Resale price maintenance in two-sided markets*

Tommy S. Gabrielsen, Bjørn Olav Johansen, Teis L. Lømo†

Department of Economics, University of Bergen, Norway

Preliminary draft, do not cite or distribute!

November 25, 2014

1 Introduction

This article analyzes the effects of price fixing schemes (i.e. RPM) in two-sided markets[1]. A central feature of two-sided markets with competing platforms is that prices to the two sides of the market are determined by the interplay between cross-group network effects between the two sides of the market and the degree of platform competition. In general, competing platforms are not able to achieve the first-best level of profit.

A second feature in many two-sided markets is that platforms do not necessarily set prices directly to both sides of the market. Often platforms determine their prices to one side directly, but use intermediaries or retailers when they sell to the other side. Examples of this feature are easy to come by. One example is when a company producing gaming consoles may contract directly with software developers, while selling hard- and software through retail stores. Another example is when newspapers and TV-channels

*We would like to thank Joe Farrell and the participants at the Sather Conference on Industrial Organization at UC Berkeley (2013), Markus Reisinger, Patrick Rey and Greg Shaffer for valuable comments to an earlier draft of this paper.

†Department of Economics, University of Bergen, Fosswinckels Gate 14, N-5007 Bergen, Norway (tommy.gabrielsen@econ.uib.no, bjorn.johansen@econ.uib.no, teis.lomo@econ.uib.no).

[1] Evans (2013) calls for more research on vertical issues in two-sided markets. There is a small literature focusing on other vertical practices and restrains in two- or multi-sided markets. Among the issues studied are tying (Rochet and Tirole (2008), Choi (2010), Amelio and Jullien (2012)), vertical integration and exclusivity (Lee (2013)) and exclusive contracts (Armstrong and Wright (2007)).

sell advertising slots directly to advertisers, but rely on distributors for sales to readers and viewers.

When a platform uses an intermediary when selling to one side of the market, she may not be able to sustain the fully integrated profit. This is easiest to explain when the platform is a monopolist. When a monopoly platform does not use intermediaries, but sells directly to both sides, she will adjust the prices to each side so as to take into account the strength and direction of the cross-groups effects from each side of the market. In general the platform will charge a lower price (a low margin) to one side of the market if more users on this side generate high demand from the other side. The platform manipulates margins earned from the two sides in a way that maximizes the profit for the platform.

On the other hand, if the platform serves one side of the market, call it side r, through a retailer, and supplies the other side, call it side d, directly, then the platform may not be able to sustain the prices that maximize overall profits. As an example, suppose the platform supplies the retailer with the monopoly quantity in exchange for a fixed fee. Whether the retailer will be able earn an appropriate margin on this quantity, depends on the quantity that the platform sells to side d of the market. However, once the contract with the retailer is signed, the platform no longer takes into account the retailer's revenue on side r. Hence, the price to each side will have to deviate from the price that maximizes overall profits, as the platform will tend to either oversupply or undersupply side d of the market compared to what is overall optimal. Of course, the retailer will anticipate this move and therefore adjusts his willingness to pay for the platform's supply contract. The monopoly platform hence faces an opportunism problem, similar to the problem faced by the monopolist in Hart and Tirole (1990). Interestingly, and identical to what was proposed by O'Brien and Shaffer (1992) as a solution to this opportunism problem in one-sided markets, we show that by imposing a fixed or maximum resale price on her retailer, the platform is able to fully appropriate the revenue from both sides of the market, realign her own incentives, and thus restore her monopoly profits.

The monopoly example above illustrates that there might be some scope for improving profits for platforms by using RPM when platforms use intermediaries at one side of the market.[2] However, from the literature on RPM is one-sided markets we know that RPM

[2]Some evidence of RPM-practices in industries characterized by two-sidedness exists. Cover pricing on newspapers and magazines is widely used across Europe and in the US. In the videogame industry during the last half of the 1990s, Sony Computer Entertainment forced prices on Playstation software upon their wholesalers and retailers, a practice for which they received long lasting attention from Japanese competition authorities. Another example is how Microsoft operated with minimum advertised prices when releasing Windows 95, granting rebates on purchases only to those retailers that committed to setting prices at, or above, the minimum level.

– specifically minimum RPM – also may serve as a device for facilitating collusive prices when there is competition between suppliers.[3] It is not so obvious whether the same can be achieved by competing platforms in a two-sided market. Moreover, it is less obvious when, *or even if*, allowing for RPM can be harmful in two-sided markets. As far as we are aware of, these questions have not yet been analyzed in the literature. However, some authors indirectly have analyzed a related issue by investigating what competing platforms may gain by colluding at one side of the market only. For instance, Armstrong and Wright (2007) and Evans and Noel (2005) argue that platform collusion to one side in a two-sided market might be less profitable than when firms collude in one-sided markets. The intuition is that platforms will compete aggressively to the non-collusive side where prices are not fixed in order to attract more customers, which in turn cause more customers at the other side to buy at the inflated price.[4] Hence it is argued that two-sided platforms gain relatively little from fixing prices at a single side of the market, compared to what can be gained by firms fixing prices in one-sided markets. This suggests that the one-sided logic on the performance of RPM clauses not necessarily carries over to a two-sided setting.

The policy debate regarding RPM is active. The competition policy towards RPM almost worldwide has been that maximum RPM largely has been considered as unproblematic, but that minimum and fixed RPM has been frowned upon by competition authorities.[5] At the same time authors have warned that two-sided markets work very different from ordinary markets, and that competition authorities should be cautious of applying a "one-sided logic in two-sided markets" (Wright, 2004). A second central aim of our paper is to investigate whether the two-sidedness of a market calls for drastic changes in current competition policy in this area.

To analyze these issues we consider a framework where two competing platforms sell directly to one side of the market - denoted the "direct" side – and where both sell through a common retailer on the other side – the "retail" side. When selling to the retail side we allow the platforms to use general non-linear tariffs and possibly maximum or minimum

[3] See for instance Rey and Vergé (2010), Innes and Hamilton (2009) and Gabrielsen and Johansen (2013).

[4] However, this insight is not fully confirmed in Ruhmer (2011). She finds that if collusion takes place at the side displaying the weakest cross-group effects in consumption or if such effects are symmetric between the two sides, firms still benefit from colluding. In some cases, platforms even make higher profits when colluding only at one side compared to the case where they collude on both sides.

[5] A recent case in the US - the Leegin case - has opened for a more lenient antitrust treatment of RPM. After this landmark case, even minimum and fixed RPM can be defended in the US, while in the EU these clauses still are considered as hardcore infringements of competition law.

RPM in their contracts with the retailer. As usual in two-sided markets we assume that there are cross-group effects between the two sides.

Our main results are as follows. First we show that a monopoly platform selling to one side of the market through a retailer will not be able to realize its monopoly profit with a non-linear contract alone. However, a non-linear contract coupled with an RPM restraint restores the platform's ability to obtain first-best prices to both sides of the market. We then turn to the case with competing platforms. Again we show that when the platforms only use general non-linear tariffs in their contracts with the retailer, it is generally not possible for them to sustain the fully collusive prices and quantities. However, if the platforms individually can impose either maximum or minimum RPM in the contract with the retailer, our results change dramatically. We then show that the individual use of RPM to the retail side of the market enables the platforms to sustain the fully collusive prices to both sides of the market. The appropriate RPM clause is either a maximum or a minimum RPM, and which one will be used in equilibrium will depend on both the cross group externalities and the degree of platform competition.

These results are derived for very general demands. To derive welfare consequences and policy implications we adopt particular utility functions. When doing this we find that, when the cross-group network externalities are positive both ways, a threshold exists for the degree of platform competition above which the buyers are hurt by the use of RPM. When platform competition is higher than the threshold, the use of RPM always benefits the buyers. Interestingly, when the use of RPM is detrimental to buyers, the equilibrium use of RPM is always a fixed or minimum RPM, and when the use of RPM benefits the buyers, it is always a fixed or maximum RPM. Thus, our results support the claim that the logic underlying competition policy towards RPM in one-sided markets also applies in two-sided markets.

The rest of the paper is organized as follows. The model is specified in Section 2. Section 3 considers the case of a monopoly platform, and highlights the main intuition behind our results. Section 4 analyzes the case with two competing platforms, and shows how the fully integrated outcome can be sustained as a subgame perfect equilibrium in an extensive form contracting game. By specifying a linear demand system, Section 5 provides some welfare analysis in the case with competing platforms. Section 6 then concludes.

2 The model

We study a setting where we have either one or two platforms that sell to two sides of the market. At one side of the market the platform(s) sells directly, whereas at the other side of the market each platform uses an intermediary or a retailer that resells the product to final consumers. The direct side of the market is denoted by d and the retail side by r. The platforms incur constant and symmetric marginal costs c_d for selling to side d and c_r for selling to side r. The retailer has no costs except the payment he makes to the platform, and all fixed costs are normalized to zero. Below we set out our assumptions of the demand at sides r and d with two platforms.[6]

Prices to each side charged by platform i are labeled p_d^i and p_r^i, $i = 1, 2$. Final demand on each side is given by $q_d^i = D_d^i\left(p_d^i, p_d^j, q_r^i\right)$ and $q_r^i = D_r^i\left(p_r^i, p_r^j, q_d^i\right)$, where q_d^i and q_r^i, $i = 1, 2$, are quantities consumed and demanded at side d and r respectively. For this demand system to be invertible and stable it is required that the feedback loops between q_d^i and q_r^i are convergent. This holds if cross-group externalities are not too strong.[7]

Let $\mathbf{p} = (\mathbf{p}_d, \mathbf{p}_r) = (p_d^1, p_d^2, p_r^1, p_r^2)$ be the vector of final prices to each side. Our demand system then has a unique solution in indirect demands $q_d^i\left(\mathbf{p}_d, \mathbf{p}_r\right)$ and $q_r^i\left(\mathbf{p}_d, \mathbf{p}_r\right)$, $i = 1, 2$, which are assumed to be continuously differentiable in all prices. Furthermore, we assume that

$$\frac{\partial q_d^i}{\partial p_d^i} - \frac{\partial q_d^i}{\partial p_d^j} < 0 < \frac{\partial q_d^i}{\partial p_d^j}$$

and

$$\frac{\partial q_r^i}{\partial p_r^i} - \frac{\partial q_r^i}{\partial p_r^j} < 0 < \frac{\partial q_r^i}{\partial p_r^j}$$

i.e. negative own-price effects dominate positive cross-price effects. Additionally, we assume that, at both sides, own-side effects dominate cross-group effects: an increase in p_d^1 will have a stronger effect on q_d^1 than on q_r^1.

Finally, the market is two-sided in the sense that consumers on each side cares about consumption at the other side. Specifically we assume that side d attaches positive value to consumption by side r, i.e. $\partial q_d^i / \partial p_r^i < 0$. On the other hand, we assume that side r can either value or dislike consumption at side d, so that this cross-group effect be either positive or negative (but not zero); $\partial q_r^i / \partial p_d^i \gtrless 0$.[8]

[6] For the case of a monopoly platform sub- and superscripts i is deleted and the corresponding price vectors to each side reduced accordingly.

[7] See Filistrucchi and Klein (2013).

[8] These assumptions corresponds to many real life markets. Advertisers and game developers value many customers on the other side. Customers on the retail side may like or dislike consumption on the

3 Monopoly platform

We start by analyzing the situation with one platform, P, that sells directly to side d and uses a retailer R when selling to side r. The structure is illustrated in Figure 1 below. We first consider the fully integrated (industry-profit maximizing) outcome as a benchmark[9]. Industry profits are given by

$$\Pi = (p_d - c_d)\, q_d + (p_r - c_r)\, q_r,$$

which reaches its maximum at prices $\mathbf{p}^M = \left(p_d^M, p_r^M\right)$. Let $\Pi^M = \Pi\left(\mathbf{p}^M\right)$ denote industry profits when prices are set equal to $\mathbf{p}^M$. The fully integrated firm's first order condition for p_d, evaluated at $\mathbf{p}^M$, is given by

$$q_d^M + \left(p_d^M - c_d\right) \left.\frac{\partial q_d}{\partial p_d}\right|_{\mathbf{p}^M} + \left(p_r^M - c_r\right) \left.\frac{\partial q_r}{\partial p_d}\right|_{\mathbf{p}^M} = 0, \tag{1}$$

and analogously for p_r

$$q_r^M + \left(p_d^M - c_d\right) \left.\frac{\partial q_d}{\partial p_r}\right|_{\mathbf{p}^M} + \left(p_r^M - c_r\right) \left.\frac{\partial q_r}{\partial p_r}\right|_{\mathbf{p}^M} = 0. \tag{2}$$

Here, q_d^M and q_r^M are quantities when $\mathbf{p}^M$ is implemented.

Absent full integration, we consider a game where the platform and the distributor first engage in bargaining over wholesale terms, where we let $T\left(q_r\right)$ denote the retailer's total payment for q_r units of the good supplied to side r. We will assume that $T\left(.\right)$ can take any non-linear form; generally it can depend on any variables that are observed by both the platform and the retailer, and can be verified by a third party. However, we assume that the latter does not apply to the price or quantity offered by the platform to side d, i.e. $T\left(.\right)$ cannot be made contingent on p_d or q_d. Given this contract the platform sets its price to side d, while the distributor sets the price (as imposed by the RPM clause, if imposed) to side r.

Let $\mathbf{p}^* = \left(p_r^*, p_d^*\right)$ be the price equilibrium at the final stage induced by the contract T^*. We then have the following useful lemma.

Lemma 1. *If p^* forms a price equilibrium at the final stage, then T^* is continuous and differentiable at the quantity q_r^* induced by p^*.*

direct side. For instance it is discussed in the two-sided literature whether or not readers or TV-viewers value advertisements.

[9] Note that this is equivalent to the situation where the platform sells directly to both sides of the market.

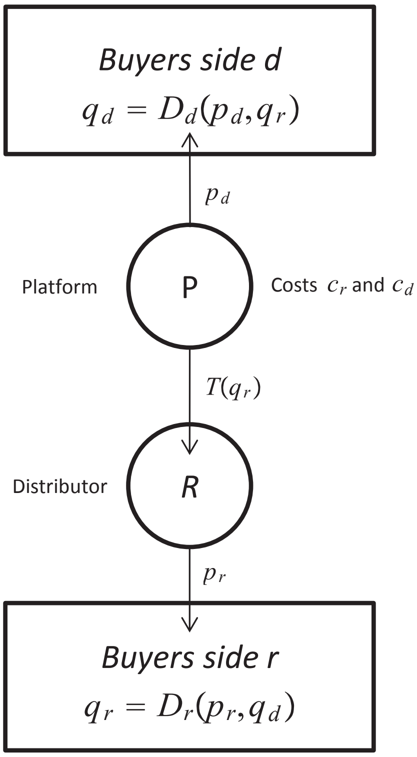


Figure 1: A monopoly platform selling through an intermediary.

Proof: See the appendix.

In addition to simplifying the rest of the analysis considerably, Lemma 1 also provides valuable insights into which contract arrangements between the platform and the retailer that are feasible. Lemma 1 says that, in equilibrium, a slight increase or decrease in the quantity q_r sold to side r cannot induce a discontinuous change in the payment from the retailer to the platform. The contract may involve discontinuities outside of equilibrium, but the point of discontinuity (the threshold value) of, say, q_r cannot be equal to the equilibrium quantity $q_r(\mathbf{p}^*)$, because then $\mathbf{p}^*$ would not be immune to profitable deviations. The intuition for this is straightforward: In a two-sided market, the quantity sold to side r, depends also on the quantity sold to the buyers on the other side of the market. Hence, if for example $T^*(.)$ were to 'jump up' at the equilibrium quantity q_r^*, then the platform could induce a discrete increase in its profit by slightly adjusting its price to side d (either up or down depending on the cross-group effect from side d to side r), so as to cause a slight increase in the quantity sold to side r. Obviously, the payment T^* cannot 'jump down', otherwise it would be profitable for the retailer to increase the quantity sold by charging a slightly lower price to side r.[10]

We can now prove the following result:

Proposition 1. A *general non-linear tariff is insufficient to sustain the fully integrated outcome.*

Proof: See the appendix.

An important insight from the one-sided literature is that a monopolist manufacturer dealing with a monopolist retailer can achieve the first-best level of profit by using a simple non-linear contract (e.g. a two-part tariff with a marginal wholesale price equal to the manufacturer's marginal cost). Such a sellout contract will avoid double marginalization, and as a result the retailer will maximize industry profits. Proposition 1 shows that in a two-sided market this does not work. The reason is that, when one side of the market is supplied by a retailer, an opportunism problem arises for the platform very similar the opportunism problem faced by the monopolist in Hart and Tirole (1990) and O'Brien and Shaffer (1992):

Suppose that the platform and the retailer have negotiated a contract that result in the first best monopoly price to side r. Absent side d, at the equilibrium quantity q_r^* the retailer's marginal wholesale price should be at the platform's marginal cost of serving side r, $T' = c_r$. However, due to the positive cross-group externality from side r to side d, the platform should subsidize the retailer's quantity to side r and hence the marginal wholesale price should be below the marginal cost. Then suppose that the platform at the same time sets the first-best monopoly price to side d. In such a situation the retailer will earn large positive quasi-rents, as $p_r^M > c_r > T'$. This also means that the platform incurs a marginal loss on each unit she sells to side r. Therefore, once the contract with side r is signed, the platform can improve its profit by shifting trade from side r to side d side of the market. When the r side dislikes consumption on side d this is done by decreasing the price (from the monopoly level) to side d. Alternatively, when side r likes more consumption from side d, the platform will increase the price to side d. The retailer will of course anticipate this and appropriately adjust his willingness to pay for the opportunity to sell the platform's product. Hence, just like the monopolist in Hart and Tirole (1990), the platform is left unable to extract her full monopoly profit.

Note that the result in Proposition 1 does not rely on our assumption that there is a positive cross-group effect from side r to side d. If this cross-group effect were negative,

[10] Note that the intuition for this result resembles the intuition for Lemma 1 in O'Brien and Shaffer (1992).

the first best contract offered to side r should have a marginal price above marginal cost. However, if the retailer accepted such a contract and all retail profits were collected by the platform with a fixed fee, the platform's incentive to deviate from the monopoly price to side d would still be the same. This result and its intuition suggest that there might be some scope for improving profits for platforms by using RPM when the platform uses an intermediary at one side of the market. Consider therefore the case where the platform imposes an RPM clause on p_r. This leads to our first main result.

Proposition 2. *The fully integrated outcome Π^M can be sustained by pairing a general non-linear tariff with fixed or maximum RPM to side r.*

Proof: See the appendix.

With reference to the one-sided example above, now maximum RPM does the job in both settings. The structure of the optimal contract T^* is here that $T^{*\prime} = p_r^M$ with a price ceiling $p_r \leq p_r^M$ which is the same that would be the optimal contract in a one-sided market. Hence, the retail margin is squeezed, but the retailer may be compensated for instance by fixed transfers in T^*. However, the intuition for the optimal contract in the two markets is very different. In a one-sided market, the purpose of the maximum RPM is to avoid marginalization at the retail level. In our two-sided setting the purpose of the contract with a high marginal price and a price ceiling is to remove the platform's opportunism problem, i.e. its incentive to deviate from the monopoly price at side d.

4 Competing platforms

In this section there are two competing platforms, $i = 1, 2$, both selling directly to side d and through a retail sector at side r. The fully integrated profits are given by

$$\Pi = \sum_i \left\{ \left(p_d^i - c_d\right) q_d^i + \left(p_r^i - c_r\right) q_r^i \right\},$$

which is maximized by the price vector $\mathbf{p}^M = \left(p_d^M, p_d^M, p_r^M, p_r^M\right)$. We will also here denote by Π^M the (horizontally and vertically) integrated industry profit, i.e. when prices are set equal to $\mathbf{p}^M$. First order conditions for p_r^i for the fully integrated firm evaluated at $\mathbf{p}^M$ are given by

$$q_r^M + \left(p_d^M - c_d\right) \sum_i \left. \frac{\partial q_d^i}{\partial p_r^i} \right|_{\mathbf{p}^M} + \left(p_r^M - c_r\right) \sum_i \left. \frac{\partial q_r^i}{\partial p_r^i} \right|_{\mathbf{p}^M} = 0, \tag{3}$$

and analogously for p_d^i

$$q_d^M + \left(p_d^M - c_d\right) \sum_i \left. \frac{\partial q_d^i}{\partial p_d^i} \right|_{\mathbf{p}^M} + \left(p_r^M - c_r\right) \sum_i \left. \frac{\partial q_r^i}{\partial p_d^i} \right|_{\mathbf{p}^M} = 0. \tag{4}$$

Here, $q_d^M = q_d^i\left(\mathbf{p}^M\right)$ and $q_r^M = q_r^i\left(\mathbf{p}^M\right)$ for $i = 1, 2$. That is, q_d^M and q_r^M are demands at side d and side r when all prices are at the fully integrated level.

We assume the following setup as illustrated in Figure 2: There exists one common distributor ('the distributor' R), with the capacity to sell both platforms' products. In addition we assume that there exist a set of alternative exclusive distributors $(a_1, a_2, ..., a_n)$ each with the capacity to sell only one product. For ease of exposition we shall assume that the platforms hold all the bargaining power when bargaining with these alternative distributors. We include the assumption of alternative distributors here to highlight the fact that in many cases it may be possible for platforms to find equally efficient alternative channels for distribution of their products. Also, by allowing the platforms to sell through alternative distributors, we avoid having to make specific assumptions about what happens if a platform is forced to sell to only one side of the market (side d).

We consider the following game with public information:

Stage 1 Bargaining.

- **1-A** The platforms and the distributor R engage in pairwise bargaining over wholesale terms T_1 and T_2.
- **1-B** The platforms simultaneously and independently decide whether or not to engage in trade with the distributor according to the wholesale terms negotiated in step 1-A.
- **1-C** If both platforms decide to trade with the distributor, the game proceeds directly to stage 2. If platform i decides not to trade with the distributor (and platform j decides to trade), then platform i negotiates a contract with its relevant alternative retailer while platform j and the distributor renegotiate their wholesale terms (from scratch). If neither platform decides to trade, then each platform simultaneously negotiates a contract with its relevant alternative retailer.

Stage 2 Competition. The platforms simultaneously set prices to side d, while the distributor(s) set prices (as imposed by the RPM clauses, if allowed) to side r.

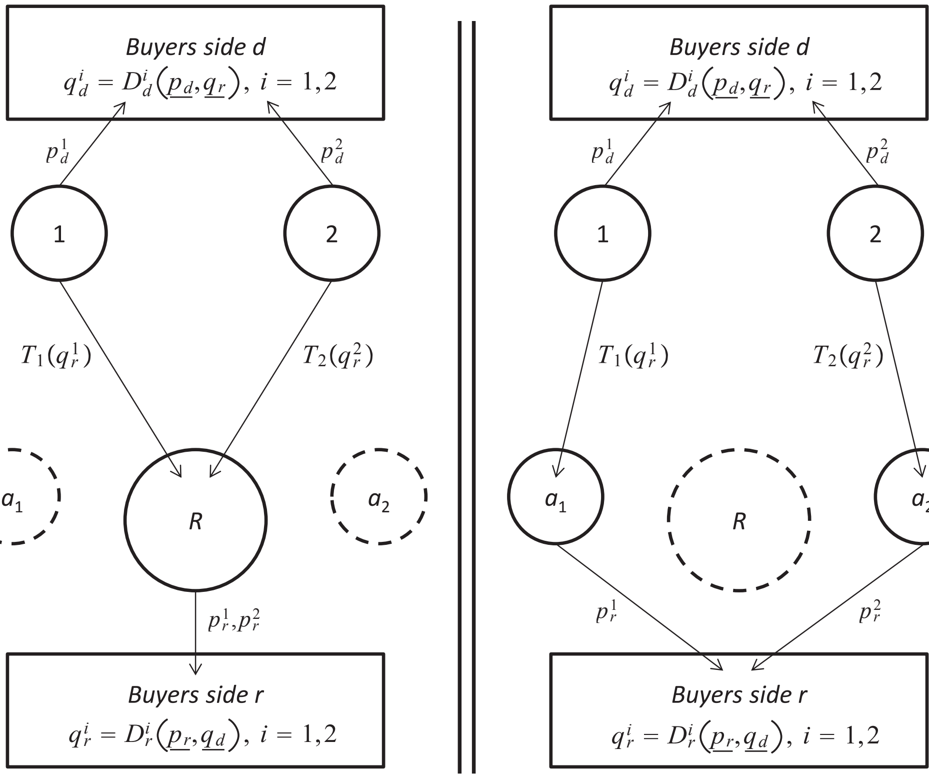


Figure 2: The platforms' choices at the bargaining stage determines whether they end up in a common agency situation, as in the left panel, or a situation with competing vertical structures (exclusive retailers), as in the right panel.

Contracts are general non-linear contracts $T_i\left(q_r^i\right)$ that may or may not include either maximum or minimum RPM. We need not specify the bargaining protocol or sharing rule used in step 1-A, but will simply assume that bargaining between the distributor and platform i results in maximization of the two firms' joint profit, subject to the platform's and the distributor's participation constraints. In general, the wholesale contract T_i negotiated in step 1-A may include any number of elements, and may for example be contingent on both the number of units sold q_r^i and on whether or not actual trade takes place – i.e., on whether or not the distributor sells a positive amount of the platform's product.[11]

At the bargaining stage we therefore have two possible continuation games, as illus-

[11] Marx and Shaffer (2007) analyze contracts that can be conditional on trade, and find that they can be used by a retailer to induce exclusion of a rival in the downstream market. Miklós-Thal et al. (2011) show how the same types of contracts (although with the added assumption that they can be contingent whether or not the retailer obtains exclusivity in the downstream market) can facilitate the fully integrated outcome by allowing rival retailers to accommodate each other.

trated in Figure 2: Either i) the platforms choose to trade with the common distributor at steps 1-A and 1-B, or ii) one or both platforms decide to trade with one of their respective exclusive distributors.

Let $\mathbf{q}^*$ denote the vector of quantities on both sides induced by the price vector $\mathbf{p}^*$, i.e. $\mathbf{q}^* = \left(q_d^i(\mathbf{p}^*), q_d^j(\mathbf{p}^*), q_r^i(\mathbf{p}^*), q_r^j(\mathbf{p}^*)\right)$. Let T_i^*, $i = 1, 2$, be the contract between platform i and the retailer that yields $\mathbf{q}^*$ as the equilibrium outcome. We then can show the following Lemma.

Lemma 2. If $\mathbf{p}^*$ forms a price equilibrium at the final stage, then for $i = 1, 2$, T_i^* is continuous and differentiable at the quantities induced by $\mathbf{p}^*$.

Proof. See the appendix.

Lemma 2 is a straightforward generalization of Lemma 1, and the interpretation is the same as discussed in the previous section.

Platform i's profit is given by

$$\pi_i = q_d^i(p_d^i - c_d) + T_i\left(q_r^i\right) - c_r q_r^i,$$

and her first-order condition for p_d^i by

$$\frac{\partial \pi_i}{\partial p_d^i} = q_d^i + \frac{\partial q_d^i}{\partial p_d^i}(p_d^i - c_d) + \frac{\partial q_r^i}{\partial p_d^i}\left(T_i' - c_r\right) = 0. \tag{5}$$

Before we proceed, it will be useful to describe the outcome if both negotiations with R break down in step 1-A and the platforms select exclusive distributors in step 1-C. We then have a market structure with two competing vertical structures: platform 1 sells through (say) a_1 and platform 2 sells through (say) a_2, where a_1 and a_2 are undifferentiated distributors.

Competing vertical structures Given that RPM is not used, and assuming an interior solution exists, the first-order conditions for the distributors a_1 and a_2 at the competition stage can be written as

$$q_r^i + \frac{\partial q_r^i}{\partial p_r^i}\left(p_r^i - T_i'\right) = 0,\ i \in \{1, 2\}, \tag{6}$$

while the platforms' first-order conditions are given by (5). Maximization then gives us a Nash equilibrium in prices $\mathbf{p}^E(\mathbf{T}')$ and a joint profit for each pair, $\Omega_i(\mathbf{T}') = \pi_i\left(\mathbf{p}^E\right) +$

$\pi_{a_i}\left(\mathbf{p}^E\right)$, as functions of the marginal wholesale terms $\mathbf{T}'$ negotiated in step 1-C. Taking $\mathbf{p}^E\left(\mathbf{T}'\right)$ as given, in step 1-C platform i will then maximize its joint profit with distributor a_i with respect to its own marginal wholesale terms, T_i', and extracts its distributor's quasi-rents through the wholesale terms it sets for the inframarginal units (e.g., through a fixed fee).[12] If the platforms are allowed to use price restraints, in step 1-C the platform will simply maximize its joint profit with the exclusive distributor with respect to both p_r^i and T_i'.

Definition 1. *We denote by* Π^E and Π_p^E *the equilibrium overall industry profit in the subgame with competing vertical structures and no RPM and with RPM, respectively.*

Because we have assumed that the platforms hold all the bargaining power vis-a-vis their alternative distributors, each platform's profit will be equal to $\Pi^E/2$ or $\Pi_p^E/2$ in the subgame after the negotiations break down between R and both platforms simultaneously.

Lemma 3. *Each platform's outside option in the negotiations in step 1-A with R is equal to $\Pi^E/2$ in the case without RPM, and $\Pi_p^E/2$ in the case with RPM, where Π^E (Π_p^E) is the overall equilibrium profit in a game with competing vertical structures. The distributor's outside option in step 1-A, ignoring sunk transfers, is equal to zero.*

Proof: See the appendix

Note that we have $\Pi^M - \Pi_p^E \geq 0$ per definition, given that Π^M is the maximum overall profit that the firms can achieve in any subgame or market structure, and given that exclusive distribution generally will involve more competition than in the common agency situation.

Common agency Next we will define the maximum overall profit that the firms can achieve in the common agency situation, given that RPM is not allowed. Given $T_i\left(q_r^i\right)$, $i = 1, 2$, the retailer sets $p_r^i, i = 1, 2$ to maximize

$$\pi_R = \sum_{i=1,2} \left\{ q_r^i p_r^i - T_i\left(q_r^i\right) \right\}.$$

[12] Note that this subgame is similar in structure to the set-up in Bonanno and Vickers (1988), with the difference that the two manufacturers (platforms) in our model simultaneously sell to two sides of the market – and sells through a distributor to one of the two sides.

Thus, his first-order condition for p_r^i is

$$\frac{\partial \pi_R}{\partial p_r^i} = q_r^i + \sum_{k=1,2} \frac{\partial q_r^k}{\partial p_r^i} \left(p_r^k - T_k'\right) = 0. \tag{7}$$

In this situation, the platforms and the distributor will set prices determined by (5) and (7) respectively at stage 2. Hence, again assuming an interior solution exists, we obtain a Nash equilibrium in prices $\mathbf{p}^C(\mathbf{T}')$ as functions of the *marginal* wholesale terms negotiated in step 1-A. We will denote by

$$\Pi(\mathbf{T}') = \pi_R\left(\mathbf{p}^C\right) + \sum_i \pi_r^i\left(\mathbf{p}^C\right)$$

the resulting overall profit as a function of the wholesale terms.

Definition 2. *Let* $\Pi^C = \max_{\mathbf{T}'} \Pi(\mathbf{T}')$ *be the maximal (but not necessarily equilibrium) overall profit in in the subgame with common agency and no RPM.*

First we show that when RPM cannot be used, general non-linear contracts are insufficient to sustain the fully integrated prices to both sides of the market. Without RPM we show that there exist a non-linear contract that will enable the parties to sustain an equilibrium in which industry profit is $\max\{\Pi^C, \Pi^E\} < \Pi^M$. Next we show that there exists an equilibrium where each platform offer the retailer non-linear contracts coupled with either maximum or minimum RPM in which the fully integrated profit Π^M is achieved.

To analyze the scope for sustaining fully integrated prices as an equilibrium outcome absent full integration it is useful to compare the first order conditions for the platforms for side d in (5) and for the retailer in (7) to those of the fully integrated firm given by (3) and (4). Evaluated at fully integrated prices $\mathbf{p}^M$, we require (3) $-$ (7) $= 0$ and (4) $-$ (5) $= 0$ for implementation of prices at the fully integrated level to both sides. Inserting and rewriting yields the following conditions for side r and d respectively:

$$T_i' - c_r = \frac{\left(p_d^M - c_d\right) \sum_{k=1,2} \left.\frac{\partial q_d^k}{\partial p_r^i}\right|_{\mathbf{p}^M}}{-\sum_{k=1,2} \left.\frac{\partial q_r^k}{\partial p_r^i}\right|_{\mathbf{p}^M}},\ i = 1, 2, \tag{8}$$

$$T_i' - c_r = \frac{\left(p_d^M - c_d\right) \left.\frac{\partial q_d^j}{\partial p_d^i}\right|_{\mathbf{p}^M} + \left(p_r^M - c_r\right) \sum_{k=1,2} \left.\frac{\partial q_r^k}{\partial p_d^i}\right|_{\mathbf{p}^M}}{\left.\frac{\partial q_r^i}{\partial p_d^i}\right|_{\mathbf{p}^M}}, \quad i = 1,2. \tag{9}$$

Condition (8) specifies the margin each platform must have in their wholesale contracts with R to induce the retailer to set the integrated prices $\mathbf{p}_r^M$ to side r. The denominator in (8) is clearly positive, hence the optimal margin depends on the cross-group effect from side r to d, which is assumed to be positive, i.e. $\partial q_d^k / \partial p_r^i < 0$. Hence, (8) requires a negative margin $T_i' - c_r < 0$. The intuition is exactly the same as for the monopoly case above. To induce the retailer to take into account the positive externality from consumption at side r to side d, the retailer should receive a marginal subsidy. Condition (9) specifies the necessary margin each platform must have in the same wholesale contract with R to induce themselves to set the integrated prices $\mathbf{p}_d^M$ to side d. The sign of (9) depends on the cross-group effect from side d to r (which is either positive or negative) and the degree of competition between the platforms for sale to side d, i.e. $\partial q_d^j / \partial p_d^i > 0$. This means that (9) cannot generally be signed. The intuition is as follows. If side r dislikes consumption at side d, both effects are positive and this will call for a positive margin. On the other hand, if side r values consumption at side d this alone will call for a negative margin, but the competitive effect on side d calls for a positive margin. The final effect on the sign of the margin will then depend on the strength of the cross-group effect from d to r and the degree of competition at side d. If competition is weak, the margin should be negative and if competition is strong, the margin according to (9) should be positive.[13,14]

The following result shows that when RPM cannot be used, the platforms and the retailer is unable to sustain the integrated prices. Moreover, it can be the case that a common agency equilibrium does not exist in this situation.

Proposition 3. *(RPM is not allowed) There exist non-linear contracts that will sustain a Pareto undominated common agency equilibrium, in which the profit $\Pi^C < \Pi^M$ is induced at stage 2, as long as $\Pi^C > \Pi^E$. Otherwise, in all equilibria the platforms will use competing distributors and industry profit will be Π^E.*

[13]Solving the numerator of (9) for $\left.\frac{\partial q_d^j}{\partial p_d^i}\right|_{\mathbf{p}^M}$ yields $\left.\frac{\partial q_d^j}{\partial p_d^i}\right|_{\mathbf{p}^M} > -\frac{(p_r^M - c_r)}{(p_d^M - c_d)} \sum_{k=1,2} \left.\frac{\partial q_r^k}{\partial p_d^i}\right|_{\mathbf{p}^M}$, $i = 1,2$. This is the threshold value of platform competition at side d for which the sign of the appropriate wholesale margin to side d given by (9) changes. If the value of $\left.\frac{\partial q_d^j}{\partial p_d^i}\right|_{\mathbf{p}^M}$ lies above the threshold, (9) yields $T_i'\left(q_r^i\right) - c_r < 0$.

[14]When $T_i' < c_r \implies T_i' < 0$, a quantity ceiling might be needed as an additional restraint to ensure stability.

Proof: See the appendix

Note that $\Pi^C - \Pi^E \lesseqgtr 0$ cannot generally be signed, as both Π^C and Π^E will be smaller than the fully integrated profit Π^M. Whether $\Pi^C > \Pi^E > 0$ or $\Pi^C < \Pi^E$ will depend on the situation, as we will discuss below. We now proceed as follows. Proposition 3 shows that there exist contracts that are general enough to allow for the maximal profit, i.e. $\max\{\Pi^C, \Pi^E\}$ to be realized, and that the maximal profit is less than the integrated profit, i.e. $\max\{\Pi^C, \Pi^E\} < \Pi^M$. The result can be understood as follows: In a one-sided market, e.g. if the platforms were to sell to side r only, we know that the platforms would i) always select a common distributor and ii) supply the distributor at cost (hence, eliminating any upstream margins and maintaining retail prices at the monopoly level). The platforms would redistribute (some of) the monopoly profits, for example through fixed fees. In a two-sided market, however, supplying the distributor at cost is not necessarily optimal, because the wholesale margins to side r also affect prices on the direct side of the market. Hence, when choosing the wholesale margins to side r the platforms have to trade off the concerns for the prices at the two sides of the market. This tradeoff can sometimes lead to too high prices of the retail side, and to such an extent that the total industry profit would be greater with exclusive distribution. The intuition is that retail competition - and elimination of double marginalization - will curtail high retail prices on the r side due to excessive high margins in the non-linear contracts that would optimally be offered to the retailer R.

The basic tradeoff can be illustrated by inspection of conditions (8) and (9) above. Absent RPM, the resulting equilibrium margin comes as a tradeoff between the two conditions. It is when condition (9) calls for a high positive margin that prices may be too high on the retail side. This is more likely to occur when side r dislikes consumption at side d, or when side r values consumption at side d and competition on the d side is strong. When r strongly dislikes consumption at side d the platforms should correct for this by increasing their margins to the retailer R. This will lower their incentives to lower their prices to side d, as they will care more about sales to the retail side. However, a high margin may cause the retail price to be too high, and in order to correct for retailer double marginalization the platforms may prefer a system with competing retailers. The second situation in which too high retail prices may be a problem is when r cares mildly about consumption on side d but competition on the d side is very strong. In this situation side d concerns will induce platforms to increase margins to side r in order to increase the prices to side d. This in turn, may cause prices on side r to rise too much, so as to make the platform prefer exclusive distribution to dampen double marginalization.

Let us then analyze our contracting game when the platforms and the retailer may impose maximum or minimum RPM in addition to a non-linear contract. Then we can show our next main result:

Proposition 4. *(RPM is allowed) There exist non-linear contracts that will sustain a Pareto undominated common agency equilibrium, in which i) the platforms impose either price floors or price ceilings equal to* p_r^M *for side* r*, ii) the wholesale margins are set according to (9), and iii) the fully integrated outcome* Π^M *is induced at stage 2. The appropriate RPM clause is*

- *always a maximum RPM if the cross-group effect from side* d *to side* r *is negative,*
- *a maximum RPM if the cross-group effect from side* d *to side* r *is positive and platform competition at side* d *is sufficiently weak, and a minimum RPM otherwise*

Proof: See the appendix

The intuition for this result is as follows. The RPM clause fixes the price to side r, and will be maximum or minimum depending on in what way the retailer needs to be restricted[15]. The margin to side r is set so as to provide optimal incentives for the platforms to set the integrated prices to side d. In general each platform will have a (positive or negative) margin in the equilibrium contract with the retailer. This opens the possibility that a pair consisting of a platform and the retailer could free-ride on this margin by charging a different marginal price T_i' or by imposing a different price than p_r^M *for side* r. However in equilibrium, the non-linear tariff from each platform to R is designed such that neither pair of a platform and R will have incentives to deviate. The exact non-linear contract used to prove both Propositions 3 and 4 is one where each platform receive an upfront payment from the retailer irrespective of whether trade takes place or not, and then a non-linear tariff that governs any positive amount of trade between the retailer and the platforms. It is well known that such contracts can be constructed so as to to make an equilibrium with non-zero wholesale margins immune to deviations.

When there is a negative externality from side d to side r, then competition between the two platforms may cause too much consumption on side d and too little consumption on side r. This can be corrected by giving the platforms higher margins on their sales to side r, which would cause them to compete less fiercely when selling to side d. In turn

[15] It follows trivially that a fixed RPM will be a perfect substitute for either a maximum og minimum RPM.

this would lead to less consumption at side d and increased consumption at side r. On the other hand, a maximum (or fixed) RPM clause is needed to prevent the retailer from marking up the price to side r too much.

When the cross-group externalities are positive both ways, the platforms would like to set their wholesale margins til R lower, because this dampens the competition between the platforms on side d. The higher the degree of substitution between the platforms on side d, the lower the marginal wholesale prices have to be in order to fully eliminate competition between the platforms. A sufficiently low wholesale price, however, in turn may cause the retailer to set too low prices to side r. Hence, the need for a price floor in this case.

5 Welfare analysis with linear demand

The purpose of this section is to provide some measure of how the use of RPM affects consumer welfare in our model, i.e. to quantify the welfare effects of moving from Proposition 3 to Proposition 4. As previously mentioned, competition policy towards RPM vary with the type of price restraint in question. Hence, we are especially interested in how the consumer welfare effect of RPM depend on which RPM-clause that is being applied.

Consider a quasi-linear utility function $V_r = y + U_r(\mathbf{q})$ for side r, where y is a composite good with price $p_y = 1$, and

$$U_r(\mathbf{q}) = \sum_{i=1,2} \left\{ q_r^i + \alpha q_d^i q_r^i - \frac{1}{2}\left(q_r^i\right)^2 \right\} - b q_r^1 q_r^2 + \alpha b \left(q_d^1 q_r^2 + q_d^2 q_r^1\right)$$

and a similar function $V_d = y + U_r(\mathbf{q})$ for side d, where

$$U_r(\mathbf{q}) = \sum_{i=1,2} \left\{ q_d^i + \beta q_d^i q_r^i - \frac{1}{2}\left(q_d^i\right)^2 \right\} - b q_d^1 q_d^2 + \beta b \left(q_d^1 q_r^2 + q_d^2 q_r^1\right)$$

$\alpha \in [-1, 0) \vee (0, 1]$ here represents the cross-group effect from side d to side r, while $\beta \in (0, 1]$ represents the cross-group effect from side r to side d. $b \in (0, 1)$ represents the degree of substitutability between the platforms, where a high b indicates a high degree of substitutability. These utility functions yield linear equilibrium direct demand functions for side r

$$q_r^i(\mathbf{p}) = \frac{1 + \alpha - b - b\alpha - p_r^i + b p_r^j - \alpha p_d^i + b\alpha p_d^j}{1 - \alpha\beta + b^2\alpha\beta - b^2}, i, j = 1, 2, i \neq j \tag{10}$$

and side d

$$q_d^i(\mathbf{p}) = \frac{1+\beta-b-b\beta-p_d^i+bp_d^j-\beta p_r^i+b\beta p_r^j}{1-\alpha\beta+b^2\alpha\beta-b^2},\ i,j=1,2,i\neq j \tag{11}$$

Using (10) and (11), we can calculate prices and quantities with and without RPM from the profit functions given in Section 3. Let

$$\begin{aligned}\Delta S \equiv\ & U_r\left(\mathbf{p}^M\right)-2q_r^M p_r^M+U_d\left(\mathbf{p}^M\right)-2q_d^M p_d^M \\ & -\left[U_r\left(\mathbf{p}^*\right)-2q_r^M p_r^M\right]-\left[U_d\left(\mathbf{p}^*\right)-2q_d^M p_d^M\right]\end{aligned}$$

be the difference in overall surplus for the buyers between the situation where prices are at the fully integrated level $\mathbf{p}^M$ (RPM) and where prices are set absent RPM, $\mathbf{p}^*$. Now, recall from the previous section that the degree of platform competition at side d and the sign of the cross-group effect from side d to side r are the factors determining the appropriate RPM-clause. If the cross-group effect from side d to side r is negative, (17) is always positive, which implies that a price ceiling is the right RPM-clause. If the cross-group effect from side d to side r is positive, (17) is positive as long as (18) holds. (18) is determined by the degree of platform competition, corresponding to b in the above demand system. For given values of α, β, c_d and c_r, we find that both (17) and ΔS switch from being positive to negative at the exact same threshold value of b, when increasing b towards 1. Since (17)<0 calls for a price floor and $\Delta S<0$, this implies that the buyers are hurt when price floors are used and that they benefit when price ceilings are used. The following proposition pins down this discussion.

Proposition 5.

- *If the cross-group effect from side d to side r is negative, then a maximum RPM is used and $\Delta S > 0\ \forall b \in (0,1)$.*
- *If the cross-group effect from side d to side r is positive, then there exists a threshold value of $b \equiv \widetilde{b} \in (0,1)$ such that*

  (i) : If $b \in \left(0,\widetilde{b}\right)$ *a maximum RPM is used and* $\Delta S > 0$

  (ii) : If $b \in \left(\widetilde{b},1\right)$ *a minimum RPM is used and* $\Delta S < 0$

Proof: See the appendix.

Proposition 5 states that when consumers at side r dislikes consumption at side d, the platforms will always use maximum (or fixed) RPM and the platforms' customers will always benefit. When the cross-group effects are positive both ways the platforms will use a maximum RPM when platform competition is weak enough, and in this case customers also benefits from RPM. When platform competition is fierce the platforms will use minimum (or fixed) RPM, and customers are hurt from RPM in this case. The policy implication from this is to forbid minimum (and fixed) RPM whereas maximum RPM should be allowed. This corresponds exactly to the current competition policy towards RPM in for instance the EU. Hence, in this case one-sided logic applies in a two-sided market.

6 Concluding remarks

The existing literature on two-sided markets holds that rival two-sided platforms have little to gain by fixing prices to one side only as this will induce them to compete more fiercely to the other side. The present paper argues that this reasoning might not hold up when platforms sell to one of the sides through a retailer. More specifically, we have shown that two rival platforms can induce prices at the fully integrated level at both sides of the market by offering contracts consisting of non-linear tariffs plus RPM to one side only. The appropriate non-linear contract and RPM-clause depend on the sign of the cross-group effects in consumption and the degree of interbrand platform competition. Under reasonable assumptions, we then argued that this outcome is sustainable as an equilibrium in an extensive form contracting game.

Our paper adds to both the literature on two-sided markets and the literature on RPM. In regards to the literature on two-sided markets, our paper is the first to specifically study the effects of contractually determined RPM. Compared to a more conventional two-sided structure, where platforms sell directly to both sides, the presence of the retailer in our framework opens the door for vertical restrains as efficient instruments. In regards to the RPM-literature, we confirm that the efficiency, and possible necessity, of RPM as an instrument for internalizing multiple externalities and restoring monopoly prices might also carry over to two-sided markets.

7 Appendix

Proof of Lemma 1 (Monopoly)

The proof is structurally identical to the proof of Lemma 2 (see below) and is proved by deleting all sub- and superscrips i in the proof of Lemma 2. Q.E.D.

Proof of Proposition 1 (Monopoly)

The platform P solves $\max_{p_d} \pi_P = q_d(p_d - c_d) + T(q_r) - c_r q_r$ yielding

$$q_d + \frac{\partial q_d}{\partial p_d}(p_d - c_d) + \frac{\partial q_r}{\partial p_d}(T' - c_r) = 0. \tag{12}$$

Given $T(q_r)$, the retailer R solves $\max_{p_r} \pi_R = q_r p_r - T(q_r)$, yielding

$$q_r + \frac{\partial q_r}{\partial p_r}(p_r - T') = 0. \tag{13}$$

For P and R to set p_d^M and p_r^M without vertical integration we require that, evaluated at $\mathbf{p}^M$, (1) − (12) = 0 and (2) − (13) = 0 respectively. For side d we have

$$\left.\frac{\partial \Pi}{\partial p_d}\right|_{\mathbf{p}^M} - \left.\frac{\partial \pi_P}{\partial p_d}\right|_{\mathbf{p}^M} = 0 \Longleftrightarrow T' = p_r^M, \tag{14}$$

For side r we have that

$$\left.\frac{\partial \Pi}{\partial p_r}\right|_{\mathbf{p}^M} - \left.\frac{\partial \pi_R}{\partial p_r}\right|_{\mathbf{p}^M} = 0 \Longleftrightarrow T' - c_r = \frac{\left(p_d^M - c_d\right) \left.\frac{\partial q_d}{\partial p_r}\right|_{\mathbf{p}^M}}{- \left.\frac{\partial q_r}{\partial p_r}\right|_{\mathbf{p}^M}}, \tag{15}$$

for which the RHS is negative whenever $\partial q_d / \partial p_r < 0$, hence according to (15) $T' < c_r$. However, from (14) we have that $T' = p_r^M > c_r$, a contradiction, and monopoly prices cannot be sustained. Q.E.D.

Proof of Proposition 2 (Monopoly)

The marginal wholesale price is set according to (**??**), so that $p_d = p_d^M$. The appropriate price restraint to side r can be determined by signing $\partial \Pi / \partial p_r - \partial \pi_R / \partial p_r < 0$ evaluated at $\mathbf{p}^M$, and given that $T' = p_r^M$. Rearranging yields

$$\left.\frac{\partial q_d}{\partial p_r}\right|_{\mathbf{p}^M} < \frac{\left(p_r^M - c_r\right) \left.\frac{\partial q_r}{\partial p_r}\right|_{\mathbf{p}^M}}{-\left(p_d^M - c_d\right)},$$

which is always satisfied given our assumption that $\partial q_d / \partial p_r < 0$. Since $\partial \Pi / \partial p_r$ is zero at $\mathbf{p}^M$, this implies that $\partial \pi_R / \partial p_r > 0$ at $\mathbf{p}^M$, which means that the appropriate restraint is a price ceiling, $p_r \leq p_r^M$. .Q.E.D.

Proof of Lemma 2 (Duopoly)

Step 1. T_i^* is continuous at the quantities induced by $\mathbf{p}^*$.

Assume that T_i^* is not continuous at the quantities induced by $\mathbf{p}^*$. Then, a marginal deviation, either positive or negative, from $q_r^i(\mathbf{p}^*) = q_r^*$ would cause a discrete change in T_i. If such a deviation causes T_i to "jump up", then, since $\partial q_r^i / \partial p_d^i \neq 0$, platform i could adjust p_d^i slightly to change q_r^i, causing a discrete increase in his profits through a larger payment from the distributor. Since the jump up can be caused by both a positive and a negative marginal deviation from q_r^i, the appropriate adjustment of p_d^i depends on how q_r^i varies with p_d^i. For instance, to get $q_r^i > q_r^*$, platform i should reduce p_d^i slightly when q_r^i falls in p_d^i and increase p_d^i slightly when q_r^i rises in p_d^i. The opposite adjustments are needed to get $q_r^i < q_r^*$. If a marginal deviation causes T_i to "jump down", platform j and the distributor could change T_j, i.e. q_r^j, slightly, resulting in a discrete increase in their bilateral profits. Thus, in both cases of discontinuity, at least one player has a profitable deviation. Hence, T_i^* must be continuous at the quantities induced by $\mathbf{p}^*$.

By step 1., T_i^* has both a right-hand $(+)$ and a left-hand $(-)$ partial derivative wrt. p_r^i at q_r^*, $T_{i+}'^*$ and $T_{i-}'^*$ respectively. For T_i^* to be differentiable in equilibrium, we require that $T_{i+}'^* = T_{i-}'^*$. We show this in two steps, as follows:

Step 2. For distributor optimality it is required for $i = 1, 2$ that

$$\left(\frac{\partial \pi_R}{\partial p_r^i}\right)_- = \frac{\partial q_r^i}{\partial p_r^i} p_r^i + q_r^i - T_{i+}' \frac{\partial q_r^i}{\partial p_r^i} \geq 0 \Leftrightarrow \left(\frac{\partial q_r^i}{\partial p_r^i} p_r^i + q_r^i\right) \left(\frac{\partial q_r^i}{\partial p_r^i}\right)^{-1} \leq T_{i+}^{*\prime},$$

$$\left(\frac{\partial \pi_R}{\partial p_r^i}\right)_+ = \frac{\partial q_r^i}{\partial p_r^i} p_r^i + q_r^i - T_{i-}' \frac{\partial q_r^i}{\partial p_r^i} \leq 0 \Leftrightarrow \left(\frac{\partial q_r^i}{\partial p_r^i} p_r^i + q_r^i\right) \left(\frac{\partial q_r^i}{\partial p_r^i}\right)^{-1} \geq T_{i-}^{*\prime}.$$

The left hand-side derivative of π_R includes the right-hand side derivative of $T_i^*(q_r^i)$ because q_r^i is decreasing in p_r^i. $T_{i+}' \geq T_{i-}'$ follows directly from the rearranged inequalities.

Step 3. For platform optimality, two cases for the cross-group externality from side d to side r must be considered.

(i) : Positive cross-group externality $(\partial q_r^i / \partial p_d^i < 0)$

$$\left(\frac{\partial \pi_i}{\partial p_d^i}\right)_- = \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right) + q_d^i + \left(T'_{i+} - c_r\right)\frac{\partial q_r^i}{\partial p_d^i} \geq 0$$

$$\Leftrightarrow$$

$$T'_{i+} \leq \left(c_r \frac{\partial q_r^i}{\partial p_d^i} - q_d^i - \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right)\right)\left(\frac{\partial q_r^i}{\partial p_d^i}\right)^{-1},$$

$$\left(\frac{\partial \pi_i}{\partial p_d^i}\right)_+ = \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right) + q_d^i + \left(T'_{i-} - c_r\right)\frac{\partial q_r^i}{\partial p_d^i} \leq 0$$

$$\Leftrightarrow$$

$$T'_{i-} \geq \left(c_r \frac{\partial q_r^i}{\partial p_d^i} - q_d^i - \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right)\right)\left(\frac{\partial q_r^i}{\partial p_d^i}\right)^{-1}.$$

Hence, $T^{*\prime}_{i-} \geq T^{*\prime}_{i+}$ by the same logic as in step 2.

(ii) : Negative cross-group externality $(\partial q_r^i / \partial p_d^i > 0)$

$$\left(\frac{\partial \pi_i}{\partial p_d^i}\right)_- = \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right) + q_d^i + \left(T'_{i-} - c_r\right)\frac{\partial q_r^i}{\partial p_d^i} \geq 0$$

$$\Leftrightarrow$$

$$T^{*\prime}_{i-} \geq \left(c_r \frac{\partial q_r^i}{\partial p_d^i} - q_d^i - \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right)\right)\left(\frac{\partial q_r^i}{\partial p_d^i}\right)^{-1},$$

$$\left(\frac{\partial \pi_i}{\partial p_d^i}\right)_+ = \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right) + q_d^i + \left(T'_{i+} - c_r\right)\frac{\partial q_r^i}{\partial p_d^i} \leq 0$$

$$\Leftrightarrow$$

$$T^{*\prime}_{i+} \leq \left(c_r \frac{\partial q_r^i}{\partial p_d^i} - q_d^i - \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right)\right)\left(\frac{\partial q_r^i}{\partial p_d^i}\right)^{-1}.$$

Note that left hand-side derivative of π_R now includes the left-hand side derivative of $T_i^*\left(q_r^i\right)$ since q_r^i is increasing in p_d^i. Again, $T^{*\prime}_{i-} \geq T^{*\prime}_{i+}$.

Combining step 2 ($T'_{i+} \geq T'_{i-}$) and step 3 ($T^{*\prime}_{i-} \geq T^{*\prime}_{i+}$) implies $T'^*_{i+} = T'^*_{i-}$, and thus T_i^* is differentiable at the quantities induced by $\mathbf{p}^*$. $Q.E.D.$

Proof of Lemma 3

Note that $\Pi_p^E/2$ or $\Pi^E/2$ (depending on whether RPM is used or not) serves as each platform's outside options in the negotiations with the distributor R in step 1-A. On the other hand, ignoring any unconditional (sunk) transfers carried out between the distributor and the platforms in step 1-A, the distributor's outside option is always zero. To see this, consider the case where the negotiations break down with platform 1 in step 1-A, while the distributor and platform 2 negotiates successfully:

Irrespective of what happens in step 1-B, we know that platform 1 will earn $\Pi^E/2$ in the continuation equilibrium (assuming RPM is not permitted). In step 1-B, platform 2 will have to decide whether to trade with the distributor. However, note that the maximum that the distributor and platform 2 can earn *together* in the continuation game is $\Pi^E/2$, and that the maximum that platform 2 can earn alone (when not trading with the distributor), again ignoring sunk transfers, is also $\Pi^E/2$. In step 1-B platform 2 therefore decides not trade with the distributor, unless, in the renegotiation with the distributor in step 1-C, the distributor earns zero profits (again ignoring sunk transfers). We can therefore conclude that the situation where the negotiation breaks down with one platform is equivalent to the situation when the negotiations break down with both platforms simultaneously.

Proof of Proposition 3.

Step 1. Equilibrium profit is less than Π^M. Assume that each platform negotiates any non-linear contract $T_i(q_r^r)$ with retailer R, with T_i'. Since (8) and (9) cannot both be satisfied for the same T_i', it follows that the integrated profit cannot be sustained.

Step 2. Suppose then that we are in a candidate equilibrium with prices $\mathbf{p}^C$ and aggregate profit Π^C. Assume first that $\Pi^C > \Pi^E$. Consider the following contract negotiated successfully in step 1-A between platform i and the distributor,

$$T_i\left(q_r^i\right) = \begin{cases} U + t_i\left(q_r^i\right) & \text{if} \quad R \text{ sells } q_r^i > 0 \\ U & \text{if} \quad R \text{ sells } q_r^i = 0 \end{cases} \tag{16}$$

where $U > 0$ is an upfront fee (unconditional on the quantity traded) paid by the distributor to platform i in step 1-A, and $t_i\left(.\right)$ is a non-linear price, constructed such that

$$\underbrace{\left(p_d^C - c_d\right) q_d^C + t_i\left(q_r^C\right) - c_r q_r^C}_{X} + U = \frac{\Pi^E}{2} + U \leq \frac{\Pi^C}{2}$$

and $t_i' - c_r$ is set according to (9). $X + U$ is then platform i's share of the overall joint profit Π^C. The only deviations we have to worry about, are deviations of the type where the distributor deviates together with platform j in step 1-A by charging a different marginal wholesale price T_j' in order to free-ride on platform i's quasi-rents. Now consider any deviation that would cause the joint profit of the pair $R-j$ to rise and, consequentially, platform i's profit to fall below $U+\Pi^E/2$. Given that platform i decides to trade with the distributor, it will then earn a profit less than $U+\Pi^E/2$. However, by negotiating a contract with one of the exclusive distributors instead, platform i can secure a profit equal to $U+\Pi^E/2$. Hence, given that the pair $R-j$ deviates, their joint profit will fall to $\Pi^E/2-U$ in the continuation game, and the deviation is therefore unprofitable. We can conclude that equilibria exists with common agency where i) platforms extract part of their profits up front in step 1-A, ii) non-linear prices $t_i(.)$ (conditional on trade) are implemented to protect each platform against opportunistic deviations on the part of the distributor and the rival platform, and iii) the maximal outcome Π^C is induced at stage 2. Obviously, when $\Pi^C\leq\Pi^E$ a common agency equilibrium cannot be sustained, and both platforms will contract with one alternative distributor each and earn $\Pi^E/2$. Q.E.D

Proof of Proposition 4.

The proof is in two steps. We first show that a Pareto undominated common agency equilibrium exists in which fixed RPM is imposed and the fully integrated outcome Π^M is induced at stage 2. Next we show that the same equilibrium can be sustained by either maximum or minimum RPM.

Step 1. Suppose we are in a candidate equilibrium with fully integrated prices where the price to side r from each platform is a fixed RPM at $p_r^i=p_r^M$. Consider the following contract negotiated successfully in step 1-A between platform i and the distributor,

$$T_i\left(q_r^i\right)=\begin{cases} U+t_i\left(q_r^i\right) & \text{if} \quad R \text{ sells } q_r^i>0 \\ U & \text{if} \quad R \text{ sells } q_r^i=0 \end{cases}$$

where $U>0$ is an upfront fee (unconditional on the quantity traded) paid by the distributor to platform i in step 1-A, and $t_i(.)$ is a non-linear price, constructed such that

$$\underbrace{\left(p_d^M-c_d\right)q_d^M+t_i\left(q_r^M\right)-c_rq_r^M}_{X}+U=\frac{\Pi_p^E}{2}+U\leq\frac{\Pi^M}{2}$$

while $t_i'-c_r$ is set according to (9). $X+U$ is then platform i's share of the overall joint profit Π^M. The only deviations we have to worry about, are deviations of the type where the distributor deviates together with platform j in step 1-A – either by charging a different marginal wholesale price T_j' or by imposing a different price p_r^j to side r – in order to free ride on platform i's quasi-rents. Now consider any deviation that would cause the joint profit of the pair $R-j$ to rise and, consequentially, platform i's profit to fall below $U+\Pi_p^E/2$. Given that platform i decides to trade with the distributor, it will then earn a profit less than $U+\Pi_p^E/2$. However, by negotiating a contract with one of the exclusive distributors instead, platform i can secure a profit equal to $U+\Pi_p^E/2$. Hence, given that the pair $R-j$ deviates, their joint profit will fall to $\Pi_p^E/2-U$ in the continuation game, and the deviation is therefore unprofitable. We can conclude that equilibria exists with common agency and RPM where i) platforms extract part of their profits up front in step 1-A, ii) non-linear prices $t_i(.)$ (conditional on trade) are implemented to protect each platform against opportunistic deviations on the part of the distributor and the rival platform, and iii) the fully integrated outcome Π^M is induced at stage 2.

Step 2. We now permit the use of either maximum or minimum RPM on p_r^i. By restricting the retailer's ability to either reduce or increase prices to side r, wholesale margins can be adjusted so as to induce the two platforms to set the integrated prices on side d. The appropriate RPM-clause is then determined by the sign of $\partial\pi_R/\partial p_r^i - \partial\Pi/\partial p_r^i \lesseqgtr 0$, when evaluated at fully integrated prices $\mathbf{p}^M$, and given that $t_i' - c_r$ are set according to (9). Rewriting the resulting expression yields

$$\frac{\left\{\begin{array}{c}\left(p_d^M-c_d\right)\left[\frac{\partial q_r^i}{\partial p_d^i}\sum_{k=1,2}\frac{\partial q_d^k}{\partial p_r^i}+\frac{\partial q_d^j}{\partial p_d^i}\sum_{k=1,2}\frac{\partial q_r^k}{\partial p_r^i}\right]_{\mathbf{p}^M}\\ +\left(p_r^M-c_r\right)\left[\sum_{k=1,2}\frac{\partial q_r^k}{\partial p_r^i}\sum_{k=1,2}\frac{\partial q_r^k}{\partial p_d^i}\right]_{\mathbf{p}^M}\end{array}\right\}}{-\frac{\partial q_r^i}{\partial p_d^i}}\lesseqgtr 0 \tag{17}$$

(17) is positive as long as the cross-group externality from side d to side r is negative, i.e. as long as $\partial q_r^i/\partial p_d^i > 0$ – which implies that the appropriate RPM clause in this case is a price ceiling. On the other hand, if both cross-group externalities are positive, then (17) may still be positive, but only as long as

$$\left.\frac{\partial q_d^j}{\partial p_d^i}\right|_{\mathbf{p}^M} < \frac{\left(p_r^M-c_r\right)\left[\sum_{k=1,2}\frac{\partial q_r^k}{\partial p_r^i}\sum_{k=1,2}\frac{\partial q_r^k}{\partial p_d^i}\right]_{\mathbf{p}^M}+\left(p_d^M-c_d\right)\left[\frac{\partial q_r^i}{\partial p_d^i}\sum_{k=1,2}\frac{\partial q_d^k}{\partial p_r^i}\right]_{\mathbf{p}^M}}{-\left(p_d^M-c_d\right)\left.\sum_{k=1,2}\frac{\partial q_r^k}{\partial p_r^i}\right|_{\mathbf{p}^M}} \tag{18}$$

for $i\neq j\in\{1,2\}$. The right-hand side of (18) is always positive, which means that the firms should use price floors as long as the degree of substitution between the platforms on side d is sufficiently strong.Q.E.D.

Proof of Proposition 5.

Step 1. With demands given by (10) and (11) and $c_r = c_d = c$, fully integrated prices to side r and side d are given by

$$p_r^M = \frac{1}{2-\alpha-\beta}(1+c-\beta-c\alpha)$$
$$p_d^M = \frac{1}{2-\alpha-\beta}(1+c-\alpha-c\beta)$$

and

$$S^M = U_r(\mathbf{p}^M) - 2q_r^M p_r^M + U_d(\mathbf{p}^M) - 2q_d^M p_d^M = \frac{2}{1+b}\frac{(1-c)^2}{(2-\alpha-\beta)^2}$$

Step 2. Assume no RPM and that each platform and the retailer negotiates over two-part tariffs with a symmetric wholesale price w. Given a wholesale price w, the platforms and the retailer maximizes profits. Straightforward maximization yields prices

$$p_d^i(w) = \frac{(\beta-2b-b\beta-2w\alpha-w\beta-\alpha\beta+bw\beta+2c+b\alpha\beta+2\alpha c+2)}{b\alpha\beta-\alpha\beta-2b+4}, i=1,2$$
$$p_r^r(w) = \frac{(2w-b+\alpha-bw-\alpha\beta+b\alpha\beta-\alpha c+w\alpha^2-\alpha^2 c+2)}{b\alpha\beta-\alpha\beta-2b+4}, i=1,2$$

$$\begin{aligned}\Delta S \equiv\; & U_r\left(\mathbf{p}^M\right) - 2q_r^M p_r^M + U_d\left(\mathbf{p}^M\right) - 2q_d^M p_d^M \\ & - \left[U_r\left(\mathbf{p}^*\right) - 2q_r^M p_r^M\right] - \left[U_d\left(\mathbf{p}^*\right) - 2q_d^M p_d^M\right]\end{aligned}$$

The wholesale price will be set to maximize industry profit, i.e. $\Pi_A = \pi_1 + \pi_2 + \pi_D$. Let $w = w^*$ be the solution to this maximization. Plugging this back into the prices above yields $\mathbf{p}^* = (\mathbf{p}_d^*, \mathbf{p}_r^*)$, and we can easily calculate $S^* := U_r(\mathbf{p}^*) - 2q_r^M p_r^M + U_d(\mathbf{p}^*) - 2q_d^M p_d^M$, by inserting the prices in the demand function and then into the utility functions at each side of the market.

$$S^{*} = \frac{(1-c)^2}{4(b+1)}\left(\frac{\left(2\beta-2b-b\beta+4\alpha^2+2\alpha^3-\alpha\beta+2\alpha^2\beta-\alpha^3\beta-b\alpha^2\beta+4\right)^2}{\begin{pmatrix} 4-b^2\alpha\beta+b^2+b\alpha^3\beta+b\alpha^2\beta^2+b\alpha\beta+b\beta^2 \\ -4b-\alpha^4-3\alpha^3\beta-\alpha^2\beta^2+4\alpha^2-2\alpha\beta-\beta^2\end{pmatrix}^2}\right.$$

$$\left.+\frac{\left(2\alpha-4b+2\beta+b\alpha-2b\beta+2\alpha^2+\alpha^3+b\alpha^2+\alpha^2\beta+b^2+b\alpha\beta-b\alpha^2\beta-b^2\alpha\beta+4\right)^2}{\begin{pmatrix} 4-b^2\alpha\beta+b^2+b\alpha^3\beta+b\alpha^2\beta^2+b\alpha\beta+b\beta^2 \\ -4b-\alpha^4-3\alpha^3\beta-\alpha^2\beta^2+4\alpha^2-2\alpha\beta-\beta^2\end{pmatrix}^2}\right)$$

We know that with RPM the firms can achieve the collusive prices. Therefore we set $\mathbf{p}_r = \mathbf{p}_r^M$ and let the platforms compete by setting $\mathbf{p}_d$ which yields

$$p_d^i(w) = \frac{\left(2c-2b-\alpha+b\alpha+c\alpha-2c\beta-2w\alpha-\alpha\beta-c\alpha^2+w\alpha^2+bc\beta+b\alpha\beta+w\alpha\beta-bc\alpha\beta+2\right)}{(b-2)(\alpha+\beta-2)}, i =$$

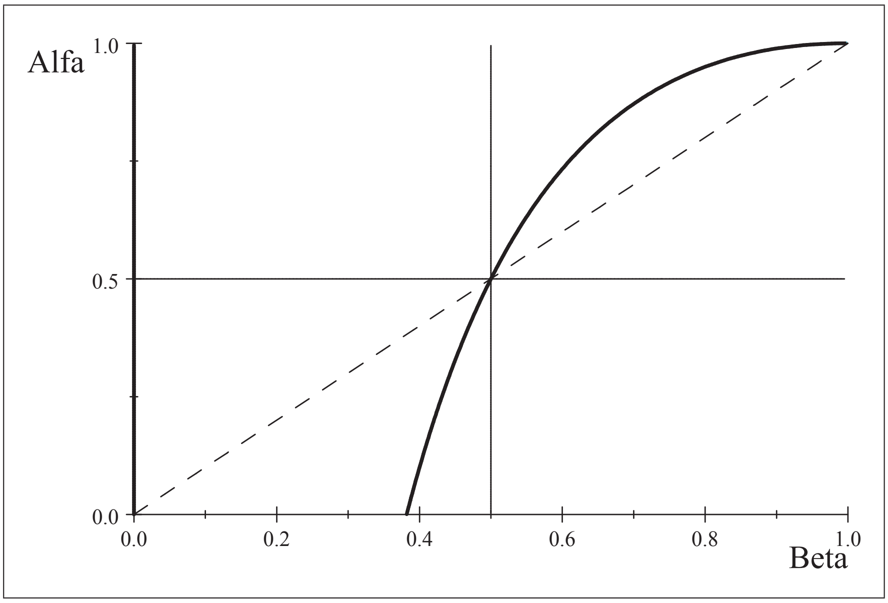
Then we look for the wholesale price that gives the collusive prices to side d, i.e.:

$$\mathbf{p}_d(\mathbf{w}) = \mathbf{p}_d^M$$
$$\Updownarrow$$
$$w = \frac{\left(b-\alpha-c\alpha+\alpha\beta+c\alpha^2-bc-b\alpha\beta+bc\alpha\beta\right)}{\alpha(\alpha+\beta-2)} := w^I$$

For $\alpha, \beta > 0$, $w^I > 0$ as long as b, α, and β are not too small, and α not too small compared to β. w^I is decreasing in b. Specifically for $\alpha, \beta > 0$, $w^I \geq 0$ as long as

$$\alpha \geq \frac{1}{2b}\left(1+b-b\beta-\sqrt{5b^2-2b\beta-2b-2b^2\beta+b^2\beta^2+1}\right),$$

which is increasing in b. The following graph plots the condition when $c = 1/2$ and $b \to 1$:



Step 3: The condition for maximum or minimum RPM:

Given that $\mathbf{p} = \mathbf{p}^M$ and $w = w^I$, we investigate if the retailer will increase or reduce any p_r^i from the collusive level (when $p_r^i = p_r^M$). This is equivalent to check the sign of (17):

$$\begin{aligned} (17) &\lesseqgtr 0 \\ &\Updownarrow \\ \frac{1}{\alpha}\frac{b-\alpha}{b+1}\frac{1-c}{\alpha+\beta-2} &\lesseqgtr 0 \end{aligned}$$

This means that the platforms will use minimum (or fixed) RPM as long as $\beta, \alpha > 0$ and when platform competition is sufficiently strong:

$$b > \alpha := \widetilde{b}$$

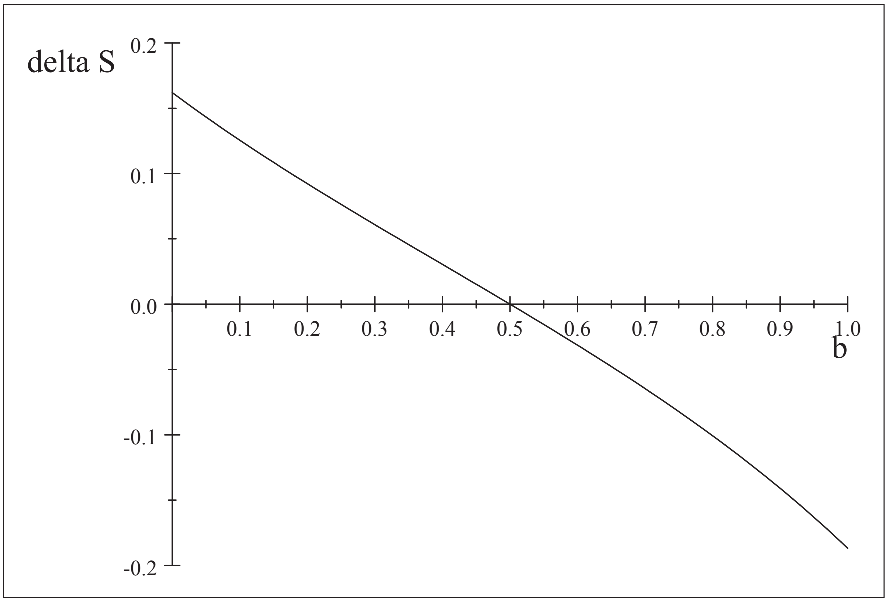
and they will use maximum (or fixed) RPM otherwise.

Finally compare consumer welfare with and without RPM when $\beta, \alpha > 0$. We calculate

$$\begin{aligned} \Delta S &= S^M - S^* \\ &\Updownarrow \\ \Delta S &= \frac{1}{4}\frac{(\alpha - b)\, K(\alpha, \beta, b)}{(2-\alpha-\beta)^2 \begin{pmatrix} 4 - b^2\alpha\beta + b^2 + b\alpha^3\beta + b\alpha^2\beta^2 + b\alpha\beta + b\beta^2 \\ -4b - \alpha^4 - 3\alpha^3\beta - \alpha^2\beta^2 + 4\alpha^2 - 2\alpha\beta - \beta^2 \end{pmatrix}^2} \end{aligned}$$

where $K(\alpha, \beta, b)$ is a long and complex function. It can be shown that $K(\alpha, \beta, b) > 0$ as longs as $\beta, \alpha, b \in (0, 1)$. Hence ΔS is positive when $b < \alpha$ (which is when maximum RPM is used) and negative when $b > \alpha$ (which is when minimum RPM is used.

Below we have plotted ΔS for $\alpha = \beta = c = 1/2$.



Q.E.D.

References

[1] Amelio, A., and Jullien, B. (2012). ”Tying and freebies in two-sided markets”. *International Journal of Industrial Organization*, 30(5), 436-446.

[2] Argentesi, E., and Filistrucchi, L. (2007). ”Estimating market power in a two-sided market: The case of newspapers”. *Journal of Applied Econometrics*, 22(7), 1247-1266.

[3] Armstrong, M., and J. Wright (2007). ”Two-sided markets, competitive bottlenecks andexclusive contracts”. *Economic Theory*, 32(2), 353-380.

[4] Bonanno, G., and Vickers, J. (1988). ”Vertical separation”. *The Journal of Industrial Economics*, 257-265.

[5] Choi, J. P. (2010). ”Tying in two-sided markets with multi-homing”. *The Journal of Industrial Economics*, 58(3), 607-626.

[6] Evans., D. (2013), ”Economics of Vertical Relations for Multi-sided Platforms”, working paper.

[7] Evans, D. S., and Noel, M. (2005). ”Defining antitrust markets when firms operate two-sided platforms”. *Colum. Bus. L. Rev.*, 667.

[8] Filistrucchi, L. and T. Klein (2013). ”Price Competition in Two-Sided Markets with Heterogeneous Consumers and Network Effects”. working paper.

[9] Gabrielsen, T.S. and B.O. Johansen (2013). "Resale price maintenance and up-front payments: Achieving horizontal control under seller and buyer power." WP 02/2013, Department of Economics, University of Bergen.

[10] Gabszewicz, J. J., Laussel, D., and Sonnac, N. (2005). ”Does advertising lower the price of newspapers to consumers? A theoretical appraisal”. *Economics Letters*, 87(1), 127-134.

[11] Innes, R., and Hamilton, S. F. (2009). ”Vertical restraints and horizontal control”. *The RAND Journal of Economics*, 40(1), 120-143.

[12] Lee, R. S. (2013). ”Vertical Integration and Exclusivity in Platform and Two-Sided Markets”. *American Economic Review*, 103 (7), 2960-3000.

[13] Marx, L. M., and Shaffer, G. (2007). "Upfront payments and exclusion in downstream markets". *The RAND Journal of Economics*, 38(3), 823-843.

[14] Miklós-Thal, J., Rey, P., and Vergé, T. (2011). "Buyer power and intrabrand coordination". *Journal of the European Economic Association*, 9(4), 721-741.

[15] O'Brien, D. P., and Shaffer, G. (1992). "Vertical control with bilateral contracts". *The RAND Journal of Economics*, 299-308.

[16] OECD (2009). "Policy Roundtables: Two-Sided Markets", DAF/COMP(2009)20.

[17] Rey, P., and Vergé, T. (2010). "Resale price maintenance and interlocking relationships". *The Journal of Industrial Economics*, 58(4), 928-961.

[18] Rochet, J. C., and Tirole, J. (2008). "Tying in two-sided markets and the honor all cards rule". *International Journal of Industrial Organization*, 26(6), 1333-1347.

[19] Ruhmer, I. (2011). "Platform Collusion in Two-Sided Markets", working paper.

[20] Wright, J. (2004). "One-sided logic in two-sided markets". *Review of Network Economics*, 3(1).