{\beta_1}{\delta_1^2}v_1\left(\gamma-1\right)\left(\delta_2-1\right) > 0$

$\frac{d\tilde{p}_1}{d\delta_2} = -\frac{1}{t\delta_1}\left(c\gamma\delta_1 - t\beta_1 v_1 - \gamma\delta_1 v_1 + t\gamma\beta_1 v_1\right) > 0$

$\frac{d\tilde{p}_1}{dv_1} = \frac{1}{t\delta_1}\left(t\delta_1 - t\beta_1 + t\gamma\beta_1 - t\gamma\delta_1 + t\beta_1\delta_2 + \gamma\delta_1\delta_2 - t\gamma\beta_1\delta_2\right) > 0$ if and only if $\beta_1 < \frac{\delta_1(t(1-\gamma+\gamma\delta_2))}{t(1-\gamma)(1-\delta_2)}$.

$\frac{d\tilde{p}_1}{dc} = \frac{1}{t}\gamma\left(t - \delta_2\right) > 0$

$\hat{p}_1 = v_1\left(1 - \left(1 - \frac{\delta_2}{t}\right)\beta_1/\delta_1\right)$

$\frac{d\hat{p}_1}{dt} = -\frac{1}{t^2}\frac{\beta_1}{\delta_1}\delta_2 v_1 < 0$

$\frac{d\hat{p}_1}{d\beta_1} = -\frac{1}{t\delta_1}v_1\left(t - \delta_2\right) < 0$

$\frac{d\hat{p}_1}{d\delta_1} = \frac{1}{t}\frac{\beta_1}{\delta_1^2}v_1\left(t - \delta_2\right) > 0$

$\frac{d\hat{p}_1}{d\delta_2} = \frac{1}{t}\frac{\beta_1}{\delta_1}v_1 > 0$

$\frac{d\hat{p}_1}{dv_1} = \frac{1}{t\delta_1}\left(t\delta_1 - t\beta_1 + \beta_1\delta_2\right) > 0$ if and only if $\beta_1 < t\delta_1/\left(t - \delta_2\right)$.

$\tilde{p}_2 = v_2\left(1 - \gamma\left(1 - \delta_1/t\right) - \frac{(1-\gamma-\delta_1+\gamma\delta_1)\beta_2}{\delta_2}\right) + c\gamma\left(1 - \delta_1/t\right)$

$\frac{d\tilde{p}_2}{dt} = \frac{1}{t^2}\gamma\delta_1\left(c - v_2\right) < 0$

$\frac{d\tilde{p}_2}{d\beta_1} = 0$

$\frac{d\tilde{p}_2}{d\beta_2} = -\frac{(1-\gamma-\delta_1+\gamma\delta_1)}{\delta_2} < 0$

$\frac{d\tilde{p}_2}{d\gamma} = \frac{1}{t\delta_2}\left(ct\delta_2 - c\delta_1\delta_2 + t\beta_2 v_2 - t\delta_2 v_2 + \delta_1\delta_2 v_2 - t\beta_2\delta_1 v_2\right) > 0$ if and only if $\beta_2 > \delta_2\frac{t-\delta_1}{t(1-\delta_1)}\frac{v_2-c}{v_2}$ which may or may not hold. Always holds if c is sufficiently close to v_2. Also always holds if β_2 is sufficiently close to δ_2 because $\frac{t-\delta_1}{t(1-\delta_1)} < 1$.

$\frac{d\tilde{p}_2}{d\delta_1} = -\frac{1}{t\delta_2}\left(c\gamma\delta_2 - t\beta_2 v_2 - \gamma\delta_2 v_2 + t\gamma\beta_2 v_2\right) > 0$ if and only if $v_2 > c\frac{\gamma\delta_2}{\gamma\delta_2+t\beta_2(1-\gamma)}$ which always holds.

$\frac{d\tilde{p}_2}{d\delta_2} = \frac{\beta_2}{\delta_2^2}v_2\left(\gamma - 1\right)\left(\delta_1 - 1\right) > 0$

$\frac{d\tilde{p}_2}{dv_2} = \frac{1}{t\delta_2}\left(t\delta_2 - t\beta_2 + t\gamma\beta_2 - t\gamma\delta_2 + t\beta_2\delta_1 + \gamma\delta_1\delta_2 - t\gamma\beta_2\delta_1\right) > 0$ if and only if $\beta_2 < \frac{t\delta_2(1-\gamma)+\gamma\delta_1\delta_2}{t(1-\gamma)(1-\delta_1)}$ which always holds because $\frac{t\delta_2(1-\gamma)+\gamma\delta_1\delta_2}{t(1-\gamma)(1-\delta_1)} > \delta_2$, and $\beta_2 < \delta_2$ by assumption.

$\frac{d\tilde{p}_2}{dc} = \frac{1}{t}\gamma\left(t - \delta_1\right) > 0$

$\hat{p}_2 = v_2\left(1 - \frac{(1-\delta_1/t)\beta_2}{\delta_2}\right)$

$\frac{d\hat{p}_2}{dt} = -\frac{1}{t^2}\beta_2\frac{\delta_1}{\delta_2}v_2 < 0$

$\frac{d\hat{p}_2}{d\beta_1} = 0$

$\frac{d\hat{p}_2}{d\beta_2} = -\frac{(1-\delta_1/t)}{\delta_2} < 0$

$\frac{d\hat{p}_2}{d\delta_1} = \frac{1}{t}\frac{\beta_2}{\delta_2}v_2 > 0$

$\frac{d\hat{p}_2}{d\delta_2} = \frac{1}{t}\frac{\beta_2}{\delta_2^2}v_2\left(t - \delta_1\right) > 0$

$\frac{d\hat{p}_2}{dv_2} = \frac{1}{t\delta_2}\left(t\delta_2 - t\beta_2 + \beta_2\delta_1\right) > 0$

$\tilde{p}_{1,2} = v_1\left(1 - \frac{\beta_1((t-1)+t\gamma(1-\delta_2))}{\gamma\delta_1(t-\delta_2)}\right)$

$\frac{d\tilde{p}_{1,2}}{dt} = -\frac{1}{\gamma}\frac{\beta_1}{\delta_1}v_1\frac{\delta_2-1}{(t-\delta_2)^2}\left(\gamma\delta_2 - 1\right) < 0$

$\frac{d\tilde{p}_{1,2}}{d\beta_1} = -\frac{1}{\gamma\delta_1}\frac{v_1}{t-\delta_2}\left(t + t\gamma - t\gamma\delta_2 - 1\right) < 0$
$\frac{d\tilde{p}_{1,2}}{d\gamma} = \frac{1}{\gamma^2}\frac{\beta_1}{\delta_1}\frac{v_1}{t-\delta_2}\left(t-1\right) > 0$
$\frac{d\tilde{p}_{1,2}}{d\delta_1} = \frac{1}{\gamma}\frac{\beta_1}{\delta_1^2}\frac{v_1}{t-\delta_2}\left(t + t\gamma - t\gamma\delta_2 - 1\right) > 0$
$\frac{d\tilde{p}_{1,2}}{d\delta_2} = \frac{1}{\gamma}\frac{\beta_1}{\delta_1}v_1\frac{t\gamma-1}{(t-\delta_2)^2}\left(t-1\right) > 0$ if and only if $\gamma > 1/t$.
$\frac{d\tilde{p}_{1,2}}{dv_1} = \left(1 - \frac{\beta_1((t-1)+t\gamma(1-\delta_2))}{\gamma\delta_1(t-\delta_2)}\right) > 0.$
$\tilde{p}_{2,2} = v_2\left(1 - \frac{\beta_2(\delta_1(t-1)+t\gamma(1-\delta_1))}{\gamma\delta_2(t-\delta_1)}\right)$
$\frac{d\tilde{p}_{2,2}}{dt} = -\frac{1}{\gamma}\beta_2\frac{\delta_1}{\delta_2}v_2\left(\gamma-1\right)\frac{\delta_1-1}{(t-\delta_1)^2} < 0$
$\frac{d\tilde{p}_{2,2}}{d\beta_1} = 0$
$\frac{d\tilde{p}_{2,2}}{d\beta_2} = -\frac{(\delta_1(t-1)+t\gamma(1-\delta_1))}{\gamma\delta_2(t-\delta_1)} < 0$
$\frac{d\tilde{p}_{2,2}}{d\gamma} = \frac{1}{\gamma^2}\beta_2\frac{\delta_1}{\delta_2}\frac{v_2}{t-\delta_1}\left(t-1\right) > 0$
$\frac{d\tilde{p}_{2,2}}{d\delta_1} = \frac{t}{\gamma}\frac{\beta_2}{\delta_2}v_2\frac{\gamma-1}{(t-\delta_1)^2}\left(t-1\right) < 0$
$\frac{d\tilde{p}_{2,2}}{d\delta_2} = -\frac{1}{\gamma}\frac{\beta_2}{\delta_2^2}\frac{v_2}{t-\delta_1}\left(\delta_1 - t\gamma - t\delta_1 + t\gamma\delta_1\right) > 0.$
$\frac{d\tilde{p}_{2,2}}{dv_2} = \left(1 - \frac{\beta_2(\delta_1(t-1)+t\gamma(1-\delta_1))}{\gamma\delta_2(t-\delta_1)}\right).$
$c_1 = v_1\left(1 - \beta_1/\delta_1\right)$
$\frac{dc_1}{d\delta_1} = \frac{\beta_1}{\delta_1^2}v_1 > 0$
$\frac{dc_1}{d\beta_1} = -\frac{1}{\delta_1}v_1 < 0$
$\frac{dc_1}{dv_1} = \left(1 - \beta_1/\delta_1\right) > 0$
$c_2 = v_2\left(1 - \left(1 - \frac{\beta_1}{t}\right)\beta_2/\delta_2\right)$
$\frac{dc_2}{d\delta_2} = \frac{1}{t}\frac{\beta_2}{\delta_2^2}v_2\left(t-\beta_1\right) > 0$
$\frac{dc_2}{d\beta_1} = \frac{1}{t}\frac{\beta_2}{\delta_2}v_2 > 0$
$\frac{dc_2}{d\beta_2} = -\frac{1}{t\delta_2}v_2\left(t-\beta_1\right) < 0$
$\frac{dc_2}{dt} = -\frac{1}{t^2}\beta_1\frac{\beta_2}{\delta_2}v_2 < 0$
$\frac{dc_2}{dv_2} = v_2\left(1 - \left(1 - \frac{\beta_1}{t}\right)\beta_2/\delta_2\right) > 0.$

5.2 Proof of Lemma 2

Proof of Lemma 2. This follows from $\hat{p}_2 - c_2 = \frac{\beta_2}{t\delta_2}v_2\left(\delta_1 - \beta_1\right) > 0$, and from

$$c_2 - \tilde{p}_{2,2} = -\frac{1}{t\gamma}\frac{\beta_2}{\delta_2}\frac{v_2}{t-\delta_1}\left(t\delta_1 - t^2\delta_1 - t\gamma\beta_1 - t\gamma\delta_1 + \gamma\beta_1\delta_1 + t^2\gamma\delta_1\right)$$

which is positive when $t = 1$. Furthermore, this expression equals 0 at the two roots

$$\frac{1}{2} - \frac{\gamma\beta_1 \pm \sqrt{\delta_1^2 + \gamma^2\beta_1^2 + \gamma^2\delta_1^2 - 2\gamma\delta_1^2 + 4\gamma\beta_1\delta_1^2 + 2\gamma^2\beta_1\delta_1 - 4\gamma^2\beta_1\delta_1^2 - 2\gamma\beta_1\delta_1}}{2\delta_1\left(1-\gamma\right)}$$

which are both less than 1. To see this note that the larger root,

$$\frac{1}{2}-\frac{\gamma\beta_1-\sqrt{\delta_1^2+\gamma^2\beta_1^2+\gamma^2\delta_1^2-2\gamma\delta_1^2+4\gamma\beta_1\delta_1^2+2\gamma^2\beta_1\delta_1-4\gamma^2\beta_1\delta_1^2-2\gamma\beta_1\delta_1}}{2\delta_1\left(1-\gamma\right)},$$

is less than 1 if and only if

$$\delta_1\left(1-\gamma\right)>-\gamma\beta_1+\sqrt{\delta_1^2+\gamma^2\beta_1^2+\gamma^2\delta_1^2-2\gamma\delta_1^2+4\gamma\beta_1\delta_1^2+2\gamma^2\beta_1\delta_1-4\gamma^2\beta_1\delta_1^2-2\gamma\beta_1\delta_1}$$

which always holds because

$$\begin{aligned}
& \left(\delta_1\left(1-\gamma\right)+\gamma\beta_1\right)^2- \\
& \left(\sqrt{\delta_1^2+\gamma^2\beta_1^2+\gamma^2\delta_1^2-2\gamma\delta_1^2+4\gamma\beta_1\delta_1^2+2\gamma^2\beta_1\delta_1-4\gamma^2\beta_1\delta_1^2-2\gamma\beta_1\delta_1}\right)^2 \\
= \;& 4\gamma\beta_1\delta_1\left(\gamma-1\right)\left(\delta_1-1\right)>0.
\end{aligned}$$

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