Tableaux* and *Systèmes

Early French Contributions to Linear Production Models

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Calculations are to the economic science what bones are to the human body. Without them it will always be a vague and confused science, at the mercy of error and prejudice.

(François Quesnay)[1]

1. Introduction

The history of linear production models begins, properly speaking, with François Quesnay's *Tableaux économiques* of the late 1750s and early 1760s. While earlier authors, like Boisguilbert and Cantillon offered accounts of circular flow processes, the *Tableaux* were the first significant attempts to model such processes by means of a formal apparatus. A main reason why the introduction of a formal apparatus was an important step is that it necessitated the making of clear modelling choices, something which may be more easily side-stepped as long as purely verbal accounts are used to describe phenomena of circulation and reproduction.

Less well known than Quesnay's contributions are the *systèmes des richesses*, developed one generation later by the engineer Achilles Nicolas Isnard (1748-1803). These *systèmes* were in fact the only significant alternative attempt during the same period to use a formal apparatus for analysing the economic processes of reproduction in an economy conceived as a single system. Although Isnard built on Quesnay's analysis, he was highly critical of a number of assumptions made by the doctor. As a result his apparatus exhibits a number of important differences with the *Tableaux*.

In a sense Quesnay's *Tableaux* and Isnard's *systèmes* belong together. They constitute the 'state of the art' of linear production models in the second half of the 18th century and they are best understood through being studied in tandem. Of course in comparison to modern techniques the mathematical content of their contributions is basic. It is true that Isnard makes an important advance by using simultaneous equations instead of Quesnay's characteristic pictorial approach. But such formal progress is perhaps of less significance than the differences in conceptions and aims motivating the modelling efforts of these pioneers. Generally speaking, Quesnay presents an idealised model of the French economy, making a number of specific 'institutional' assumptions to study phenomena of economic growth and decline. Isnard's approach is more abstract, only preserving the essentials of the doctor's conception of the economy in order to focus on the interplay between

[1] Remark in a letter from Quesnay to Mirabeau (*Archives Nationales*, Ms. 779, 4 bis, p.2 note; quoted in Weulersse, 1910, II: 124). It was adopted with minor changes in the preface of *Philosophie Rurale* (1763: xix-xx).

reproduction, prices and distribution. When looking at these different conceptions and aims one gets an impression of the various directions that were possible within a nascent branch of economic analysis which we now call linear production modelling. Even at the very outset of this new branch of analysis one gets the impression that, rather than a unidirectional path of successive improvements, the history of linear production models itself is far from linear.

In this paper we will compare the following aspects of the different approaches of the two pioneers from the French Enlightenment. First we will look at the differences in the formal presentation of their conceptions of economic interdependence, that is Quesnay's pictorial approach versus Isnard's use of simultaneous equations. Second, we consider their different ways of incorporating the concept of 'surplus' in their formal apparatus. Third, we discuss the strikingly different uses the two men made of their models. While Quesnay was principally interested in the analysis of economic growth and decline, Isnard focussed on the interrelation of market prices, reproduction and distribution. We finish with a number of observations from a 'modern' point of view.

2. What form of presentation to use?

The fact that Quesnay's *Tableaux économiques* are pictorial representations of the economy makes them quite unique in the history of linear production models. While later economists have acknowledged an affinity between their own work and that of the 18th century forebear, most famously Leontief's description of his own work as 'a *Tableau économique* of the United States' (Leontief 1936: 105), they did not follow him in literally providing a 'picture' of the economy as Quesnay had done. Since they were first published, both the original 'zigzag' version of the *Tableau* and the final simplified 'Analysis' version have fascinated and baffled students in equal measure.[2] Indeed, a certain suspicion persists until the present day: besides a compelling but basic impression of interdependence between different 'sectors' of the economy, what are the *Tableaux* actually supposed to demonstrate? In this context it is worth considering the question what motivated Quesnay to use pictures.

Charles (2003, 2004) puts forward interesting views on this point. He argues that Quesnay consciously decided against mathematical expressions when constructing his visual aids for the study of economic circulation. One reason for this was that Quesnay judged that the use of images would be more striking and spark the imagination and interest of the uninitiated reader more directly. Charles (2003: 529-36) shows that the original zigzag picture, first drawn in 1758, was most likely inspired by existing mechanical devices, such as rolling-ball clocks and hydraulic machines that were designed primarily for

[2] For a discussion of the responses of Quesnay's contemporaries to the *Tableaux* see van den Berg 2002. There it is shown that at the time the intermediate 'Précis' version of the *Tableau* in the *Philosophie rurale* did not arouse any interest.

the entertainment of the viewer.[3] As a courtier Quesnay understood the importance of such gentle means of persuasion for people with limited attention spans. Hence the zigzag '[…] paints ideas to the eyes that are very intertwined and which [someone of] simple intelligence would have difficulties to grasp, untangle and agree with if they were to be presented by means of discourse'[4] In short, a picture is worth a thousand words and the *Tableau* was therefore conceived in the first place as a tool of instruction and persuasion.

A second reason is perhaps that the use of a picture allowed Quesnay to express his specific ideas about socio-political and economic primacy. Of the three social 'classes', the land proprietors were literally given the central place. They were presented as initiating a new round of spending on goods produced by farmers, on the one side, or artisans, on the other side. The spending round was concluded with the payment by the 'productive' class of farmers of the *produit net*, in the form of a contractual rent payment to the class of land proprietors. The idea that one particular 'intersectoral' payment, and subsequent spending of this payment flowing from one specific class, was the prime motivating force of circulation and reproduction could not have been more effectively conveyed than in a picture.

At the same time, this picture was not meant to be contemplated by itself. All *Tableaux* were accompanied with various notes and text explaining the finer detail of the multiple exchanges and the cycle of reproduction that the picture purported to show. Especially this accompanying text makes clear that even though Quesnay did not claim in so many words to be estimating the size of the French economy, under various assumption and conditions, the 'agricultural kingdom' he had in mind clearly was a nation of a similar size and composition. In chapter VII of the *Philosophie rurale*, for instance, one of the most extended and detailed explanations of the *Tableaux*, the totals of *milliards* of *livres* used in those pictures were justified by detailed breakdowns of costs and charges in various branches of agriculture, sizes of populations employed (or idle) in different parts of the economy and estimates of average incomes and spending patterns.[5] Of course such figures were based on based on heroic assumptions and generalisations rather than actual statistical work, but this should not obscure what may be called their statistical intent. In addition to the pretended empirical realism of his approach, another reason why Quesnay avoided mathematical techniques beyond simple arithmetic was that as a physician, he had received little training in mathematics. With this came a limited confidence in the uses of mathematics in the economic sciences. Basically, Quesnay's ideas for the uses of mathematics were limited to the firm conviction that (arithmetical) calculations were a necessary tool for estimating the size of the effects which (changes in) some 'empirically' estimated economic quantities would have on others. But Quesnay did not

[3] In particular a book published in 1719 by Gaspard Grollier about a cabinet of curiosity machines in Lyon that were designed and built by his grandfather may have inspired Quesnay. Rieter (1990) earlier pointed out parallels between pictures contained in this work and the zigzag, as acknowledged by Charles (2003:533).

[4] Quesnay (2005: 1183) in a letter to Mirabeau, written in the first half of 1759.

[5] In the 2005 edition of Quesnay's works this chapter is admirably presented together with transcripts of various manuscript drafts. This provides the reader with fascinating insights into the process of the composition of this crucial contribution.

pretend that the calculations that were carried out on the figures with the aid of the *Tableaux* were any more than arithmetical computations of which the 'truth' depended solely on the empirical validity of the data used.[6] He did not see the merits of mathematical proof or algebraic solution for economic theory. In a work admittedly written when his intellectual capacities were on the decline, he appeared to reject the kind of 'evidence' derived from algebraic demonstrations:

> Metaphysical geometry and metaphysical calculations, or calculations with abstract numbers, give abstract and general notions, which are not always applicable with *évidence*, to the positive truths of demonstrative geometry. And abstract notions, severed from concrete ideas, always beget sophisms, even metaphysical axioms (Quesnay 1773: 8-9).[7]

One can doubt whether this attempt at criticising the uses of 'abstract numbers' in arguments is based on any real understanding of more advanced mathematics. But at least it shows Quesnay's reservations to go beyond basic arithmetic.

Isnard's attitude towards formal modelling and the use of mathematics in economics was very different. He may be said to strip away much of Quesnay's cumbersome apparatus and preserves only the essentials of the conception of circulation and reproduction. One obvious, but partial, explanation for this reductive approach is that Isnard, in contrast to Quesnay, did receive a thorough training in mathematics. From what we know about his education at the *École royale des Pont et Chaussées*, at the time the pre-eminent institution of technical education in France, it is clear that he completed several advanced courses such as the 'application of algebra to geometry and mechanics'. Moreover, from the fact that in 1773, in the absence of the mathematics professor, Isnard took over the classes and taught his fellow students algebra, geometry and differential and integral calculus, it is clear that he achieved a good level of proficiency in several branches of mathematics (see van den Berg 2006:11).

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[6] In one interesting footnote, he offered the following reflections on the limits of his quantitative approach: '[…] calculations are neither causes nor effects; thus in the sciences they never constitute the objects of our researches. Now in all the sciences certainty consists in the fact that the objects are made self-evident. If we do not attain to these self-evident objects, which supply calculation with facts or data capable of being counted and measured, calculation will not rectify our errors. […] This certainty, it is true, can be extended by means of calculation to cover quantities which only calculation can compute, and in this case it is always essentially infallible, *i.e.* it always infallibly and consistently presents us with either errors or realities, according to whether we apply it to errors or realities. Whence it follows that in research into truth by means of calculation, the whole certainty lies in the self-evidence of the data' (Translation in Meek 1962:183;.for the French original see Quesnay 2005: 614).

[7] This translation is the one given by Charles (2003: 537) apart from the double occurrence of the term 'calculations' in the first line. Charles instead translates the French *calcul* with 'calculus'. I am not convinced that Quesnay's (rather vague) criticism was directed at the use of calculus. If anything, he appears to aim his criticism at the use of algebra.

Of course, the actual mathematics Isnard uses in his *Traité des richesses* of 1781 is not particularly advanced.[8] But this should not obscure the fact that his analyses of reproduction and exchange are profound formal innovations. In both cases Isnard uses systems of simultaneous equations, a first in the history of economic analysis.

Significantly, Isnard refers to his examples of sets of equations describing the relations of reproduction within an economy, not as *Tableaux* but as *systèmes*. This abandonment of Quesnay's pictorial approach in favour of the use of simultaneous equations signifies more than a purely formal innovation: it is part of a critique of Quesnay's biased depiction of the landowners as the prime movers in the process of circulation. In a rare direct comment on the *Tableau* the engineer writes:

> One could reproach [the physiocrats] for having represented the landowners as being seated on a throne and distributing to both sides the salaries of the two classes according to how they value their advances, their talents, their activity, etc.

This, according to Isnard, is fundamentally the wrong picture, because it leaves out what is truly the 'arbiter' in the process of reproduction, namely the exchange of products in the market. He continues:

> They have not realised that according to their own principles about the freedom of exchange wheat is no more the salary of labour than labour is the salary of wheat, interest is no more the salary of the capitalist than the things which he produces are the salary of interest. There is exchange everywhere and it is in exchange that values are determined; it is through the values that each proprietor of labour, of products and capital attracts a part of the disposable wealth (Isnard 1781, I, 42 n; translation in van den Berg 2006: 124).

The simultaneous equations describing exchange and those describing production are specifically designed to express this alternative conception. The simplest example of equations describing production provided by the engineer, reads:

> Let there be two types of commodities 40M and 60M'. Suppose that to produce 40M, a consumption of 10M + 10M' is necessary, and to produce 60M', 5M + 10M'. Thus, to produce the total of the two products, a consumption of 15M and 20M' is required, and the value of disposable wealth is equal to 25M + 40M'. To know what each producer receives of that disposable wealth, one has to suppose values to those two products (Isnard 1781, I, 36; translation in van den Berg 2006: 118).

The surprisingly modern nature of this conception is attested to by the fact that one can straightforwardly write this example in familiar input-output format:[9]

[8] In this particular respect the *Theoretical considerations about sinking funds* of 1802 demonstrates his ability to apply mathematics to economic subject more fully (see van den Berg 2006: 344-417).

[9] This table is identical to table 1 in Steenge and van den Berg 2001.

Table 1

	M	M'	Disp	Tot
M	10	5	25	40
M'	10	10	40	60

An important difference with Quesnay's *Tableaux* is that the units in Isnard's example are physical units of 'heterogeneous' commodities (hence in table 1 numbers can only added up horizontally). This was a deliberate choice of the engineer who in a later comment clarified that

[s]ince commodities or products of different kinds enter into the costs of production, a relation of homogeneity has to be given to those commodities or products that allows them to be compared to one another. This relation is obtained from the values those commodities or products obtain in exchange, or from the comparison made between all commodities or products to one commodity, which serves as common measure (Isnard 1789: 7-8; translation in van den Berg 2006:278).

Once various inputs would be expressed in 'homogeneous' value terms one could add their quantities (vertically, in table 1). But this would at the same time fix the distribution of the 'disposable wealth' between the two sectors.

Isnard's novel mathematical expression, distinguishing the "quantity *système*" from the "price *système*", must be understood in the first place as a tool to expose an assumption with regards to relative prices that was only implicit in the *Tableaux*. This was, as Gilibert (1989:93) expresses it, the assumption of the 'one set of prices [that] allows the *Tableau* to reproduce itself and to guarantee the desired outcome (a net revenue appearing only in agriculture)'.[10]

It should be noted that while Isnard's intention was therefore to provide a mathematical demonstration in order to expose a kind of slight of hand in Quesnay's analysis of reproduction and distribution, by the same token he also sacrificed the empirical aspirations of the doctor's device. As was noted above, the numbers used in the *Tableaux* were a thinly veiled attempt to estimate the size of the French economy under certain assumptions. Isnard jettisons this empirical intent by not pretending in the least that the numerical examples he uses in any way measure a real economy. [11]

Thus, one may say, the engineer's intent in the use of mathematics is more analytical. This is especially clear in his discussion of market exchange. This

[10] See section 4 below for a further discussion.

[11] This particular difference between Quesnay and Isnard reminds one of a similar change in approach, but in reverse, that occurred early in Wassily Leontief's career. As Kurz and Salvadori (2000: 170-2) recount, in his early work, partly published in 1928, Leontief developed ideas about economic interdependence by clearly distinguishing between physical measurements and value measurements – much like Isnard. However, once he turned towards the empirical study of economic interdependence (Leontief 1936) he opted for value measurements, mainly because they allowed for easy aggregation – not unlike Quesnay.

rightly famous piece of analysis takes the form of an algebraic proof of the proposition that in order to find a set of equilibrium market prices, 'one would have to formulate as many equations as there are commodities' (Isnard 1781, I, 19; van den Berg 2006: 100). Isnard only provides a solution for the case of three commodities and acknowledges that 'such calculations would be very complicated in a system with a great number of commodities' (Isnard 1781, I, 26-7; van den Berg 2006: 108). In other words, Isnard recognises the need for mathematical techniques to solve larger sets of simultaneous equations. In this context the following observation in a later pamphlet about what he calls 'a great unsolved problem' is interesting:

> No Dupont, no Condorcet … not even a Lagrange has solved it [*i.e.*, the great unsolved problem] in a manner that is applicable to the system of the Economists, because the complication of the givens is too great: it is the problem of the value of commodities or of things (Isnard 1789: 305; van den Berg 2006:331).

The mention of J.F. Lagrange is significant because it suggests that Isnard was at least familiar with the former's work on solutions for systems of simultaneous equations in classical mechanics.[12] While Lagrange's work of 1776/1777 was mathematically much more advanced than Isnard's efforts in economics, the very fact that the engineer appears to have recognised the relevance to economic analysis of techniques for solving systems of simultaneous equations in mechanics is unique for its time and several generations of economists after him. It contrasts starkly with Quesnay's strong reservations about the use of any mathematics in economics beyond simple arithmetic.

3. How to deal with the surplus?

The most fundamental similarity between Quesnay's *Tableau* and Isnard's *système* is that both models of reproduction are irreducibly circular and social: outputs produced by one 'sector' of the economy are used as inputs in other 'sectors'. It is of course this fundamental interdependence within their models that allows them to be rewritten into input-output format. Such transcriptions do however throw up difficulties with regards to the entries that represent the 'surplus'.

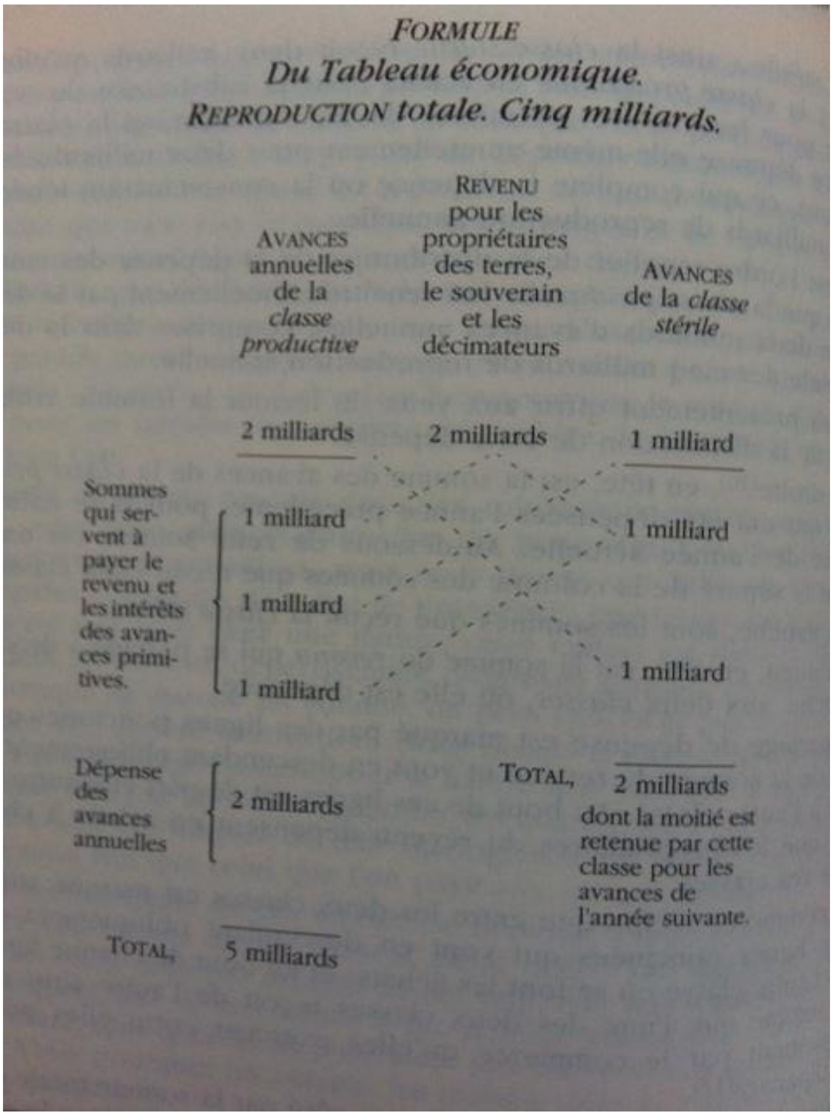
Prior to Quesnay, the notion of a social 'surplus', that is to say a production or payment in excess of reproductive requirements, had already figured in a theoretical context in the work Petty and Cantillon. Especially the latter's descriptions of the payment of a *surplus* by farmers to landowners and the subsequent circulation of this sum between economic classes, probably exercised an important influence on the formation of Quesnay's concept of *produit net*. However, the adoption of a formal apparatus for studying reproduction and circulation posed a problem that was new, namely how to model the net product as a quantity distinct from the exchanges to replace

[12] The names of Dupont and Condorcet are mentioned because the pamphlet is directed against specific economic writings of these authors. This is not the case with Lagrange, whose name appears to be mentioned because of his mathematical work .

materials used up in production. The same problematic applies to Isnard's notion of surplus, what he calls the *richesses disponibles*.

In order to see the problem clearly we may rewrite Quesnay's *Tableau* in input-output format. This is most easily done with the consolidated 'Analysis' version of 1766. We are referring to the following famous picture:

Picture 1. The Tableau of the *Analysis*



Following Phillips (1955) the presentation of this *Tableau* in closed Leontief format is:[13]

[13] This table is identical to table 2 in Steenge and van den Berg 2001 and primarily differs from the original table of Phillips in the sense that the artisans and proprietors have changed places. A separate point is if we indeed can interpret the economy of Table 2 in terms of a

Table 2	F	A	P	T
F	2	2	1	5
A	1	0	1	2
P	2	0	0	2
T	5	2	2	

So the notion of a net product appears in two ways: Column P gives the composition of landlords' consumption, row P gives rent payment. A difference with Isnard is the role of monetary values, but if prices are assumed unity, the similarity with the *système* becomes obvious.

Quesnay is interested primarily in phenomena of economic growth and decline. His preoccupation is with the revival of agriculture, which he considers the crucial sector for achieving a position of optimal wealth for the economy as a whole. The structure of Table 2 reflects an optimal situation in the sense that the agricultural sector is the productive one that generates all rents necessary to guarantee an uninterrupted repetition the next year, the year after that, and so on.

So, the notion of uninterrupted circularity is very much capable of illustrating the interconnections in Quesnay's economy. The big problem for Quesnay was to use this model as a starting point for showing the detrimental effects of a different spending behaviour of the ruling classes. A shift to the more luxurious commodities would mean that the optimal situation would be disturbed with negative consequences for the agricultural sector and, consequently, for the country as a whole.

However, how to model that in a Phillips type closed Leontief model? To that end, one has to open up the closed model, to turn it into an open (Leontief) model. However, shifts in (what then must be called) 'final demand' have no effect on the total surplus as registered in the p-row. This fact was by several analysts seen as an error on the part of Quesnay. However, as shown in Steenge and van den Berg (2007) there is another, and entirely consistent way to interpret (and model) a decline in importance of agriculture.

A demonstration of his ideas with regards to the disturbance of the ideal proportions between various expenditures requires a complicated apparatus consisting of subsystems and special assumptions (see Steenge and van den Berg 2007). In fact, Quesnay's numerical figures can be interpreted as an

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closed Leontief model. One reason is that the columns of a closed Leontief represent fixed coefficients production functions while the entries in Table 2 stand for a complex mix of historical, political, economic and technological data. Opening up the model with respect to one category thereby may attribute properties solely to that sector which are shared by all sectors; see further Steenge and van den Berg (op. cit.).

early form of Stone's famous 'RAS-method', many years later developed for updating and finalizing input-output tables.[14]

The complex underlying structure of the *Tableaux* consisting of specific proportions between the various expenditures within the economic system, is almost entirely missing from Isnard's work. On the other hand, what is almost entirely missing from the analysis illustrated by means of the *Tableaux* is the role of prices in encouraging (or discouraging) production. Without much justification, prices are simply assumed to be unchanging during the analysis of changes in spending patterns.[15] This is surprising given the fact that elsewhere in the physiocatic literature the importance of free competition and the opening up of markets for the revival of (agricultural) production is repeatedly stressed. It has even been argued that the notion of *bons prix* expressed a consistent theory of prices that included motivational surplus incomes for farmers (see Vaggi 1987). In the *Tableaux*, however, Quesnay hardly analyses these notions.

4. The role of prices in reproduction and distribution

The focus in Isnard's analysis is precisely on what figures only implicitly in the *Tableaux*, namely the role of prices in ensuring the reproduction of commodities, and in the distribution of the surplus. The immediate reason for the engineer to concentrate on this aspect is that he believes there to be a slight of hand in the *Tableaux* that guarantees the occurrence of a net income in agriculture only. According to Isnard it can easily be demonstrated that this is a very arbitrary distributional outcome:

> Assume, for example, taking a common measure M", that M is worth M", and that M' is worth 2M". The producers of 40M would have to incur the costs of 30M or 30M", and could freely dispose of 10M or 10M". The producers of 60M' would have to incur the costs of 12½M' or 25M", and could freely dispose of 47½M' or 95M" (Isnard 1781, I, 36; translation in van den Berg 2006: 118).

This assumption about the relative price between units of the two commodities allows one to rewrite table 1 above in value terms, units of M", (the second row is simply multiplied by two, which gives the distribution of disposable income in the third row:

	M	M'	Disp	Tot
M	10	5	25	40
M'	20	20	80	120
Disp	10	95		
Tot	40	120		160

[14] For the details we refer to Steenge and van den Berg (2007).

[15] For example, in the 'Analysis' Quesnay remarks that the figures he uses 'imply certain conditions *sine quabus non*. They assume that freedom of trade maintains sales of products at a proper price – a price of 18 livres per *setier* of corn, for example' (in Meek 1962: 153).

Isnard continued:

If one [alternatively] supposes that M = M", and that M' = 3M", the disposable revenue of the proprietors or of the producers of 40M will be equal to zero, and that of the producers of 60M' will be equal to the total mass of disposable wealth, 25M + 40M', or 481/3M', or 145M" (Isnard 1781,I, 36-7; translation in van den Berg 2006: 118).

This time, table 1 expressed in value terms (the second row being multiplied by three) becomes:

	M	M'	Disp	Tot
M	10	5	25	40
M'	30	30	120	180
Disp	0	145		
Tot	40	180		220

In the second case only one class of producers receive the whole disposable income in the system, very much like the farmers in the *Tableaux*. However, in Isnard's demonstration this was merely the effect of a particular assumption about relative prices and 'one could make an infinite number of other suppositions' (*ibid*. 42; 124) that each led to a different distributional outcome, while still allowing reproduction to continue. How to think about Table 1 in "modern terms"? First of all, we can easily derive a general expression for the relation between prices (and, thus, relative prices) and the disposable revenue of the producers. If we denote these revenues by the symbols A and B, respectively, we have

$10p_1 + 10\,p_2 + A = 40p_1$

$5p_1 + 10p_2 + B = 60p_2$.

Substituting $p_1 = 1$ and $p_2 = 2$ and adding up, we find A – B = -85. Simultaneously, A + B = 105, Combining, we find A = 10 and B = 95. In this way, for each set of values for p_1 and p_2 we find a different set of values for A and B, and vice versa. Isnard's point thus is clear: relative prices and distribution of the surplus are interdependent, one determines the other. (And, in fact, an infinite number of outcomes is possible. For a further discussion see Steenge and van den Berg (2001: 137-9) where a procedure is presented that generates Isnard prices via a reallocation of shares of the net output bundle to the two producers.

Within this context and, of course, in hindsight, in terms of modelling a most important question concerns whether we may interpret the first two columns in terms of production functions with fixed input proportions and the 2 x 2 matrix they form together in terms of a matrix of intermediate deliveries. This depends to a large extent on whether we can interpret the surplus column (the *richesses disponibles*) as a final demand vector that "somehow" is limited in size by the 2 x 2 matrix M, where the term "somehow" refers to the fact that there is no "visible" constraint on the size (and composition) of the surplus.

Looking back, Isnard can be said to stand at a crossroads. All modern interpretations of economic interdependence in terms of a set of simultaneous equations contain some rule that links the deliveries among the producers to a productivity concept. In Isnard's work such a notion is (still) lacking. Let us illustrate some of the consequences via a concept that is not found in Isnard's conceptualization, i.e the notion of intermediate input coefficients. This notion makes it possible to link an economy's surplus to a number of technologically determined properties, and led ultimately to concepts like Leontief's multiplier and von Neumann types of duality.

If we interpret Table 1 in terms of intermediate inputs into production processes and if we calculate input coefficients "in the Leontief way", we obtain as input coefficients matrix:

$$M_c = \begin{matrix} 1/4 & 1/12 \\ 1/4 & 1/6 \end{matrix}$$

Given matrix M_c, do we have alternatives for the surplus vector? In fact, we do. For example, a doubling or tripling is possible. The reason is that there is no single factor (which can be a primary factor in the Leontief sense) that puts an upper limit on the size of the surplus. Thus, accepting the presence of the two *systèms*, if prices change, the value and distribution of the surplus change too, and *vice versa*. Quesnay can be said to have found a solution for this, by selecting one set of prices (and quantities), and by sticking to them. (As said, he combined this with a 'way out' based on a separate algorithm to capture the structural changes following a shift in spending behaviour).

Translated again in modern language, Quesnay was faced with the problem of using a rigid mathematical model (a closed Leontief model) to show that deviations of the optimal configuration would have a negative impact on the total economy (and he did this by devising a separate algorithm). Isnard, on the other hand, was faced with the problem of having developed an overly flexible model that could accommodate (almost) any set of prices and quantities, but lacked a constraint that would make it more realistic to describe real world situations.

Realizing the fact that prices and quantities should be treated as 'dual' to each other, makes Isnard a forerunner of, particularly, Leontief and von Neumann who both suggested methods to make the price-quantity relation transparent *and* to introduce constraints that made the models much more realistic and suitable for empirical work. Leontief accomplished this by introducing a model in two variants, a quantity and a price variant. The quantity model is given by the well-known formula,

$$x = Ax + f$$

where A is a matrix of input coefficients, and x and f, respectively, vectors of total output and exogenous final demand. The price model is given by

$$p = pA + wl,$$

where p is the vector of equilibrium prices, l the vector of labour input coefficients and w the wage rate, in money terms. Built-in is the famous property that the value of final demand f is equal to the total remuneration of the labour factor.[16] We recognize here the two ways of registering the surplus in the Phillips table. We also recognize here a consistent formulation of the duality property that Isnard struggled with.

Finally, we would like to refer to John von Neumann (1945) who developed, independently of Leontief and other scholars, an optimality criterion that provides an alternative "missing constraint". Von Neumann assumed knowledge of a set of production processes quite like the two processes in Isnard's Table 1, but adding an optimality criterion. In fact, optimality came in by posing the question how to organize production and the price system to guarantee that the economy at hand could grow as fast as possible, where growth was defined in terms of 'balanced growth', a situation where the proportions between the total quantities of the various goods being produced does not change over time. Von Neumann's mathematics basically consist of the two 'dual' parts, prices and quantities, each with their own properties, but firmly connected in the economy's objective, i.e. maximal balanced growth. So, and again looking back, we may recognize here a central role for a Quesnay type of fixed output proportions and an Isnard-like price-quantity duality.

5. Conclusion

In the above we hope to have shown that Quesnay and Isnard can rightfully be considered as forerunners of linear approaches to macro-economic modelling. They can, as we have argued, be seen as belonging together, wrestling with issues that have become part of standard modelling only much later. At the risk of some exaggeration, we may say that Quesnay started from an economy characterized by fixed or 'rigid' mutual proportions and was faced with the problem to address the question what would happen if one or more of these proportions would change (and, as we have pointed out) he found a most interesting solution).

Isnard, on the other hand, dissatisfied with the perceived inflexibility caused by many 'built-in' fixed proportions, ended up with the problem that his solution –simultaneous equations- were too 'flexible' and too far removed from real use and application. This means that to obtain more specific statements on prices and surplus, additional assumptions have to be brought in. If these are adopted with an eye on historical or institutional factors, we encounter

[16] More complex cases can be found in modern textbooks such as Miller and Blair (2009).

Quesnay again, who captured many features reminiscent of contemporaneous France. However, in terms of adding assumptions –or structure- Isnard encountered difficulties that only were solved much later. His problem was, in hindsight, that his system lacked a vision on the role of specific constraints which only were solved firmly embedded in the notion of fixed coefficients productions functions. This was a task only much later performed by two great scholars, Wassily Leontief and John von Neumann.

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