Appeals Process, Judicial Errors and Crime Deterrence

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Abstract

We build on the intuitive result of Shavell (1995) that the appeals process may improve trial court accuracy by inducing trial court judges who fear reversal to devote more effort to obtaining correct decisions than they would in the absence of an appeals process. To test the robustness of this result, we extend Shavell's framework in three ways. First, we add on the possibility that judges are sensitive to either the social harm caused by the erroneous decisions they take or to their reputation. Second, we take into account the impact of crime deterrence on trial court judges' effort at equilibrium. Third, we consider simultaneously type-I errors (wrongful convictions) and type-II errors (wrongful acquitals) in this setting. We show that the appeals process may lead trial court judges to decrease their effort even if they are strongly reputation-concerned. We explore how this result depends on the sanction, the probability of detection, the efficiency of the judicial system and the distribution of the private benefit of crime. We show that an appeals process does not necessarily reduce the expected social cost of judicial errors but may induce a perverse effect such that it leads to an increase in the expected social cost of judicial errors.

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1 Introduction

Appeals courts exist in most adjudication systems. They allow for losing party in a lawsuit at the first order to seek reconsideration of their arguments. Several authors have put forward possible justifications for implementing such procedures. For instance, the appeals process may reduce the occurrence of legal errors and enable uniform rules of law to be created and maintained. Without neglecting the second issue, Shavell (1995) has focused his analysis on judicial errors.[1] He argues that appeals process can be viewed as a tool to correct errors. This is the case for two reasons: first, an appeal is likely to be brought if the first-order tribunal is mistaken. Therefore, not only does the appeals process allow for a review of trial court decisions, but it also gives an additionnal information to the appeals court concerning the occurence of errors at the trial court level. Second, the appeals process may help to prevent errors at trial when trial court judges fear reversal. In this case, the appeals process provides trial court judges with incentives to devote more effort to obtain accurate decisions.

In this paper, we focus on this second virtue of the appeals process: we investigate whether this procedure encourages first-order judges to devote more effort to decide cases better than they would in absence of the appeals process. To this end, we develop a model based on Shavell (1995), but we depart from Shavell's framework in several respects. First, we do not study the optimal allocation of state expenditures between

[1] Shavell (2010) explores the social desirability of another aspect of the appeals process: the discretionary court's power to decide to hear an appeal.

the first-order tribunal and the higher tribunal. Instead, we focus on the sole first-order judge's effort, which still remains unobservable. By the way, we consider that the occurence of errors at trial depends only on the judge's effort (and not on the state expenditures). Second, we add on the possibility that judges are sensitive to either the social harm caused by the erroneous decisions they take or to their reputation. Third, we take into account the impact of crime deterrence on trial court judges' effort at equilibrium. To study this second issue, we introduce some heterogeneity in the population of potential criminals concerning their private benefit of crime. Individuals are assumed to choose their action as in the standard model *à la* Polinsky and Shavell (2009), by comparing the expected benefit and cost of committing a crime. Fourth, we introduce both type-I errors (wrongful convictions) and type-II errors (wrongful acquitals) in this setting.

The closest paper to ours is the one of Shavell (1995) since we extend this model. However, other papers have studied the impact of the appeals process on trial court judges' incentives. Some of them assume that the judge is careerist (Levy, 2005; Shavell, 2006) while others assume that the judge is solely motivated by the social welfare (Daughety and Reinganum, 1999, 2000). We depart from this existing literature first by considering a judge that can be both socially-motivated (i.e. sensitive to judicial errors) and career(reputation)-concerned. Intuitively, this permits to qualify Shavell's result: on the one hand establishing an appeals court provides repution-concerned judges with incentives to increase their effort at trial, but on the other hand it also weakens the pressure on socially-motivated judges, encouraging them to exert less effort, since judicial errors are likely to be corrected by the higher court. Further, we show that whether trial court judges exert more or less effort when an appeals process is implemented depends not only on their type but also on the impact of their effort on the relative probability of type-I and type-II errors (which are antagonistic) and on the effect of legal errors on the incentive to obey the law (Png, 1986 ; Polinsky and Shavell, 1989). Finally, our main result is that trial court judges may exert less effort when the appeals process is established, even if they are solely concerned about their reputation. As a consequence, the appeals process may either increase or decrease the expected social cost of legal errors whatever the judge's type.

The paper is organized as follows. Section 2 presents the framework with only type-II legal errors. In section 3, we extend our model to the case where there exists simultaneously type-I and type-II errors. In section 4, we run the welfare analysis. Section 5 concludes.

2 Appeals process, type-II errors and crime deterrence

We consider that judges have enough influence to reduce the probability of legal type-II errors. As the appeals process may also reduce the occurrence of type-II errors, we study the interaction between the judge's choice of a level of effort (in reducing the risk of type-II error) and the correction of errors by the appeals court.

2.1 Framework and assumptions

Following Shavell (1995), we consider that the risk neutral first order judge chooses some effort level $e > 0$, at convex cost $\psi(e)$ in terms of disutility of effort, time, or any other resources. By the way, he may reduce $p(e)$ the likelihood of type-II errors (relaxing a guilty defendant), so that $p'(e) < 0$ and $p''(e) > 0$. At the same time, the judge is paid a gross wage w but suffers a reputational cost or a disutility (z) if he is reversed, reflecting that judges do not appreciate when appeals courts remove/cancel their decision, or that judges are careerist (and that the appeals process is a means of judges' selection based on their performance, Levy, 2005). In addition to Shavell (1995), we introduce two ingredients. First, judges suffer a private loss associated with their own mistakes αh, where h represents the social harm due to judicial errors and α refers to the judge's sensitivity to social harm. By the way, the judge may be either socially-motivated ($\alpha h > z$) or reputation-concerned ($\alpha h < z$) according to whether the judge is more sensitive to either social cost of legal errors or his reputation cost. Second, we consider a continuum of risk neutral agents each defined by a private benefit of crime. We denote the latter b and take it to be distributed on a finite support $V \equiv [0, B]$, with cumulative distribution function $F(.)$. The corresponding density $f(.)$ is continuously differentiable and positive in the interval $(0, B)$, and equal to zero outside this interval. We denote $\varepsilon(b) = \frac{f'(b)}{f(b)}b$ the elasticity of the density, and we assume that:

Assumption 1:

$$\varepsilon(b) < 2\frac{1 - p(e)}{p(e)}$$

where the right-hand term equals two times the odds ratio of the probability of type-II error. This is a sufficient condition to ensure that a maximum exists. In other words, the probability of type-II error must be sufficiently low to deter crime for a given distribution of the private benefit of crime.

Given e, an agent chooses to commit a crime if $b > tS(1 - p(e)) \equiv \tilde{b}_1(e)$, defining a cutoff rule, where S defines the agent's penalty, and t is the probability of detection. If $b < \tilde{b}_1(e)$, then the agent does not become a criminal. The value $\tilde{b}_1(e)$ will be referred to as the borderline type of an effort level e.

In this basic setting, the judge chooses e to solve:

$$\max_e \left\{ u_1(e) = w - \psi(e) - (1 - F(\tilde{b}_1(e)))p(e)\alpha h \right\}.$$

where the last term of $u_1(e)$ represents the expected cost for the judge of wrongly acquitting a guilty defendant, weighted by the probability of crime. Differentiating $u_1(e)$ with respect to e leads to the lemma 1 below.

Lemma 1 *There exists a unique effort level at equilibrium e_1^* defined by:*

$$\psi'(e_1^*) = -\alpha h p'(e_1^*) \left[(1 - F(\tilde{b}_1(e_1^*))) + p(e_1^*) tSf(\tilde{b}_1(e_1^*)) \right]. \tag{1}$$

The first order condition (1) can be rewritten:

$$\underbrace{\psi'(e)}_{C} = \underbrace{-\alpha h p'(e)(1 - F(\tilde{b}_1(e)))}_{B_p^{NA}} \underbrace{-\alpha h p(e) \frac{\partial\ (1 - F(\tilde{b}_1(e)))}{\partial e}}_{B_d^{NA}}.$$

Term C represents the judge's disutility of raising e by one unit. Together, terms B_p^{NA} and B_d^{NA} are the judge's total marginal benefit of raising e by one unit (when there is *no appeals* process) under two effects: the reduction of the probability of type-II errors and crime deterrence. We first explore the term B_p^{NA}: it represents the marginal benefit due to the fact that, by increasing his effort by one unit, the judge reduces the *probability* of type-II errors by $-p'(e)$ units, and by the way reduces his private loss in case of a type-II error by $-\alpha h p'(e)$ units, under the condition that a crime has occured with a probability $(1 - F(\tilde{b}_1(e)))$. Second, term B_d^{NA} captures the *deterrence* effect: by increasing his effort by one unit, the judge reduces the likelihood of crime by $-\frac{\partial\ (1-F(\tilde{b}_1(e)))}{\partial e}$ units and then reduces his own expected loss associated with type-II errors by $-\alpha h p(e)\frac{\partial\ (1-F(\tilde{b}_1(e)))}{\partial e}$ units.

The static comparative results are summarized in Proposition 1 below:

Proposition 1 *An increase in either the sanction (S) or the probability of detection (t) implies a decrease in the trial court judge's effort if and only if*

$$\varepsilon(\tilde{b}_1(e)) < \frac{1 - 2p(e)}{p(e)}. \tag{2}$$

An increase in either the social harm caused by a judicial error (h) or the trial court judge's sensitivity to social harm of errors (α) implies an increase in the trial court judge's effort.

The effect of the judge's private loss associated with a type-II error (αh) on the equilibrium effort is quite intuitive. An increase in it contributes to rise both B_p^{NA} and B_d^{NA}, thus encouraging him to increase his effort in order to avoid type-II errors. The effect of the penalty (S) and the probability of detection (t) on the optimal level of effort is ambiguous. Indeed, a variation of one of these parameters may affect B_p^{NA} and B_d^{NA} in a different way.

First, an increase in the penalty or the probability of detection decreases the probability of crime (by increasing $\tilde{b}_1(e)$). Since the probability of crime becomes lower, B_p^{NA} decreases, which clearly reduces the judge's incentives to increase his effort. Intuitively, the judge has a lower incentive to reduce the probability to commit a type-II error simply because there are fewer crimes. In this sense, the penalty and the probability of detection can be considered as substitutes for the trial court judge's effort.

Second, raising the penalty or the probability of detection contributes to either increase or decrease the marginal benefit of raising e in term of crime deterrence (B_d^{NA}). Indeed, notice that

$$B_d^{NA} = -\alpha h p(e) \frac{\partial \left(1 - F(\tilde{b}_1(e))\right)}{\partial e} = \alpha h p(e) f(\tilde{b}_1(e)) \frac{d\tilde{b}_1(e)}{de} = -\alpha h p(e) f(\tilde{b}_1(e)) tSp'(e) > 0.$$

It appears that the penalty and the probability of detection modify the magnitude of the positive effect of the judge's effort on the probability of crime through two terms $\frac{d\tilde{b}_1(e)}{de}$ and $f(\tilde{b}_1(e))$. On the one side, $\frac{d\tilde{b}_1(e)}{de}$ is increasing with the penalty and the probability of detection. Thus, an increase in one of these two parameters strengthens the positive effect of the judge's effort on crime deterrence. On the other side, $f(\tilde{b}_1(e))$ may either increase or decrease with the penalty or the probability of detection. In particular, if $\varepsilon(b) < 0$, $f(\tilde{b}_1(e))$ decreases when one of these two parameters increases, the judge is thus discouraged to increase his effort. Intuitively, $\varepsilon(b) < 0$ means that due to the distribution of the private benefit of crime, the effect of the judge's effort on deterrence is decreasing when $\tilde{b}_1(e)$ takes higher values, thereby discouraging the judge to increase his effort. The reverse is true if $\varepsilon(b) > 0$. Finally, remark that the impact of the previous parameters is weighted by the probability of type-II error. Therefore, the lower is the probability of type-II error, the lower is the effect of the penalty and the probability of detection on the extent to which the judge's effort affects criminal decision making. Put together, condition (2) resumes all these effects and indicates whether the penalty and the probability of detection can be viewed as complements or substitutes for the judge's effort to reduce the occurrence of type-II errors.

2.2 Appeals process as a means of type-II error correction

When a type-II error occurs, victims have now the right to bring an appeal. For sketch of simplicity, we assume that (i) a plaintiff do not file an appeal if no error has occured at trial and (ii) a plaintiff do file an appeal with a probability of 1 if an error has occured at trial. Following Shavell (1995), we also consider that the appeals court may reverse the trial court's decision with a probability $q \in (0, 1)$ and that this probability is common knowledge. Finally, we replace assumption 1 by:

Assumption 2:

$$\varepsilon(b) < \frac{2[1 - p(e)(1 - q)]}{p(e)(1 - q)} = \overline{\varepsilon}.$$

Under this new setting, the borderline type equals

$$\tilde{b}_2(e) = tS(1 - p(e)(1 - q))$$

where $\tilde{b}_2(e) \geq \tilde{b}_1(e)$. The judge chooses e to solve

$$\max_e \left\{ u_2(e) = w - \psi(e) - (1 - F(\tilde{b}_2(e)))p(e)[(1-q)\alpha h + qz] \right\}.$$

Differentiating $u_2(e)$ with respect to e leads to the lemma 2 below.

Lemma 2 *There exists a unique effort level at equilibrium e_2^* defined by:*

$$\psi'(e_2^*) = -[(1-q)\alpha h + qz]p'(e_2^*)[(1 - F(\tilde{b}_2(e_2^*))) + f(\tilde{b}_2(e_2^*))p(e_2^*)tS(1-q)]. \tag{3}$$

The first order condition can be rewritten:

$$\underbrace{\psi'(e)}_{C} = \underbrace{-\,[(1-q)\alpha h + qz]\,p'(e)(1 - F(\tilde{b}_2(e)))}_{B_p^A} \underbrace{-\,[(1-q)\alpha h + qz]\,p(e)\frac{\partial\,(1 - F(\tilde{b}_2(e)))}{\partial e}}_{B_d^A}$$

where B_p^A and B_d^A are the judge's marginal benefits of raising e by one unit when there is an *appeals* process. The intuition for this result is quite similar to the one for Lemma 1 except that now, we replace the judge's private loss associated with his own errors (αh), by a linear combination of this cost and the judge's reputational cost (z) that is $(1-q)\alpha h + qz$. Moreover, the establishement of an appeals court modifies the probability of crime, which is mathematically reflected by the fact that $\tilde{b}_1$ is replaced by $\tilde{b}_2$. Finally, we also drive the static comparative analysis as follows:

Proposition 2 *An increase in either the sanction (S) or the probability of detection (t) implies a decrease in the trial court judge's effort if and only if*

$$\varepsilon(\tilde{b}_2(e)) < \frac{1 - 2p(e)(1-q)}{p(e)(1-q)}.$$

An increase in either the social harm of judgement errors (h) or the trial court judge's sensitivity to social harm of errors (α) implies an increase in the trial court judge's effort.

The intuition for this proposition is quite similar to the intuition for Proposition 1. First, increasing the penalty (t) or the probability of detection (S) makes the judicial system more deterrent. Since the probability of crime is lower (or B_p^A is lower), the judge has less incentives to exert an effort. This effect, which is also at stake when there is no appeals process, is reinforced when there is an appeals process. Indeed, an increase in the penalty or the probability of detection is even more deterrent since a potential type-II error at first instance may be corrected subsequently. Second, increasing the penalty or the probability of detection may still encourage or discourage the judge to exert more effort according to the distribution of the private benefit of crime.

2.3 Appeals process, judge's incentives and type-II errors

Now, we explore the conditions under which the appeals process may increase (or not) the judge's effort according to the judge's type (socially-motivated or reputation-concerned) when we take into account crime deterrence. Following Shavell (1995) (i.e. without any crime deterrence nor judge's sentivity to legal errors), the appeals process (in fact the reputational cost associated with it) quite naturally gives judges a strong incentive to increase their effort at the margin. Here, the reverse may be true for two reasons. First, the judge is not solely concerned about his reputation: he may also be sensitive to errors. Second, we show that the deterrence issue also tends to qualify Shavell's result. Our results are summarized in the proposition below:

Proposition 3 *Assume that there does exist only type-II errors: (i) if the judge is socially-motivated ($\alpha h - z > 0$) then establishing an appeals court leads to a decrease in the trial court judge's equilibrium effort ; (ii) if the judge is reputation-concerned ($\alpha h - z < 0$) then establishing an appeals court has an ambiguous effect on the trial court judge's equilibrium effort. But, the higher is the risk of type-II error ($p(.)$), the probability of detection (t), the penalty (S), and the term $\left(\frac{\overline{\varepsilon}-\varepsilon(\tilde{b}_2(.))}{\overline{\varepsilon}}\right)$, the higher is the likelihood that the introduction of an appeals process induces a decrease in the trial court judge's equilibrium effort.*

To better understand the intuition of Proposition 3, notice that the appeals process leads to a decrease in the judge's effort if (but not only if) $\Delta B_p = B_p^A - B_p^{NA} < 0$ and $\Delta B_d = B_d^A - B_d^{NA} < 0$. Intuitively, if the appeals process reduces the marginal benefits of raising e by one unit (associated with the reduction of the *probability* of type-II errors ($\Delta B_p < 0$) and *deterrence* ($\Delta B_d < 0$)) then the judge will reduce his effort at equilibrium.

Let us assume that the judge is socially motivated (that is $\alpha h > z$ or equivalently $\alpha h > (1-q)\alpha h + qz$). As a consequence, by correcting type-II errors, the appeals process reduces the judge's private loss associated with his own errors. First, we explore ΔB_p i.e. the way that the appeals process affects the judge's marginal benefit resulting from the fact that, by increasing his effort, the judge reduces the probability of type-II error. We know that B_p^A and B_p^{NA} depend on two elements: the judge's loss associated with a type-II error (αh in B_p^{NA} and $(1-q)\alpha h + qz$ in B_p^A) and the marginal effect of the judge's effort on the probability of type-II error weighted by the probability of crime at the threshold value ($p'(e)(1 - F(\tilde{b}_1(e))$ in B_p^{NA} and $p'(e)(1 - F(\tilde{b}_2(e))$ in B_p^A). We know also that the appeals process allows for a reduction of the probability of crime for a given effort ($(1 - F(\tilde{b}_2(e)) < (1 - F(\tilde{b}_1(e)))$). As a consequence, if the judge is rather socially-motivated ($\alpha h > z$) then $\Delta B_p = B_p^A - B_p^{NA} < 0$. The decrease in the marginal benefit of his effort (in terms of the reduction of the probability of type-II errors) encourages the judge to reduce his effort when the appeals process is introduced.

Now, we study ΔB_d i.e. the way that the appeals process affects the judge's marginal benefit resulting from the fact that, by increasing his effort by one unit, the judge deters crime at the margin. As before, the appeals process reduces the judge's loss associated with a type-II error. Further, B_d^{NA} and B_d^A depend on the marginal effect of the judge's effort on the probability of crime weighted by the probability of type-II error ($-p(e)\frac{\partial\ (1-F(\tilde{b}_1(e)))}{\partial e}$ in B_d^{NA} and $-p(e)\frac{\partial\ (1-F(\tilde{b}_2(e)))}{\partial e}$ in B_d^A). We show in appendix (proof of Remark 1) that when $\varepsilon(b) < 0$ (or $\varepsilon(b)$ weakly positive), the appeals process reduces the marginal effect of the judge's effort on the probability of crime (that is $\frac{\partial\ (1-F(\tilde{b}_1(e)))}{\partial e} < \frac{\partial\ (1-F(\tilde{b}_2(e)))}{\partial e}$). In such a case, we have $\Delta B_d = B_d^A - B_d^{NA} < 0$ when the judge is socially-motivated ($\alpha h > z$). Such a decrease in the marginal benefit of effort in terms of deterrence again encourages the judge to decrease his effort.

To sum up, a socially-motivated judge has an incentive to reduce his effort when the appeal process is introduced (Proposition 3(i)) because the total marginal benefit of effort ($B_p^A + B_d^A < B_p^{NA} + B_d^{NA}$) is reduced for three reasons. First, the judge's expected loss associated with a type-II error is lower. Second, the appeals process deters crime (recall that the lower is the probability of crime, the less the judge is exposed to a type-II error). Third, because the appeals process reduces the deterrence power of the judge's marginal effort as long as crimes do not bring criminals too much benefit.

Now let us assume that the judge is reputation-concerned (that is $\alpha h < z$ or equivalently $\alpha h < (1 - q)\alpha h + qz$). The sole difference with the previous reasonning is that the appeals process now contributes to increase the judge's private loss associated with type-II errors. So, holding everything else constant, this should encourage the judge to increase his effort. We can easily verify that the effects of the appeals process on both the probability of crime ($(1 - F(\tilde{b}_2(e)) - (1 - F(\tilde{b}_1(e)) > 0$) and the deterrent effect of the judge's effort ($\frac{\partial\ (1-F(\tilde{b}_1(e)))}{\partial e} - \frac{\partial\ (1-F(\tilde{b}_2(e)))}{\partial e} < 0$) are unchanged. As a consequence, when the judge is rather reputation-concerned ($\alpha h < z$), the signs of ΔB_d and ΔB_p are undetermined (see the first part of Proposition 3ii). However, it is possible to derive some sufficient conditions under which the introduction of an appeals process leads to a decrease of the judge's effort (see the second part of Proposition 3ii). Intuitively, we have to identify some factors which have a negative impact on ΔB_p and ΔB_d in order to compensate for the positive impact of the increase in the judge's private loss associated with type-II errors on the same terms ΔB_p and ΔB_d. As ΔB_p is decreasing with the probability of detection (t), the penalty (S) and the risk of type-II error ($p(.)$), then high values of these parameters contribute to reinforce the effect of the implementation of the appeals process on the probability of crime. Further, ΔB_d is also decreasing with the probability of detection (t) and the penalty (S). Indeed, we know that the appeals process reduces the effect of the judge's effort on crime deterrence at the margin : it is all the more true that the probability of detection (t) and the penalty (S) take high values. Finally, notice that the negative effect of the probability of detection (t), the penalty (S) and the risk of type-II error ($p(.)$) on ΔB_p and/or ΔB_d is all the stronger that $\varepsilon(b)$ is negative or weakly positive: indeed, in such cases, the deterrent effect of the judge's effort decreases when $\tilde{b}$ increases, which is the case when the appeals process is introduced.

To resume, in Shavell's setting (1995), if the representative judge is only reputation-concerned, then implementing an appeals process creates an incentive for him to increase his effort due to his disutility of being overturned. In our setting, if the judge is still reputation-concerned then the appeals process may now discourage him to exert more effort because his disutility of being overturned may be counterbalanced by the two following effects. First, the appeals process reduces the probability of crime (whatever the judge's effort) and by the way the occurence of a type-II error. Second, the appeals process diminishes the judge's capacity to deter crime through its effort's impact on the probability of type-II error at the margin. Finally, if the judge is socially-motivated (that is, rather sensitive to the reduction of social cost of errors than to his reputation cost), he has an additional incentive to reduce his effort. Not only does the judge reduces his effort due to the two effects depicted above, but he also relies on the appeals court to correct his mistakes.

3 Appeals process, type-I/type-II errors and crime deterrence

In this section, we check the robustness of our results by assuming that there now exists both type-I and type-II legal errors. We first present the case in which there is no appeals process.

3.1 Framework and assumptions

Up to now, by choosing his effort e, the judge also influences the probability of type-I error denoted $\beta(e)$ with $\beta'(e) > 0$ and $\beta''(e) > 0$ meaning that the risks of type-I and type-II errors are wholly antagonistic. Further we assume that the probability of type-II error is superior to the probability of type-I error because of the high standard of evidence that is usually required in criminal procedure (Rizolli and Saraceno, 2013):

Assumption 3: $p(e) > \beta(e)$.

Under this setting, the threshold value of the benefit of crime equals:

$$\tilde{b}_3(e) = tS[1 - p(e) - \beta(e)].$$

Since $\tilde{b}_3(e) \leq \tilde{b}_1(e)$ the probability of crime becomes higher when innocents may be wrongly convicted. This is consistent with the conventional result of the theory of the public enforcement of law stating that wrongful convictions of innocents are detrimental to deterrence (Garoupa and Rizzolli, 2012). We also control for the fact that an increase in the judge's effort has a deterrent effect. It means analytically that $\frac{d\tilde{b}_3(e)}{de} = -tS(\beta'(e) + p'(e)) > 0$ or:

Assumption 4:

$$-\frac{p'(e)}{\beta'(e)} > 1.$$

We interpret the ratio $-\frac{p'(e)}{\beta'(e)}$ as a measure of the trial court efficiency. Indeed, recall that an increase of the judge's effort by one unit leads to a decrease of the probability of type-II error by $-p'(e)$ units and an increase of the probability of type-I error by $\beta'(e)$ units. As a consequence, $\frac{-p'(e)}{\beta'(e)}$ indicates how much larger the effect of the judge's effort on the probability of type-II error is compared to the effect of judge's effort on the probability of type-I error. In other words, it means that an increase of the judge's effort by one unit leads to a decrease of the probability of type-II error $\frac{-p'(e)}{\beta'(e)}$ times higher than the corresponding increase of the probability of type-I error. Finally, a sufficient condition for our problem's concavity is:

Assumption 5:

$$\varepsilon(b) < \left(\frac{2(p'(e) - \beta'(e))}{(p'(e) + \beta'(e))}\right)\left(\frac{1 - (p(e) + \beta(e))}{(p(e) - \beta(e))}\right).$$

Under these assumptions, the judge chooses e to solve

$$\max_e \left\{u_3(e) = w - \psi(e) - (1 - F(\tilde{b}_3(e)))p(e)\alpha h - F(\tilde{b}_3(e))\beta(e)\alpha h\right\}.$$

Remark that in this setting, neither the social harm of judgement errors (h) nor the trial court judge's sensitivity to social harm of errors (α) differ from type-I errors to type-II errors. Differentiating $u_3(e)$ with respect to e leads to the lemma 3 below.

Lemma 3 *There exists a unique effort level at equilibrium e_3^* defined by:*

$$\psi'(e_3^*) = \alpha h\left(-p'(e_3^*)[1 - F(\tilde{b}_3(e_3^*))] - \beta'(e_3^*)F(\tilde{b}_3(e_3^*)) - f(\tilde{b}_3(e_3^*))tS[p'(e_3^*) + \beta'(e_3^*)][p(e_3^*) - \beta(e_3^*)]\right). \quad (4)$$

The first order condition can be rewritten:

$$\underbrace{\psi'(e)}_{C} + \underbrace{\alpha h F(\tilde{b}_3(e))\beta'(e)}_{C_p^{NA}} + \underbrace{\alpha h\beta(e)\frac{\partial F(\tilde{b}_3(e))}{\partial e}}_{C_d^{NA}} = \underbrace{-\alpha h p'(e)(1 - F(\tilde{b}_3(e)))}_{B_p^{NA}}\underbrace{-\alpha h p(e)\frac{\partial(1 - F(\tilde{b}_3(e)))}{\partial e}}_{B_d^{NA}}. \quad (5)$$

As in section 2.1., the first-order condition highlights the trade-off faced by the trial court judge between the marginal cost (on the left-hand side of equation 5) and the marginal benefit (on the right-hand side of equation 5) of raising e by one unit. The two components of the marginal benefit of the judge's effort (B_p^{NA} and B_d^{NA}) can be interpreted in a manner similar to the case with one type of errors (see the interpretation following Lemma 1). The marginal cost is composed of three terms. First, as previously, the judge is subject to a disutility of effort which is reflected in C. The second term (C_p^{NA}) is the marginal cost resulting from the fact that the probability of type-I errors increases by $\beta'(e)$ units as the judge increases his effort by one unit. It induces a loss equals to $\alpha h\beta'(e)$ weighted by the probability that no crime has been committed $F(\tilde{b}_3(e))$. The third term (C_d^{NA}) captures the fact that, by increasing his effort by one unit, the judge increases the likelihood of not committing a crime by $\frac{\partial F(\tilde{b}_3(e))}{\partial e}$ which increases his expected loss associated with type-I errors by $\alpha h\beta(e)\frac{\partial F(\tilde{b}_3(e))}{\partial e}$ units.

The results of the comparative statics analysis are summarized in the proposition below.

Proposition 4 *Assume that there does exist both type-I and type-II judicial errors. An increase in either the sanction (S) or the probability of detection (t) implies a decrease in the trial court judge's effort if and only if*

$$\varepsilon(\tilde{b}_3(e)) < \frac{(p'(e) - \beta'(e)) + 2(\beta'(e)\beta(e) - p'(e)p(e))}{(p'(e) + \beta'(e))(p(e) - \beta(e))}.$$

An increase in either the social harm due to a judicial errors (h) or the trial court judge's sensitivity to social harm of errors (α) implies an increase in the trial court judge's effort.

As in the framework with type-II errors, an increase in either the social harm due to judicial errors or the judge's sensitivity to social harm of errors encourages the judge to exert more effort. An increase in the penalty or the probability of detection can still either encourage or discourage the judge to exert more effort according to the comparison between the elasticity of the density of the private benefit of crime and a term depending on the probabilities of errors and the efficiency of the trial court. Thus, according to the magnitude of this elasticity, the effect of an increase in the penalty or the probability of detection on $B_p^{NA} - C_p^{NA}$ will either reinforce (if the elasticity is highly negative) or counterbalance (if the elasticity is positive or weakly negative) the negative effect of an increase in the penalty or the probability of detection on $B_p^{NA} - C_p^{NA}$.

3.2 Appeals process as a means of type-I and type-II error correction

When the trial court judge makes a mistake (whatever the nature of error), the loosing party now brings an appeal. The appeals process then corrects mistakes that have been made at trial, and for sketch of simplicity we assume that the probability of correcting legal errors q does not depend on the nature of errors. As a consequence, the threshold value of the benefit of crime equals:

$$\tilde{b}_4(e) = tS[1 - (1-q)(\beta(e) + p(e))].$$

Notice that $\tilde{b}_4(e) > \tilde{b}_3(e)$, which means that the probability of crime has decreased with the establishment of an appeals court. The judge chooses e to solve

$$\max_e \left\{ u_4(e) = w - \psi(e) - (1 - F(\tilde{b}_4(e)))p(e)((1-q)\alpha h + qz) - F(\tilde{b}_4(e))\beta(e)((1-q)\alpha h + qz) \right\}$$

under assumptions 3 and 4. We replace assumption 5 by assumption 6 below in order to control for our problem's concavity.

Assumption 6:

$$\varepsilon(b) < \left(\frac{2(p'(e) - \beta'(e))}{(p'(e) + \beta'(e))} \right) \left(\frac{1 - (p(e) + \beta(e))(1-q)}{(p(e) - \beta(e))(1-q)} \right) \equiv \bar{\varepsilon}.$$

Lemma 4 *There exists a unique effort level at equilibrium e_4^* defined by:*

$$\begin{aligned} \psi'(e_4^*) \quad = \quad & f(\tilde{b}_4(e_4^*)))(-tS)(1-q)(\beta'(e_4^*) + p'(e_4^*))[p(e_4^*) - \beta(e_4^*)][(1-q)\alpha h + qz] - \\ & [(1-q)\alpha h + qz][(1 - F(\tilde{b}_4(e_4^*)))p'(e_4^*) + F(\tilde{b}_4(e_4^*))\beta'(e_4^*)] \end{aligned} \quad (6)$$

First order condition (4) may be rewritten as follows:

$$\begin{aligned} & \underbrace{\psi'(e)}_{C} + \underbrace{((1-q)\alpha h + qz)F(\tilde{b}_4(e))\beta'(e)}_{C_p^A} + \underbrace{((1-q)\alpha h + qz)\beta(e)\frac{\partial F(\tilde{b}_4(e))}{\partial e}}_{C_d^A} \\ = \quad & \underbrace{-((1-q)\alpha h + qz)p'(e)(1 - F(\tilde{b}_4(e)))}_{B_p^A} \underbrace{-((1-q)\alpha h + qz)p(e)\frac{\partial(1 - F(\tilde{b}_4(e)))}{\partial e}}_{B_d^A}. \end{aligned}$$

The intuition for Lemma 4 is quite similar to the one for Lemma 3. When he chooses his effort, the judge faces a trade-off: on the one hand, an additional effort contributes to reduce the probability of type-II errors and the probability of crime. This reduces the cost of type-II errors inducing two marginal benefits (B_p^A and

B_d^A). On the other hand, it increases the cost of type-I errors because the probability of type-I errors is higher and the probability of not committing a crime higher. It induces two marginal costs (C_p^A and C_d^A). Below, we show that the comparative statics analysis is similar to the one exposed in Proposition 4.

Proposition 5 *Assume that there does exist both type-I and type-II judicial errors. An increase in either the sanction (S) or the probability of detection (t) implies a decrease in the trial court judge's effort if and only if*

$$\varepsilon(\tilde{b}_4(e)) < \frac{(p'(e) - \beta'(e)) + 2(1-q)(\beta'(e)\beta(e) - p'(e)p(e))}{(p'(e) + \beta'(e))(p(e) - \beta(e))(1-q)}.$$

An increase in either the social harm due to a judicial errors (h) or the trial court judge's sensitivity to social harm of errors (α) implies an increase in the trial court judge's effort.

3.3 Appeals process, judge's incentives and type-I/type-II errors

As in the framework with only type-II errors, we discuss the effect of the appeals process on judge's incentives. We resume our results in the proposition below.

Proposition 6 *Assume that there does exist both type-I and type-II errors. Establishing an appeals court has an ambiguous effect on the trial court judge's equilibrium effort (whatever the judge's type). But, the higher is the gap between the probability of type-II error and the probability of type-I error ($p(e) - \beta(e)$), the probability of detection (t), the penalty (S), the term $\left(\frac{\bar{\varepsilon}-\varepsilon}{\bar{\varepsilon}}\right)$, and the lower is the efficiency of the trial court (measured by $-\frac{p'(e)}{\beta'(e)}$), the higher is the likelihood that the introduction of an appeals process induces a decrease in the trial court judge's equilibrium effort.*

Proposition 6 shows that the effect of the appeals process on the trial court judge's incentives is now ambiguous whatever the judge's type. The reason is that the appeals process has an influence on the judge's effort at three levels: (i) the judge's private loss associated with a legal error (the appeals process may either increase or decrease this loss according to the judge's type, (ii) the effect of the judge's effort on the reduction in the probability of type-II error given the corresponding increase in the probability of type-I error (for a given value of probability of crime), and (iii) the effect of the judge's effort on crime deterrence for a given probability of type-I and type-II error.

In order to analyze the effect of the appeals process on the trial court judge's incentives, we now study the changes in the marginal benefits and costs due to the introduction of an appeals process. First of all, the explanation regarding

$$\Delta B_p = B_p^A - B_p^{NA} = p'(e)\left(-((1-q)\alpha h + qz)(1 - F(\tilde{b}_4(e))) - (-\alpha h(1 - F(\tilde{b}_3(e))))\right)$$

and

$$\Delta B_d = B_d^A - B_d^{NA} = p(e)\left(-((1-q)\alpha h + qz)\frac{\partial(1 - F(\tilde{b}_4(e)))}{\partial e} - (-\alpha h\frac{\partial(1 - F(\tilde{b}_3(e)))}{\partial e})\right)$$

is identical to the one exposed in section 2.3. For summary, whatever the judge's type, the appeals process diminishes both the occurence of type-II errors (by deterring crime) and the judge's capacity to deter crime at the margin. As a consequence, the judge is rather motivated to reduce his costly effort. It is all the more true than the judge is socially-motivated because in such a case the appeals process reduces also his private loss associated with a type-II error. If the judge is rather reputation-concerned then the reverse may be true as the judge suffers a disutility when he is overturned at appeal.

The introduction of type-I errors in our setting generates two additional costs (in terms of crime deterrence and probability of legal errors) which are also influenced by the appeals process. To be more precise, we successively analyse the variations in the marginal costs

$$\Delta C_p = C_p^A - C_p^{NA} = \beta'(e)\left(((1-q)\alpha h + qz)F(\tilde{b}_4(e)) - \alpha h F(\tilde{b}_3(e))\right)$$

and

$$\Delta C_d = C_d^A - C_d^{NA} = \beta(e)\left(((1-q)\alpha h + qz)\frac{\partial F(\tilde{b}_4(e))}{\partial e} - \alpha h \frac{\partial F(\tilde{b}_3(e))}{\partial e}\right)$$

for each judge's type.

To start with, we assume that the judge is socially motivated ($\alpha h > z$) and we explore the sign of ΔC_p. Two opposite effects are at stake. On the one side, due to the deterrent effect of the appeals process, we know that the probability of crime is lower for a given level of effort ($F(\tilde{b}_4(e)) > F(\tilde{b}_3(e))$). It increases the expected cost of convicting an innocent person thus encouraging the judge to reduce his effort (that leads to increase the probability of a type-II error). On the other side, the judge's private loss associated with his own type-I errors is lower ($(1-q)\alpha h + qz < \alpha h$). This is due to the fact that the appeals process is a means of error correction thereby encouraging the judge to increase his effort. Put together, the sign of ΔC_p is ambiguous. We now explore the sign of ΔC_d. As before, the judge's private loss associated with type-II errors encourages him to increase his effort (because $(1-q)\alpha h + qz < \alpha h$). Further, the appeals process reduces the adverse effect of the judge's marginal effort on the probability of not committing a crime ($\frac{\partial F(\tilde{b}_4)}{\partial e} < \frac{\partial F(\tilde{b}_3)}{\partial e}$)[2]. As a result, ΔC_d is unambiguously negative which encourages the judge to increase his effort.

Now let us assume that the judge is rather reputation-concerned ($\alpha h < z$). We have $\Delta C_p > 0$ because not only is the probability of crime lower due to the appeals process (meaning that the probability facing an innocent person is higher) but also the judge's private loss associated with a type-I error is higher (due to the reversal cost in case of appeals). Further, ΔC_d may be either positive or negative. The reduction of the marginal effect of the judge's effort on the probability of crime (if ε is small enough) makes ΔC_d higher but this effect is counterbalanced by the fact that the judge's private loss associated with a type-I error increases. We resume these results in table 1 below.

	Type of the judge	
	reputation-concerned	socially-motivated
ΔB_p	$+$ or $-$	$-$
ΔC_p	$+$	$+$ or $-$
ΔB_d	$+$ or $-$	$-$
ΔC_d	$+$ or $-$	$-$

Table 1: study of the sign of ΔB_p, ΔC_p, ΔB_d and ΔC_d according to the judge's type.

Finally, remark that the appeals process encourages the judge to reduce his effort whatever his type if $\Delta C_d > \Delta B_d$ and $\Delta C_p > \Delta B_p$.

The condition $\Delta C_d > \Delta B_d$ is equivalent to:

$$\beta(e)\left(((1-q)\alpha h + qz)\frac{\partial F(\tilde{b}_4(e))}{\partial e} - \alpha h \frac{\partial F(\tilde{b}_3(e))}{\partial e}\right) > p(e)\left(((1-q)\alpha h + qz)\frac{\partial F(\tilde{b}_4(e))}{\partial e} - \alpha h \frac{\partial F(\tilde{b}_3(e))}{\partial e}\right).$$

[2] We show in appendix that if $\varepsilon(b)$ is small enough, then $\frac{\partial F(\tilde{b}_4)}{\partial e} < \frac{\partial F(\tilde{b}_3)}{\partial e}$.

Recall that a lower judge's capacity to deter crime ($\frac{\partial F(\tilde{b}_4)}{\partial e} < \frac{\partial F(\tilde{b}_3)}{\partial e}$) has two effects. On the one side, for a given risk of type-II error ($p(e)$), the judge is less motivated to reduce the risk to commit a type-II error. On the other side, for a given risk to commit a type-I error ($\beta(e)$), the judge is rather motivated to reduce the risk to commit a type-II error because the opportunity cost to increase his effort at the margin (in term of type-I error) is also reduced by the appeals process. Put together, if we consider solely the marginal effect on crime deterrence (i.e. the inequality $\Delta C_d > \Delta B_d$), the higher is the risk of type-II error relatively to the risk of type-I error (or the higher is the difference $p(e) - \beta(e)$), the higher is the judge's incentive to reduce his effort due to the presence of an appeals process (see the second part of Proposition 6).

The second condition $\Delta C_p > \Delta B_p$ is equivalent to:

$$((1-q)\alpha h + qz)\left((1 - F(\tilde{b}_4(e)))p'(e) + F(\tilde{b}_4(e))\beta'(e)\right) > \alpha h\left((1 - F(\tilde{b}_3(e)))p'(e) + F(\tilde{b}_3(e))\beta'(e)\right).$$

Now recall that under assumption 4 ($p'(e) + \beta'(e) < 0$), a one-unit increase of the judge's effort leads to a reduction of the probability of type-II error which is higher than the corresponding increase of the probability of type-I error (as explained before, it is a sufficient condition to ensure that the judge's effort deters crime). As a consequence, if the judge increases his effort by one unit, then the judge (i) reduces his private expected cost of type-II error by "$-p'(e)(1-F(\tilde{b}(e)))$ times the private cost of error" units and (ii) increases his private expected cost of type-I error by "$\beta'(e)F(\tilde{b}(e))$ times the private cost of error" units. Put together, it generates a (positive or negative) variation of the private expected cost of errors which equals "$\left(p'(e)(1 - F(\tilde{b}(e))) + \beta'(e)F(\tilde{b}(e))\right)$ times the judge's private cost of error" because we assume that the judge's cost does not depend on the nature of error. Further, as the appeals process deters crime ($\tilde{b}_4(e) > \tilde{b}_3(e)$), the appeals process reduces the judge's incentive to exert an effort at the first level for a given value of the judge's cost of error. Here we do as if the judge's loss in case of error remains the same even if we introduce an appeals: $-p'(e)(1 - F(\tilde{b}_4(e))) < -p'(e)(1 - F(\tilde{b}_3(e)))$ and $\beta'(e)F(\tilde{b}_4(e)) > \beta'(e)F(\tilde{b}_3(e))$, implying that $p'(e)(1 - F(\tilde{b}_4(e))) + \beta'(e)F(\tilde{b}_4(e)) > p'(e)(1 - F(\tilde{b}_3(e))) + \beta'(e)F(\tilde{b}_3(e))$. Finally, we can show that this effect is all the higher that the appeals process increases the judge's private cost of error.

Based on the preceeding reasoning, we are now able to discuss the judge's incentives according to his type.

First, when the judge is socially-motivated, we have $\Delta C_d > \Delta B_d$, thus encouraging the judge to reduce his effort. This effect due to crime deterrence is all the larger that the gap between the two probabilities of errors $p(e) - \beta(e)$ is wide. However, the effect due to the reduction of the probability of errors is unclear ($\Delta C_p \gtrless \Delta B_p$) because the appeals process distorts the probability of crime through $\frac{F(\tilde{b}_4(e))}{(1-F(\tilde{b}_4(e)))}$ for specific values of the probability of reversal q and the trial court efficiency $-\frac{p'(e)}{\beta'(e)}$. Notice that if the appeals process is sufficiently deterrent (that is, the difference $\tilde{b}_4(e) - \tilde{b}_3(e)$ is sufficiently large) then it is more likely that $\Delta C_p > \Delta B_p$ thereby encouraging the judge to reduce his effort.

Second, when the judge is reputation-concerned, we have $\Delta C_d \gtrless \Delta B_d$, thus encouraging or discouraging the judge to increase his effort. Whatever it is positive or negative, this effect due to crime deterrence is all the larger that the gap between the two probabilities of errors is wide. Regarding the effect due to the reduction of the probability of errors on the judge's incentive, the appeals process encourages the judge to reduce his effort ($\Delta C_p > \Delta B_p$) if $\left((1 - F(\tilde{b}_4(e)))p'(e) + F(\tilde{b}_4(e))\beta'(e)\right) > 0$ or equivalently

$$-\frac{p'(e)}{\beta'(e)} < \frac{F(\tilde{b}_4(e))}{(1 - F(\tilde{b}_4(e)))}.$$

In some sense, the more the trial court is inefficient and the crime is unprofitable, the more the reputation-concerned judge is encourage to reduce his effort.

To resume, recall that in Shavell's seminal paper, the judge is only reputation-concerned and there is neither type-I errors nor crime deterrence. Therefore, the judge is supposed to increase his effort when the appeals process is introduced because he wants to avoid a reversal. In Proposition 3, we have shown that adding on the fact that reputation-concerned judges can internalize the impact of both their own effort and the appeals process on the probability of crime qualifes this result. More precisely, a reputation-concerned judge is more likely to reduce his effort when the probability of type-II errors, the probability of detection and the penalty are high and when crimes do not bring criminals too much benefit. We also have shown that a socially-motivated judge reduces his effort because the appeals process corrects his own judicial errors.

When we simultaneously consider type-I and type-II errors, new results appear. First, a socially-motivated judge may increase his effort if the appeals process is not too much deterrent. Indeed, in this case, an increase in the judge's effort allows for a reduction of type-II errors without inducing too much adverse effect in terms of type-I errors, since the probability of facing an innocent person has not increased too much with the appeals process. This effect is all the larger that the probability of type-II error is superior to the probability of type-I error. Second, a reputation-concerned judge may still decrease his effort under the condition that a one unit decrease in the judge's effort leads to a decrease in type-I errors larger than the corresponding increase in type-II errors, that is, if the legal system is sufficiently efficient.

4 Welfare analysis

To measure social welfare implications, we supplement the previous analysis with the calculus of the expected social costs of legal errors in two cases: with only type-II errors and with type-II and type-I errors.

4.1 Appeals process and expected social cost of type-II errors

We compare the expected social cost of type-II errors without any appeals process (namely $ESCE_1(e_1^*)$ at equilibrium) with the expected social cost of type-II errors in presence of an appeals process ($ESCE_2(e_2^*)$ at equilibrium) where $ESCE_1(e_1^*) = [1 - F(\widetilde{b_1}(e_1^*))]p(e_1^*)h$ and $ESCE_2(e_2^*) = [1 - F(\widetilde{b_2}(e_2^*))]p(e_2^*)(1-q)h$. By the way, we explain whether and how appeals process may reduce the expected social cost of legal errors when we take into account not only the effect of the appeals process on the correction of legal errors, but also its effect on the judge's incentives and crime deterrence.

Logically, if $q = 0$ we have $e_1^* = e_2^*$ and $ESCE_1(e_1^*) = ESCE_2(e_2^*)$. Moreover, we know that $p'(e) < 0$, $\widetilde{b_2}(e)$ is increasing with q for given values of e, and $\frac{d\widetilde{b}(e)}{de} > 0$. It means that the parameter q (that is the appeals process) intervenes at three levels on the expected social cost of type-II errors. First, there exists a direct effect on the correction of legal errors (that is the term $1 - q$). Second, there exists an indirect effect (i.e. through the judge's effort) on the probability of type-II error (that is the term $p(e)$). Third, the appeals process influences the crime deterrence through simultaneously the correction of errors and the judge's effort (that is the term $1 - F(\widetilde{b}(e^*))$).

As a consequence, the appeals process, by correcting legal errors, directly reduces the expected social cost of errors. Further, a one-unit increase in the judge's effort reduces the risk of type-II error. Finally, an increase in either the judge's effort or the probability to correct errors in appeals has a positive effect on crime deterrence. This reduction of the probability of crime then reduces the risk to commit a type-II error. However, it is not sufficient to conclude that the appeals process unambigously reduces the expected social cost of type-II errors. Indeed, we show in Proposition 3 that the appeals process may either discourage or encourage the first order judge to increase his effort at equilibrium. So, even if we consider only type-II errors, the appeals process has an ambiguous effect on the expected social cost of type-II errors, especially when the judge is socially-motivated.

4.2 Appeals process and expected social cost of type-I/type-II errors

We compare the expected social cost of type-I and type-II errors without any appeals process (namely $ESCE_3(e_3^*)$ at equilibrium) with the expected social cost of type-I and type-II errors in presence of an appeals process ($ESCE_4(e_4^*)$ at equilibrium) where $ESCE_3(e_3^*) = \Big([1 - F(\widetilde{b_3}(e_3^*))]p(e_3^*) + F(\widetilde{b_3}(e_3^*))\beta(e_3^*)\Big)\, h$ and $ESCE_4(e_4^*) = \Big([1 - F(\widetilde{b_4}(e_4^*))]p(e_4^*) + F(\widetilde{b_4}(e_4^*))\beta(e_4^*)\Big)\,(1-q)h$.

If $q = 0$ then $e_3^* = e_4^*$ and $ESCE_3(e_3^*) = ESCE_4(e_4^*)$. Further, we have $p'(e) < 0$, $\beta'(e) > 0$, $\widetilde{b_2}(e)$ is increasing with q for given values of e, and $\frac{d\widetilde{b}(e)}{de} > 0$. In comparison to the previous case (with only type-II errors), the appeals process influences the expected social cost of legal errors at two additional levels. First, it indirectly (i.e. through its impact on the judge's effort) determines the risk of type-I error (that is the term $\beta(e)$). Second, it has an influence on the probability that the judge faces an innocent person ($F(\widetilde{b}(e^*))$), i.e. the necessary condition to commit a type-I error, either directly (via q) or indirectly (via the judge's effort).

According to Proposition 6, we know that the appeals process may either encourage or discourage the judge to increase his effort. As a consequence, the effect of the appeals process on the expected social cost of errors is still unclear. As in the previous case, an increase of the judge's effort simultaneoulsy reduces the probability of type-II error and deters crime (and by the way reduces also the risk to commit a type-II error). But now, the same variation in the judge's effort increases the risk to commit a type-I error not only because risks are antagonistic but also because a lower probability of crime means a higher probability to commit a type-I error.

5 Conclusion

Our paper suggests that we shall add nuance to the result that the existence of an appeals process leads judges to devote a higher effort to reduce the occurrence of type-II errors than they would in the absence of such a process. Following Shavell (1995), we first have considered the case in which only type-II errors may occur. We have shown that if a representative judge is more sensitive to this type of errors than to his repulation, he will decrease his effort and relies on the appeals court to reduce the risk of type-II errors. Then, we have found that the judge may also reduce his effort even if he's strongly motivated to preserve his reputation and to avoid reversal, and that it is all the more true that the judicial system strongly punishes crime. Further, when simultaneously considering type-I and type-II errors, we have shown that whatever the judge's type (concerned by errors or by their reputation), the appeals process may either encourage or discourage them to reduce the risk of type-II error. As a consequence, by considering the impact of an appeals process on crime deterrence and judge's incentives, it appears that an appeals process does not necessarily reduce the expected social cost of judicial errors but may induce a perverse effect such that it leads to an increase in the expected social cost of judicial errors.

Appendix

Proof of Lemma 1. Differentiating $u_1(e)$ with respect to e shows that $u_1'(e)$ is equal to

$$-\psi'(e) - \alpha h p'(e)\left[(1 - F(\tilde{b}_1(e))) + p(e)tSf(\tilde{b}_1(e))\right].$$

We know that $\psi''(e) > 0$, $p''(e) > 0$, $-\alpha h p'(e) > 0$, and $\left[(1 - F(\tilde{b}_1(e))) + p(e)tSf(\tilde{b}_1(e))\right] > 0$. This implies that a sufficient condition for having $u''(e) < 0$ is

$$\frac{\partial((1 - F(\tilde{b}_1(e))) + p(e)tSf(\tilde{b}_1(e)))}{\partial e} < 0.$$

The inequality above is equivalent to

$$tSp'(e)\left[-f'(\tilde{b}_1(e))p(e)tS + 2f(\tilde{b}_1(e))\right] < 0.$$

As $tSp'(e) < 0$, we need to have $\left[-f'(\tilde{b}_1(e))p(e)tS + 2f(\tilde{b}_1(e))\right] > 0$ or equivalently $\frac{2}{p(e)tS} > \frac{f'(\tilde{b}_1(e))}{f(\tilde{b}_1(e))}$. By definition, $\varepsilon(\tilde{b}_1) = \frac{f'(\tilde{b}_1)}{f(\tilde{b}_1)}\tilde{b}_1$. So, the previous inequality may be rewritten

$$\varepsilon(\tilde{b}_1(e)) < \frac{2(1 - p(e))}{p(e)}$$

which is satisfied under Assumption 1. Now, notice for the existence of an interior solution (the function u_1 being continuously differentiable twice) that

$$\lim_{e\to 0} u_1'(e) = -\alpha h a\left[(1 - F(\tilde{b}_1(0))) + p(0)tSf(\tilde{b}_1(0))\right] > 0$$

with $a = \lim_{e\to 0} p'(e) < 0$ and $\left[(1 - F(\tilde{b}_1(0))) + p(0)tSf(\tilde{b}_1(0))\right] > 0$. Finally, we have

$$\lim_{e\to +\infty} u_1'(e) = -\infty$$

because $\lim_{e\to+\infty}\psi'(e) = +\infty$ and $\lim_{e\to+\infty} p'(e) = 0$. ■

Proof of Proposition 1. Since

$$u_1'(e, x) = -\psi'(e) - \alpha h p'(e)\left[(1 - F(\tilde{b}_1(e))) + p(e)tSf(\tilde{b}_1(e))\right] = 0$$

for $x = \{S, \alpha, h, t\}$, then by applying the implicit function theorem, we have:

$$\frac{\partial e}{\partial S} = -\frac{\frac{\partial u_1'}{\partial S}}{\frac{\partial u_1'}{\partial e}}\ ;\ \frac{\partial e}{\partial \alpha} = -\frac{\frac{\partial u_1'}{\partial \alpha}}{\frac{\partial u_1'}{\partial e}}\ ;\ \frac{\partial e}{\partial h} = -\frac{\frac{\partial u_1'}{\partial h}}{\frac{\partial u_1'}{\partial e}} \text{ and } \frac{\partial e}{\partial t} = -\frac{\frac{\partial u_1'}{\partial t}}{\frac{\partial u_1'}{\partial e}}$$

where the derivatives $\frac{\partial u_1'}{\partial x}$ for $x = \{S, \alpha, h, t\}$ and the derivative $\frac{\partial u_1'}{\partial e}$ are calculated respectively at the points (e, x) such that $u_1'(e, x) = 0$ for $x = \{S, \alpha, h, t\}$. So,

$$\frac{\partial e}{\partial S} = -\frac{-\alpha h p'(e)\left[f'(\widetilde{b}_1(e))(1 - p(e))p(e)tS + 2f(\widetilde{b}_1(e))p(e) - f(\widetilde{b}_1(e))\right]}{\frac{\partial u_1'}{\partial e}}.$$

We know that $-\alpha h p'(e) > 0$ and $\frac{\partial u_1'}{\partial e} < 0$ (by the second order condition). This implies that the sign of $\frac{\partial e}{\partial S}$ equals the sign of the term $\left[f'(\widetilde{b}_1(e))(1 - p(e))p(e)tS + 2f(\widetilde{b}_1(e))p(e) - f(\widetilde{b}_1(e))\right]$, which is positive if and only if $\varepsilon(\tilde{b}_1(e)) > \frac{1-2p(e)}{p(e)}$. By a very similar token, we can show that $\frac{\partial e}{\partial t} > 0$ if and only if $\varepsilon(\tilde{b}_1(e)) > \frac{1-2p(e)}{p(e)}$. Finally, we also have:

$$\frac{\partial e}{\partial \alpha} = -\frac{-hp'(e)\left[(1 - F(\tilde{b}_1(e))) + p(e)tSf(\tilde{b}_1(e))\right]}{\frac{\partial u_1'}{\partial e}} > 0$$

and

$$\frac{\partial e}{\partial h} = -\frac{-\alpha p'(e)\left[(1 - F(\tilde{b}_1(e))) + p(e)tSf(\tilde{b}_1(e))\right]}{\frac{\partial u_1'}{\partial e}} > 0.$$

■

Proof of Lemma 2. Differentiating $u_2(e)$ with respect to e shows that $u_2'(e)$ is equal to

$$-\psi'(e) - [(1-q)\alpha h + qz]p'(e)[(1 - F(\tilde{b}_2(e))) + f(\tilde{b}_2(e))p(e)tS(1-q)].$$

We know that $\psi''(e) > 0$, $p'(e) < 0$, $p''(e) > 0$,$[(1-q)\alpha h+qz] > 0$, and $[(1-F(\tilde{b}_2(e)))+f(\tilde{b}_2(e))p(e)tS(1-q)] > 0$. This implies that a sufficient condition for having $u_2''(e) < 0$ is

$$\frac{\partial((1 - F(\tilde{b}_2(e))) + f(\tilde{b}_2(e))p(e)tS(1-q))}{\partial e} < 0.$$

The inequality above is equivalent to

$$tSp'(e)\left[-f'(\tilde{b}_2(e))p(e)tS(1-q) + 2f(\tilde{b}_2(e))\right] < 0.$$

As $tSp'(e) < 0$, we need to have $\left[-f'(\tilde{b}_2(e))p(e)tS(1-q) + 2f(\tilde{b}_2(e))\right] > 0$ or equivalently $\frac{2}{p(e)tS(1-q)} > \frac{f'(\tilde{b}_2(e))}{f(\tilde{b}_2(e))}$. By definition, $\varepsilon(\tilde{b}_2) = \frac{f'(\tilde{b}_2)}{f(\tilde{b}_2)}\tilde{b}_2$. So, the previous inequality may be rewritten

$$\varepsilon(\tilde{b}_2(e)) < \frac{2(1-p(e)(1-q))}{p(e)(1-q)}$$

which is satisfied under Assumption 2. Now, notice for the existence of an interior solution (the function u_2 being continuously differentiable twice) that

$$\lim_{e\to 0} u_2'(e) = -[(1-q)\alpha h + qz]a\left[(1 - F(\tilde{b}_2(0))) + p(0)tSf(\tilde{b}_2(0))\right] > 0$$

with $a = \lim_{e\to 0} p'(e) < 0$ and $\left[(1 - F(\tilde{b}_2(0))) + p(0)tSf(\tilde{b}_2(0))\right] > 0$. Finally, we have

$$\lim_{e\to+\infty} u_2'(e) = -\infty$$

because $\lim_{e\to+\infty} \psi'(e) = +\infty$ and $\lim_{e\to+\infty} p'(e) = 0$. ■

Proof of Proposition 2. Since

$$u_2'(e,x) = -\psi'(e) - [(1-q)\alpha h + qz]p'(e)[(1 - F(\tilde{b}_2(e))) + f(\tilde{b}_2(e))p(e)tS(1-q)] = 0$$

for $x = \{S, \alpha, h, t\}$, then by applying the implicit function theorem, we have

$$\frac{\partial e}{\partial S} = -\frac{\frac{\partial u_2'}{\partial S}}{\frac{\partial u_2'}{\partial e}}\ ;\ \frac{\partial e}{\partial \alpha} = -\frac{\frac{\partial u_2'}{\partial \alpha}}{\frac{\partial u_2'}{\partial e}}\ ;\ \frac{\partial e}{\partial h} = -\frac{\frac{\partial u_2'}{\partial h}}{\frac{\partial u_2'}{\partial e}} \text{ and } \frac{\partial e}{\partial t} = -\frac{\frac{\partial u_2'}{\partial t}}{\frac{\partial u_2'}{\partial e}}$$

where the derivatives $\frac{\partial u_2'}{\partial x}$ for $x = \{S, \alpha, h, t\}$ and the derivative $\frac{\partial u_2'}{\partial e}$ are calculated respectively at the points (e, x) such that $u_2'(e, x) = 0$ for $x = \{S, \alpha, h, t\}$. So

$$\frac{\partial e}{\partial S} = -\frac{-[(1-q)\alpha h + qz]p'(e)\begin{bmatrix} f'(\tilde{b}_2(e))t(1-p(e)(1-q))tS(1-q)p(e) \\ +2f(\tilde{b}_2(e))tp(e)(1-q) - f(\tilde{b}_2(e))t \end{bmatrix}}{\frac{\partial u_2'}{\partial e}}.$$

We know that $-[(1-q)\alpha h+qz]p'(e) > 0$ and $\frac{\partial u_2'}{\partial e} < 0$ (by the second order condition). This implies that the sign of $\frac{\partial e}{\partial S}$ equals the sign of the term $f'(\tilde{b}_2(e))t(1-p(e)(1-q))tS(1-q)p(e)+2f(\tilde{b}_2(e))tp(e)(1-q)-f(\tilde{b}_2(e))t$, which is positive if $\varepsilon(\tilde{b}_2(e)) > \frac{1-2(1-q)p(e)}{(1-q)p(e)}$. By a very similar token, we can show that $\frac{\partial e}{\partial t} > 0$ if $\varepsilon(\tilde{b}_2(e)) > \frac{1-2(1-q)p(e)}{(1-q)p(e)}$. Finally, we also have:

$$\frac{\partial e}{\partial \alpha} = -\frac{-(1-q)hp'(e)\left[(1 - F(\tilde{b}_2(e))) + (1-q)p(e)tSf(\tilde{b}_2(e))\right]}{\frac{\partial u_2'}{\partial e}} > 0$$

and

$$\frac{\partial e}{\partial h} = -\frac{-(1-q)\alpha p'(e)\left[(1-F(\tilde{b}_2(e))) + (1-q)p(e)tSf(\tilde{b}_2(e))\right]}{\frac{\partial u_2'}{\partial e}} > 0.$$

■

Proof of Proposition 3. If $q = 0$ then the first order condition of the judge's program is identical in both settings: with and without any appeals process (or $u_1'(e) = u_2'(e,q) \mid_{q=0}$). As a consequence, the following is true: if $\frac{\partial u_2'(e,q)}{\partial q} < 0$ for $q \in (0,1)$ (and implicitely for e being constant) then establishing an appeals process will induce a decrease in effort at equilibrium (in comparison to the case where there is no appeals process). Further, notice that

$$\frac{\partial u_2'(e,q)}{\partial q} = p'(e)\left(\begin{array}{c} (\alpha h - z)f(\tilde{b}_2(e))p(e)tS(1-q) + (\alpha h - z)(1-F(\tilde{b}_2(e))) - \\ ((1-q)\alpha h + qz)(p(e)f'(\tilde{b}_2(e))\frac{\partial \tilde{b}_2(e,q)}{\partial q}tS(1-q) - \\ f(\tilde{b}_2)tSp(e) - f(\tilde{b}_2)\frac{\partial \tilde{b}_2(e,q)}{\partial q}) \end{array}\right).$$

Since we define

$$A(e) = 3f(\tilde{b}_2(e))p(e)tS(1-q) + (1-F(\tilde{b}_2(e))) - (1-q)^2p^2(e)t^2S^2f'(\tilde{b}_2(e))$$

and

$$B(e) = f(\tilde{b}_2(e))tSp(e)(1-3q) + (1-F(\tilde{b}_2(e))) + (1-q)qp^2(e)t^2S^2f'(\tilde{b}_2(e)),$$

with $A(e) \geq 0$ and $A(e) \geq B(e)$ under assumption 2, then we rewrite

$$\frac{\partial u_2'(e,q)}{\partial q} = p'(e)(\alpha hA(e) - zB(e)).$$

Finally, remark that

$$\begin{aligned} A(e) - B(e) &= p(e)tS\left(2f(\tilde{b}_2(e)) - f'(\tilde{b}_2(e))p(e)tS(1-q)\right) \\ &= p(e)tSf(\tilde{b}_2(e))\left(\frac{\overline{\varepsilon} - \varepsilon(\tilde{b}_2(e))}{\overline{\varepsilon}}\right). \end{aligned}$$

As a consequence, if $\alpha h - z > 0$ then we have $\alpha hA(e) - zB(e) > 0$ or equivalently $\frac{\partial u_2'(e,q)}{\partial q} < 0$ because $\frac{\partial u_2'(e,q)}{\partial q} = p'(e)(\alpha hA(e) - zB(e))$. In other words, if $\alpha h - z > 0$ then establishing an appeals process leads to a decrease in the judge's equilibrium effort whatever $p(e)$, t and S. If the reverse is true (or $\alpha h < z$), then establishing an appeals process has an ambiguous effect on the judge's equilibrium effort. More precisely, assume that $\alpha h - z < 0$: it is possible to find either a gap $(A(e) - B(e))$ sufficiently high to ensure that $\alpha hA(e) - zB(e) > 0$, or a gap $(A(e) - B(e))$ sufficiently low to ensure that $\alpha hA(e) - zB(e) < 0$. As $A(e) - B(e) = p(e)tSf(\tilde{b}_2(e))\left(\frac{\overline{\varepsilon} - \varepsilon(\tilde{b}_2(e))}{\overline{\varepsilon}}\right)$, the gap $A(e) - B(e)$ is all the lower that $p(e)$, t or S tend to zero, or $\varepsilon(\tilde{b}_2(e))$ tends to $\overline{\varepsilon}$. In other words, we may find that the introduction of an appeals process leads to an increase in the judge's effort, even if the reputational cost is superior to the judge's sensitivity to judicial errors, under the condition that t, $p(e)$ or S take sufficiently low values, or that $\varepsilon(\tilde{b}_2(e))$ tends to $\overline{\varepsilon}$. ■

Proof of Lemma 3. Differentiating $u_3(e)$ with respect to e shows that $u_3'(e)$ is equal to

$$-\psi'(e) - \alpha hp'(e)[1 - F(\tilde{b}_3(e))] - \alpha h\beta'(e)F(\tilde{b}_3(e)) - \alpha hf(\tilde{b}_3(e))tS[p'(e) + \beta'(e)][p(e) - \beta(e)].$$

Differentiating twice $u_3(e)$ with respect to e gives

$$\begin{aligned}
u_3''(e) &= -\psi''(e) + f'(\tilde{b}_3(e))(-t^2S^2)(p'(e)+\beta'(e))^2(p(e)-\beta(e))\alpha h + \\
&\quad f(\tilde{b}_3(e))(-tS)(p''(e)+\beta''(e))(p(e)-\beta(e))\alpha h + \\
&\quad f(\tilde{b}_3(e))(-tS)(p'(e)+\beta'(e))(p'(e)-\beta'(e))\alpha h - \\
&\quad \alpha h \left[\begin{array}{c} -f(\tilde{b}_3(e))(-tS)(p'(e)+\beta'(e))p'(e) + (1-F(\tilde{b}_3(e))p''(e)+ \\ f(\tilde{b}_3(e))(-tS)(p'(e)+\beta'(e))\beta'(e)) + F(\tilde{b}_3(e))\beta''(e) \end{array} \right].
\end{aligned}$$

We know that $\psi''(e) > 0$, $p'(e) < 0$, $p''(e) > 0$, $\beta'(e) > 0$, $\beta''(e) > 0$, $[p'(e) + \beta'(e)] < 0$ and $[p(e) - \beta(e)] > 0$. This implies that a sufficient condition for having $u_3''(e) < 0$ is

$$\begin{aligned}
& f'(\tilde{b}_3(e))(-t^2S^2)(p'(e)+\beta'(e))^2(p(e)-\beta(e)) + f(\tilde{b}_3(e))tS(p'(e)+ \\
& \beta'(e))(p'(e)-\beta'(e)) - f(\tilde{b}_3(e))(-tS)(p'(e)+\beta'(e))(p'(e)-\beta'(e)) \\
> \; & 0.
\end{aligned}$$

The inequality above is equivalent to

$$\varepsilon(\tilde{b}_3(e)) < \left(\frac{2(p'(e)-\beta'(e))}{(p'(e)+\beta'(e))}\right)\left(\frac{1-(p(e)+\beta(e))}{(p(e)-\beta(e))}\right)$$

which is satisfied under Assumption 5. Finally, notice for the existence of an interior solution (the function u_3 being continuously differentiable twice) that

$$\lim_{e\to 0} u_3'(e) = -\alpha h a(1-F(\tilde{b}_3(0))) - \alpha h c F(\tilde{b}_3(0)) - \alpha h f(\tilde{b}_3(0)) tS[a+c][p(0)-\beta(0)]$$

must be positive. Since $a = \lim_{e\to 0} p'(e) < 0$, $c = \lim_{e\to 0} \beta'(e) > 0$ and $a + c < 0$ (according to Assumption 4), then it will be the case if the negative term $-\alpha h c F(\tilde{b}_3(0))$ is sufficiently low in absolute value in comparison to the other positive terms in the equation above. Further, we have

$$\lim_{e\to +\infty} u_3'(e) = -\infty$$

because $\lim_{e\to+\infty} \psi'(e) = +\infty$ and $\lim_{e\to+\infty} p'(e) = \lim_{e\to+\infty} \beta'(e) = 0$. ■

Proof of Lemma 4. Differentiating $u_4(e)$ with respect to e shows that $u_4'(e)$ is equal to

$$\begin{aligned}
& -\psi'(e) - ((1-q)\alpha h + qz)\left(p'(e)[1-F(\tilde{b}_4(e))] + \beta'(e)F(\tilde{b}_4(e))\right) \\
& -(1-q)f(\tilde{b}_4(e))tS[p'(e)+\beta'(e)][p(e)-\beta(e)]((1-q)\alpha h + qz).
\end{aligned}$$

Differentiating twice $u_4(e)$ with respect to e gives

$$\begin{aligned}
u_4''(e) &= -\psi''(e) + f'(\tilde{b}_3(e))(-t^2S^2)(1-q)^2(p'(e)+\beta'(e))^2(p(e)-\beta(e))((1-q)\alpha h + qz) + \\
&\quad f(\tilde{b}_4(e))(-tS)(1-q)(p''(e)+\beta''(e))(p(e)-\beta(e))((1-q)\alpha h + qz) + \\
&\quad f(\tilde{b}_4(e))(-tS)(1-q)(p'(e)+\beta'(e))(p'(e)-\beta'(e))((1-q)\alpha h + qz) - \\
&\quad ((1-q)\alpha h + qz) \left[\begin{array}{c} -f(\tilde{b}_4(e))(-tS)(1-q)(p'(e)+\beta'(e))p'(e) + (1-F(\tilde{b}_4(e))p''(e)+ \\ f(\tilde{b}_4(e))(-tS)(1-q)(p'(e)+\beta'(e))\beta'(e)) + F(\tilde{b}_4(e))\beta''(e) \end{array} \right].
\end{aligned}$$

We know that $\psi''(e) > 0$, $p'(e) < 0$, $p''(e) > 0$, $\beta'(e) > 0$, $\beta''(e) > 0$, $[p'(e) + \beta'(e)] < 0$ and $[p(e) - \beta(e)] > 0$. This implies that a sufficient condition for having $u_4''(e) < 0$ is

$$\begin{aligned}
& f'(\tilde{b}_4(e))(-t^2S^2)(1-q)^2(p'(e)+\beta'(e))^2(p(e)-\beta(e)) + \\
& f(\tilde{b}_4(e))tS(1-q)(p'(e)+\beta'(e))(p'(e)-\beta'(e)) - \\
& f(\tilde{b}_4(e))(-tS)(1-q)(p'(e)+\beta'(e))(p'(e)-\beta'(e)) \\
> \; & 0.
\end{aligned}$$

The inequality above is equivalent to

$$\varepsilon(\tilde{b}_4(e)) < \left(\frac{2(p'(e)-\beta'(e))}{(p'(e)+\beta'(e))}\right)\left(\frac{1-(p(e)+\beta(e))(1-q)}{(p(e)-\beta(e))(1-q)}\right)$$

which is satisfied under Assumption 6. Finally, notice for the existence of an interior solution (the function u_4 being continuously differentiable twice) that

$$\begin{aligned}\lim_{e\to 0} u_4'(e) &= -((1-q)\alpha h+qz)a(1-F(\tilde{b}_4(0)))-((1-q)\alpha h+qz)cF(\tilde{b}_4(0))- \\ &\quad ((1-q)\alpha h+qz)f(\tilde{b}_4(0))tS[a+c][p(0)-\beta(0)]\end{aligned}$$

must be positive. Since $a=\lim_{e\to 0} p'(e)<0$, $c=\lim_{e\to 0}\beta'(e)>0$ and $a+c<0$ (according to Assumption 4), then it will be the case if the negative term $-((1-q)\alpha h+qz)cF(\tilde{b}_4(0))$ is sufficiently low in absolute value in comparison to the other positive terms in the equation above. Further, we have

$$\lim_{e\to+\infty} u_4'(e)=-\infty$$

because $\lim_{e\to+\infty}\psi'(e)=+\infty$ and $\lim_{e\to+\infty} p'(e)=\lim_{e\to+\infty}\beta'(e)=0$. ■

Proof of Proposition 4. Since

$$\begin{aligned}u_3'(e,x) &= -\psi'(e)-(\alpha h)\left(p'(e)[1-F(\tilde{b}_3(e))]+\beta'(e)F(\tilde{b}_3(e))\right) \\ &\quad -f(\tilde{b}_3(e))tS[p'(e)+\beta'(e)][p(e)-\beta(e)](\alpha h) \\ &= 0\end{aligned}$$

for $x=\{S,\alpha,h,t\}$, then by applying the implicit function theorem, we have

$$\frac{\partial e}{\partial S}=-\frac{\frac{\partial u_3'}{\partial S}}{\frac{\partial u_3'}{\partial e}}\ ;\ \frac{\partial e}{\partial \alpha}=-\frac{\frac{\partial u_3'}{\partial \alpha}}{\frac{\partial u_3'}{\partial e}}\ ;\ \frac{\partial e}{\partial h}=-\frac{\frac{\partial u_3'}{\partial h}}{\frac{\partial u_4'}{\partial e}} \text{ and } \frac{\partial e}{\partial t}=-\frac{\frac{\partial u_3'}{\partial t}}{\frac{\partial u_3'}{\partial e}}$$

where the derivatives $\frac{\partial u_3'}{\partial x}$ for $x=\{S,\alpha,h,t\}$ and the derivative $\frac{\partial u_3'}{\partial e}$ are calculated respectively at the points (e,x) such that $u_3'(e,x)=0$ for $x=\{S,\alpha,h,t\}$. So

$$\frac{\partial e}{\partial S}=-\frac{\alpha h\begin{pmatrix}-f'(\widetilde{b}_3(e))t^2S(1-(p(e)+\beta(e))) \\ (p'(e)+\beta'(e))(p(e)-\beta(e)) \\ -f(\widetilde{b}_3(e))t(p'(e)+\beta'(e))(p(e)-\beta(e)) \\ +f(\widetilde{b}_3(e))t(1-(p(e)+\beta(e)))(p'(e)-\beta'(e))\end{pmatrix}}{\frac{\partial u_3'}{\partial e^*}}.$$

We know that $\alpha h>0$ and $\frac{\partial u_3'}{\partial e}<0$ (by the second order condition). This implies that the sign of $\frac{\partial e}{\partial S}$ equals the sign of the bracketed term in the numerator of the expression above. After some manipulations, we find that:

$$\begin{pmatrix}-f'(\widetilde{b}_3(e))t^2S(1-(p(e)+\beta(e)))(p'(e)+\beta'(e))(p(e)-\beta(e))- \\ f(\widetilde{b}_3(e))t(p'(e)+\beta'(e))(p(e)-\beta(e))+ \\ f(\widetilde{b}_3(e))t(1-(p(e)+\beta(e)))(p'(e)-\beta'(e))\end{pmatrix}>0$$

is equivalent to:

$$\varepsilon(\tilde{b}_3(e)) > \frac{(p'(e)-\beta'(e))+2(\beta'(e)\beta(e)-p'(e)p(e))}{(p'(e)+\beta'(e))(p(e)-\beta(e))}.$$

By a very similar token, we can show that under the same condition we have $\frac{\partial e}{\partial t} > 0$. Finally, we also have:

$$\frac{\partial e}{\partial \alpha} = -\frac{\begin{array}{c} -h\left(p'(e)[1-F(\tilde{b}_3(e))] + \beta'(e)F(\tilde{b}_3(e))\right) - \\ hf(\tilde{b}_3(e))tS[p'(e)+\beta'(e)][p(e)-\beta(e)] \end{array}}{\frac{\partial u_3'}{\partial e}} > 0$$

and

$$\frac{\partial e}{\partial h} = -\frac{\begin{array}{c} -\alpha\left(p'(e)[1-F(\tilde{b}_3(e))] + \beta'(e)F(\tilde{b}_3(e))\right) - \\ \alpha f(\tilde{b}_3(e))tS[p'(e)+\beta'(e)][p(e)-\beta(e)] \end{array}}{\frac{\partial u_3'}{\partial e}} > 0.$$

■

Proof of Proposition 5. Since

$$\begin{aligned} u_4'(e,x) &= -\psi'(e) - ((1-q)\alpha h + qz)\left(p'(e)[1-F(\tilde{b}_4(e))] + \beta'(e)F(\tilde{b}_4(e))\right) \\ &\quad -(1-q)f(\tilde{b}_4(e))tS[p'(e)+\beta'(e)][p(e)-\beta(e)]((1-q)\alpha h + qz) \\ &= 0 \end{aligned}$$

for $x = \{S, \alpha, h, t\}$, then by applying the implicit function theorem, we have

$$\frac{\partial e}{\partial S} = -\frac{\frac{\partial u_4'}{\partial S}}{\frac{\partial u_4'}{\partial e}} ; \frac{\partial e}{\partial \alpha} = -\frac{\frac{\partial u_4'}{\partial \alpha}}{\frac{\partial u_4'}{\partial e}} ; \frac{\partial e}{\partial h} = -\frac{\frac{\partial u_4'}{\partial h}}{\frac{\partial u_4'}{\partial e}} \text{ and } \frac{\partial e}{\partial t} = -\frac{\frac{\partial u_4'}{\partial t}}{\frac{\partial u_4'}{\partial e}}$$

where the derivatives $\frac{\partial u_4'}{\partial x}$ for $x = \{S, \alpha, h, t\}$ and the derivative $\frac{\partial u_4'}{\partial e}$ are calculated respectively at the points (e, x) such that $u_4'(e, x) = 0$ for $x = \{S, \alpha, h, t\}$. So

$$\frac{\partial e}{\partial S} = -\frac{[(1-q)\alpha h + qz]\left(\begin{array}{c} -f'(\tilde{b}_4(e))t^2S(1-(1-q)(p(e)+\beta(e))) \\ (1-q)(p'(e)+\beta'(e))(p(e)-\beta(e)) \\ -f(\tilde{b}_4(e))t(1-q)(p'(e)+\beta'(e))(p(e)-\beta(e)) \\ +f(\tilde{b}_4(e))t(1-(1-q)(p(e)+\beta(e)))(p'(e)-\beta'(e)) \end{array}\right)}{\frac{\partial u_4'}{\partial e^*}}.$$

We know that $[(1-q)\alpha h + qz] > 0$ and $\frac{\partial u_4'}{\partial e} < 0$ (by the second order condition). This implies that the sign of $\frac{\partial e}{\partial S}$ equals the sign of the bracketed term in the numerator of the expression above. After some manipulations, we find that:

$$\left(\begin{array}{c} -f'(\tilde{b}_4(e))t^2S(1-(1-q)(p(e)+\beta(e)))(1-q)(p'(e)+\beta'(e))(p(e)-\beta(e)) - \\ f(\tilde{b}_4(e))t(1-q)(p'(e)+\beta'(e))(p(e)-\beta(e)) + \\ f(\tilde{b}_4(e))t(1-(1-q)(p(e)+\beta(e)))(p'(e)-\beta'(e)) \end{array}\right) > 0$$

is equivalent to:

$$\varepsilon(\tilde{b}_4(e)) > \frac{(p'(e)-\beta'(e)) + 2(1-q)(\beta'(e)\beta(e) - p'(e)p(e))}{(p'(e)+\beta'(e))(p(e)-\beta(e))(1-q)}.$$

By a very similar token, we can show that under the same condition we have $\frac{\partial e}{\partial t} > 0$. Finally, we also have:

$$\frac{\partial e}{\partial \alpha} = -\frac{\begin{array}{c} -(1-q)h\left(p'(e)[1-F(\tilde{b}_4(e))] + \beta'(e)F(\tilde{b}_4(e))\right) - \\ (1-q)^2hf(\tilde{b}_4(e))tS[p'(e)+\beta'(e)][p(e)-\beta(e)] \end{array}}{\frac{\partial u_4'}{\partial e}} > 0$$

and

$$\frac{\partial e}{\partial h} = -\frac{\begin{array}{c} -(1-q)\alpha\left(p'(e)[1-F(\tilde{b}_4(e))] + \beta'(e)F(\tilde{b}_4(e))\right) - \\ (1-q)^2\alpha f(\tilde{b}_4(e))tS[p'(e)+\beta'(e)][p(e)-\beta(e)] \end{array}}{\frac{\partial u_4'}{\partial e}} > 0.$$

■

Proof of Proposition 6. if $q = 0$ then $u_3'(e) = u_4'(e,q) \mid_{q=0}$. As a consequence, if $\frac{\partial u_4'(e,q)}{\partial q} < 0$ for $q \in (0,1)$ then establishing an appeals court process will induce a decrease in effort at equilibrium in comparison to the equilibrium effort level in absence of any appeals process. Further remark that:

$$\begin{aligned} \frac{\partial u_4'(e,q)}{\partial q} &= \alpha h \left(\begin{array}{c} -(1-q)^2 f'(\tilde{b}_4(e))t^2S^2[\beta(e)+p(e)][\beta'(e)+p'(e)][p(e)-\beta(e)] + \\ 2f(\tilde{b}_4(e))tS(1-q)[\beta'(e)+p'(e)][p(e)-\beta(e)] + \\ (1-F(\tilde{b}_4(e)))p'(e) + F(\tilde{b}_4(e))\beta'(e) - \\ (1-q)f(\tilde{b}_4(e))tS[p(e)+\beta(e)][\beta'(e)-p'(e)] \end{array} \right) \\ & -z \left(\begin{array}{c} q(1-q)f'(\tilde{b}_4(e))t^2S^2[\beta(e)+p(e)][\beta'(e)+p'(e)][p(e)-\beta(e)] + \\ f(\tilde{b}_4(e))tS(1-2q)[\beta'(e)+p'(e)][p(e)-\beta(e)] + \\ (1-F(\tilde{b}_4(e)))p'(e) + F(\tilde{b}_4(e))\beta'(e) + \\ qf(\tilde{b}_4(e))tS[p(e)+\beta(e)][\beta'(e)-p'(e)] \end{array} \right). \end{aligned}$$

Since we define

$$\begin{array}{c} A(e) = -(1-q)^2 f'(\tilde{b}_4(e))t^2S^2[\beta(e)+p(e)][\beta'(e)+p'(e)][p(e)-\beta(e)] + \\ 2f(\tilde{b}_4(e))tS(1-q)[\beta'(e)+p'(e)][p(e)-\beta(e)] + \\ (1-F(\tilde{b}_4(e)))p'(e) + F(\tilde{b}_4(e))\beta'(e) - \\ (1-q)f(\tilde{b}_4(e))tS[p(e)+\beta(e)][\beta'(e)-p'(e)] \end{array}$$

and

$$\begin{array}{c} B(e) = q(1-q)f'(\tilde{b}_4(e))t^2S^2[\beta(e)+p(e)][\beta'(e)+p'(e)][p(e)-\beta(e)] + \\ f(\tilde{b}_4(e))tS(1-2q)[\beta'(e)+p'(e)][p(e)-\beta(e)] + \\ (1-F(\tilde{b}_4(e)))p'(e) + F(\tilde{b}_4(e))\beta'(e) + \\ qf(\tilde{b}_4(e))tS[p(e)+\beta(e)][\beta'(e)-p'(e)] \end{array}$$

then we may rewrite

$$\frac{\partial u_4'(e,q)}{\partial q} = \alpha h A(e) - zB(e).$$

By the way, we have the following. If $\alpha h A(e) - zB(e) < 0$, then the introduction of an appeals process leads to a decrease in the first order judge's effort. At the opposite, if $\alpha h A(e) - zB(e) > 0$ then the introduction of an appeals process leads to an increase in the first order judge's effort. The effect of the appeals process on the first order judge's decision then depends on the comparison between αh and z, the comparison between $A(e)$ and $B(e)$ and the sign of $A(e)$. This can be resumed in the following table:

	$A(e) > B(e)$		$A(e) < B(e)$	
	$A(e) > 0$	$A(e) < 0$	$A(e) > 0$	$A(e) < 0$
$\alpha h > z$	$e \nearrow$	$e \nearrow$ or $e \searrow$	$e \nearrow$ or $e \searrow$	$e \searrow$
$\alpha h < z$	$e \nearrow$ or $e \searrow$	$e \nearrow$	$e \searrow$	$e \nearrow$ or $e \searrow$

Table 2: effect of the appeals court on the trial court judge's effort.

Compared to the case with one type of error, the introduction of an appeals process may either lead to an increase or a decrease in the judge's equilibrium effort whatever the sign of the gap $\alpha h - z$. Recall that, as emphasized in Proposition 3, when considering only type-II errors, implementing an appeals process systematically leads the judge to decrease his effort when he is *socially-motivated* ($\alpha h - z > 0$), whereas it may lead the judge either to increase or decrease his effort when he is *reputation-concerned* ($\alpha h - z < 0$). When considering both type-I and type-II errors, Table 1 show us that, in response to the establishment of an appeals court, the judge may now increase or decrease his effort not only when he is *reputation-concerned*, but also when he is *socially-motivated*, depending on the sign of $A(e)$ and of $A(e) - B(e)$.

Now, for a richer discussion, we concentrate our analysis on the case defined by $A(e) > 0$ and $A(e) < B(e)$ because in such a case, considering two types of errors (instead of type-II errors only) significantly modificates the result summarized in Proposition 3. Indeed, if we simultaneously consider both type-I and type-II errors and we assume that $A(e) > 0$ and $A(e) < B(e)$, instead of having a decreasing equilibrium effort of the judge in response to the establishment of an appeals process when $\alpha h - z > 0$, we have an ambiguous effect of the establishment of an appeals process on the judge's equilibrium effort (it may either increase or decrease). Then, instead of having an ambiguous effect of the establishment of an appeals process on the equilibrium effort of the judge when $\alpha h - z < 0$, we have a clear negative effect of introducing an appeals process on the judge's effort (he decreases his effort).

To resume the case $\{A(e) > 0 \text{ and } A(e) - B(e) < 0\}$ establishing an appeals process has now 1) an ambiguous effect on trial court judge's equilibrium effort even if $\alpha h - z > 0$, and 2) a clear (decreasing) effect on the judge's effort if $\alpha h - z < 0$. Below, we explore these two cases.

First of all, we find after some basic calculations that $A(e) - B(e)$ equals

$$\frac{-2f(\tilde{b}_4(e))tS}{\beta'(e)}\left(\frac{-p'(e)}{\beta'(e)}\left((p(e)+\beta(e))(\frac{\overline{\varepsilon}-\varepsilon(\tilde{b}_4(e))}{\overline{\varepsilon}})-\beta(e)\right)+\left((p(e)+\beta(e))(\frac{\overline{\varepsilon}-\varepsilon(\tilde{b}_4(e))}{\overline{\varepsilon}})-p(e)\right)\right).$$

Since $p(e) > \beta(e)$ (by assumption 3) and $\frac{-p'(e)}{\beta'(e)} > 1$ (by assumption 4), then the following is true: if $\left((p(e)+\beta(e))(\frac{\overline{\varepsilon}-\varepsilon(\tilde{b}_4(e))}{\overline{\varepsilon}})-p(e)\right) > 0$ (or equivalently $(\frac{\overline{\varepsilon}-\varepsilon(\tilde{b}_4(e))}{\overline{\varepsilon}}) > \frac{p(e)}{p(e)+\beta(e)}$) then $A(e) - B(e) < 0$. Second, we explore the sign of $A(e)$. Under assumption 3, we have $\frac{p(e)}{p(e)+\beta(e)} > \frac{1}{2}$. By the way, under the condition $(\frac{\overline{\varepsilon}-\varepsilon(\tilde{b}_4(e))}{\overline{\varepsilon}}) > \frac{p(e)}{p(e)+\beta(e)}$ (i.e. the condition under which $A(e) - B(e) < 0$, see above), the following inequality holds: $(\frac{\overline{\varepsilon}-\varepsilon(\tilde{b}_4(e))}{\overline{\varepsilon}}) > \frac{1}{2}$. Then, notice that $A > 0$ iff at equilibrium

$$\begin{aligned}
&(1-F(\tilde{b}_4(e)))p'(e)+F(\tilde{b}_4(e))\beta'(e)\\
>&\\
&f(\tilde{b}_4(e))tS(1-q)\begin{pmatrix}(p'(e)-\beta'(e))(p(e)+\beta(e))(2(\frac{\overline{\varepsilon}-\varepsilon(\tilde{b}_4)}{\overline{\varepsilon}})-1)+\\ 2(p'(e)+\beta'(e))(p(e)-\beta(e))\end{pmatrix}.
\end{aligned} \tag{7}$$

Since, under assumptions $3-4$, we have:

$$f((\tilde{b}_4(e)))tS(1-q) > 0$$

$$2(p'(e)+\beta'(e))(p(e)-\beta(e)) < 0$$

and

$$(p'(e)-\beta'(e))(p(e)+\beta(e)) < 0$$

then a sufficient condition for the RHS term in (5) to be negative is $(\frac{\overline{\varepsilon}-\varepsilon(\tilde{b}_4(e))}{\overline{\varepsilon}}) > \frac{1}{2}$.

To sum up, if $(\frac{\overline{\varepsilon}-\varepsilon(\tilde{b}_4(e))}{\overline{\varepsilon}}) > \frac{p(e)}{p(e)+\beta(e)}$ then i) $A(e) - B(e) < 0$ and ii) $A(e) > 0$ if the LHS in (5) is positive, that is

$$(1-F(\tilde{b}_4(e)))p'(e)+F(\tilde{b}_4(e))\beta'(e) > 0.$$

This inequality may be rewritten as

$$-\frac{p'(e)}{\beta'(e)} < \frac{F(\tilde{b}_4(e))}{(1-F(\tilde{b}_4(e)))}.$$

Overall, according to Table 1, we know that if $\alpha h < z$ then the judge's effort will decrease after the introduction of an appeals process. In other words, if i) the judge is relatively reputation-concerned ($\alpha h < z$), ii) individuals are not too much sensitive to the benefit of crime ($\varepsilon(\tilde{b}_4(e))$ is sufficiently small) and iii) the judiciary is not too much efficient at the trial court level ($-\frac{p'(e)}{\beta'(e)}$ relatively small compared to $\frac{F(\tilde{b}_4(e))}{(1-F(\tilde{b}_4(e)))}$), then the trial court judge unambiguously decreases his effort when the appeals process is introduced in presence of type I and II errors (whereas the final effect is unclear in presence of type 2 errors). We explore the rationale of these results in the text. ■

Proof of Remark 1. Note that

$$\frac{\partial(1-F(\tilde{b}_4(e)))}{\partial e} = f(\tilde{b}_4(e))tS(1-q)\left(p'(e)+\beta'(e)\right).$$

It follows that

$$\frac{\partial\left[\frac{\partial(1-F(\tilde{b}_4(e)))}{\partial e}\right]}{\partial q} = tS\left(p'(e)+\beta'(e)\right)\left[(1-q)f'(\tilde{b}_4(e))tS\left(p(e)+\beta(e)\right) - f(\tilde{b}_4(e))\right].$$

Since $\left(p'(e)+\beta'(e)\right) < 0$ (assumption 4), we have

$$\begin{aligned}
\frac{\partial\left[\frac{\partial(1-F(\tilde{b}_4(e)))}{\partial e}\right]}{\partial q} &> 0 \Leftrightarrow \frac{f'(\tilde{b}_4(e))}{f(\tilde{b}_4(e))}tS\left(p(e)+\beta(e)\right)(1-q) > 1 \\
&\Leftrightarrow \frac{f'(\tilde{b}_4(e))}{f(\tilde{b}_4(e))}\tilde{b}_4(e) > \frac{\tilde{b}_4(e)}{tS\left(p(e)+\beta(e)\right)(1-q)} \\
&\Leftrightarrow \varepsilon(\tilde{b}_4(e)) > \frac{1-(1-q)\left[p(e)+\beta(e)\right]}{(1-q)\left(p(e)+\beta(e)\right)} \equiv X \geq 0
\end{aligned}$$

with $X < \bar{\varepsilon}$. If $\varepsilon(\tilde{b}_4(e)) < X$, then $\left|\frac{\partial(1-F(\tilde{b}_4(e)))}{\partial e}\right| < \left|\frac{\partial(1-F(\tilde{b}_3(e)))}{\partial e}\right|$, which means that the deterrence effect of a one-unit increase in the judge's effort is weakened when there is an appeals process. Conversely, if $X < \varepsilon(\tilde{b}_4(e)) < \bar{\varepsilon}$, then $\left|\frac{\partial(1-F(\tilde{b}_4(e)))}{\partial e}\right| > \left|\frac{\partial(1-F(\tilde{b}_3(e)))}{\partial e}\right|$: the deterrence effect of the judge's effort is reinforced by the appeals process. ■

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