Determinants of multiple states business exit in Europe

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Background and Motivation

- Since the late 1960s, the relationship between the financial status of a company and its probability of failure has been investigated by means of statistical methods that aim at discriminate firms with high probability of failure from those considered to be healthy.
- In recent years, the number of corporate collapses has increased and the study of financial distress has attracted renewed attention from researchers and practitioners.
- Starting from the seminal papers of Beaver (1966) and Altman (1968), a number of authors have focused on the failing and non-failing dichotomy, (Altman, 1968; Ohlson, 1980; Zmijewsky, 1984; Gilbert *et al.*, 1990; Kease and McGuiness, 1990; Lennox, 1999; Shumway, 2001; Dickerson *et al.*, 2003).
- A company may exit the market for several reasons, such as through merger, acquisition, voluntary liquidation or bankruptcy, and each type of exit is likely to be affected by different factors (Harhoff *et al.*, 1998; Prantl, 2003; Rommer, 2005).

Background and Motivation

- Theoretical evidence for considering multi-exits was first provided by Schary (1991), which analyzed acquisition and bankruptcy as alternative routes.
- Multiple states of corporate financial distress have been examined by various studies (Bhattacharjee *et al.*, 2004; Headd, 2003; Jones and Hensher, 2004, 2007; Rommer, 2005; Hensher, Jones and Greene, 2007; Chancharat *et al.*, 2012).

However, in this context several issues still need further investigation:

1. the similarities or the differences in the factors determining the various exit routes;
2. the variable selection methods in define the best set of predictors;
3. the contribution of each variables to the performance of a model;
4. the evaluation of the model's ability in predicting the firm's exit.

Research Questions

The research questions here addressed are:

- Is the competing risks approach better than the single-risk model?
- How can we efficiently select the optimal predictors subset?
- How to evaluate the forecasting performance in terms of predictive accuracy?
- Which accuracy measures can be considered for evaluating the ability of a model, when the outcome is not binary?
- How to estimate the optimal cut-off for computing the accuracy measures, in case of non-binary data?

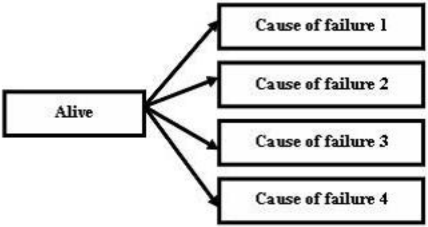
Some recent results .....

- Amendola, A., Restaino, M., & Sensini, L. (2011). Variable selection in default risk model. *Journal of Risk Model Validation*, 5, 3-19.
- Amendola, A., Restaino, M., & Sensini, L. (2011). Competing risks analysis of the determinants of business exit. Isforges.
- Amendola, A., Restaino, M., & Sensini, L.(2012). Dynamic Statistical Models for Corporate Failure Prediction in Italy. *Journal of Modern Accounting and Auditing*, 8,1214-1224.
- Amendola, A., Restaino, M., & Sensini, L.(2013). Corporate Financial Distress And Bankruptcy: A Comparative Analysis In France, Italy And Spain. *Global Economic Observer*,1,131-142.
- Amendola, A., Restaino, M., & Sensini, L. (2014). An analysis of the determinants of financial distress in Italy: A competing risks approach. *International Review of Economics & Finance*.

Outline

Competing Risks model

- Suppose that firm i $(i = 1, \ldots, N)$ is at risk for K different kinds of events, $(k = 1, \ldots, K)$.
- Let D_i be the cause of failure for the firm i.
- Assume the independent competing risks, i.e. the competing risks are mutually exclusive and exhaustive.



Competing Risks model

- The *cause-specific hazard function* is the instantaneous risk of failing at a given time t from a given cause, among all individuals at risk at that time:

$$\lambda_k(t) = \lim_{\Delta t \to 0} \frac{P[t \leq T \leq t + \Delta t, D = k | T \geq t]}{\Delta t}. \quad (1)$$

- It represents the rate of occurrence of failure k.
- The overall hazard function can be written as:

$$\lambda(t) = \sum_{k=1}^{K} \lambda_k(t) \quad (2)$$

Competing Risks model

- The cause-specific hazard function of cause k for a firm i is estimated by Cox proportional-hazards Model, where the conditional hazard rate is given by:

$$\lambda_{ki}(t|\underline{x}_i(t), \underline{\beta}_k) = \lambda_{0k}(t)\exp\{\underline{\beta}_k^{'}\underline{x}_{ik}(t)\}, \tag{3}$$

  where
  - $\lambda_{0k}(t)$ is the baseline cause-specific hazard of cause k at time t, that measures the effect of time and the event k on the hazard rate for a firm whose variables all have values of zero;
  - $\underline{x}_{ik}(t)$ is a vector of p covariates for firm i;
  - $\underline{\beta}_k$ is the vector of p unknown regression parameters to be estimated for the cause k.
- Since the same variables could have different effects on the different risks, it is reasonable to assume that β_k's are independent of each other.

Competing Risks model

- The partial likelihood function for each hazard k is given by:

$$L_k(\underline{\beta}_k) = \prod_{i=1}^{n_k} \frac{\exp[\underline{\beta}_k^{'} \underline{x}_{ik}(t_{ik})]}{\sum_{l \in R(t)} \exp[\underline{\beta}_k^{'} \underline{x}_{lk}(t_{lk})]} \tag{4}$$

where n_k refers to the number of firms in specific hazard k, $t_{1k} < \cdots < t_{n_k k}$ denotes the n_j ordered failures of hazard k, $R(t_{ik})$ is the set of firms that have not exited the market at time t_{ik}.

- The likelihood function for the Competing Risks model is:

$$L(\underline{\beta}_1, \ldots, \underline{\beta}_K) = \prod_{k=1}^{K} L_k(\underline{\beta}_k) = \prod_{k=1}^{K} \prod_{i=1}^{n_k} \frac{\exp[\underline{\beta}_k^{'} \underline{x}_{ik}(t_{ik})]}{\sum_{l \in R(t)} \exp[\underline{\beta}_k^{'} \underline{x}_{lk}(t_{lk})]} \tag{5}$$

Variable selection

- A major problem for the analysts who attempt to forecast the risk of failure is identifying the *optimal subset* of predictive variables.
- Different selection methods can be considered. They mainly based on:
  - * personal judgment
  - * empirical and theoretical evidence
  - * metaheuristic strategies
  - * statistical methods

Variable selection - Subset Regression

Try to determine the set of the most important regressors removing the noise regressors from the model.

Methods

- All subsets
- Backward selection
- Forward selection
- Stepwise selection

Limits and drawbacks

- Small change in the data can lead to very different solutions.
- It does not work well in presence of multicollinearity.
- Predictors are included one by one, significant combinations and iterations of variables could be easily missed.

Variable selection - Shrinkage Methods

Try to find a stable model that fits the data well via constrained least squares optimization.[1]

- Lasso.
- Least Angle Regression.
- Elastic Net.
- . . . .

[1] Least Absolute Shrinkage and Selection Operator, Lasso (Tibshirani, 1996, 1997)
Variable selection in default risk model. (Amendola et. al. 2011)

Variable selection - Lasso in Competing Risks model (1)

- The lasso estimator for the failure k is given by:

$$\hat{\underline{\beta}}_k = \underset{\underline{\beta}_k}{\operatorname{argmax}} \ \log(L(\underline{\beta}_k)) =$$

$$\underset{\underline{\beta}_k}{\operatorname{argmax}} \sum_{i}^{n_k} \left[\exp[\underline{\beta}_k^{'} \underline{x}_i(t_{ik})] - \log \sum_{l \in R(t)} \exp[\underline{\beta}_k^{'} \underline{x}_l(t_{lk})] \right] \tag{6}$$

subject to

$$||\underline{\beta}_k||_1 \leq s_k$$

where

- $||\underline{\beta}_k||_1 = |\beta_k^1| + |\beta_k^2| + \cdots + |\beta_k^p|$ is the L_1 norm of the coefficients vector $\underline{\beta}_k$ for the failure cause k, and
- s_k is the *tuning parameter* which quantifies the magnitude of the constraints on the L_1 norm of the coefficients vectors and determines the number of coefficients estimated as zero in the model.

Variable selection - Lasso in Competing Risks model (2)

- This is equivalent to:

$$\hat{\underline{\beta}}_k = \underset{\underline{\beta}_k}{\operatorname{argmin}} \left[-\log(L_k(\underline{\beta}_k)) + \lambda_k ||\underline{\beta}_k||_1 \right] \quad (7)$$

where $\lambda_k \geq 0$ is the *tuning parameter* which determines the magnitude of penalty on the log partial likelihood.

Variable selection - Lasso in Competing Risks model (3)

- The lasso estimator for the Competing Risks model is given by:

$$(\hat{\underline{\beta}}_1, \ldots, \hat{\underline{\beta}}_k) = \underset{\underline{\beta}_k}{\operatorname{argmax}}\ l(\underline{\beta}_1, \ldots, \underline{\beta}_k) =$$

$$\underset{\underline{\beta}_k}{\operatorname{argmax}} \sum_{K=1}^{k} \sum_{i}^{n_k} \left[\exp[\underline{\beta}'_K \underline{x}_i(t_{iK})] - \log \sum_{l \in R(t)} \exp[\underline{\beta}'_K \underline{x}_l(t_{lK})] \right] \tag{8}$$

subject to

$$||\underline{\beta}_1||_1 \leq s_1$$
$$||\underline{\beta}_2||_1 \leq s_2$$
$$\ldots$$
$$||\underline{\beta}_k||_1 \leq s_k$$

Accuracy measures

The Forecasting Performance of the models for multiple causes is evaluated by some measures of accuracy, based on the confusion matrix.

Table 1: $K \times K$ Confusion Matrix

		Actual Class				
		1	*2*	*3*	...	*K*
Predicted Class	*1*	TP(1)	$E(1\|2)$	$E(1\|3)$	...	$E(1\|K)$
	2	$E(2\|1)$	TP(2)	$E(2\|3)$	...	$E(2\|K)$
	3	$E(3\|1)$	$E(3\|2)$	TP(3)	...	$E(3\|K)$
	...	...	...	...	...	...
	K	$E(K\|1)$	$E(K\|2)$	$E(K\|3)$	...	TP(K)

Accuracy measures

The confusion matrix in case of binary data is given by:

Table 2: Confusion Matrix for binary data

		Actual Class		
		Non-Distressed	*Distressed*	
Predicted	*Non-Distressed*	True Positives	False Positives	TP+FP
Class	*Distressed*	False Negatives	True Negatives	FN+TN
		TP+FN	FP+TN	TP+FN+FP+TN

Accuracy measures

The measures of accuracy for binary data can be calculate as:

- Accuracy: i.e. the correct classification rate: it is the proportion of firms classified correctly.
- FP rate (Type I error): proportion of distressed firms misclassified as a non-distressed firm.
- FN rate (Type II error): proportion of non-distressed firms wrongly assigned to the distressed group.
- TP rate (Sensitivity): proportion of non-distressed firms classified correctly.
- TN rate (Specificity): proportion of distressed firms classified correctly.

Accuracy measures for multiple exit

Two main research questions arise in this context.

- How to extend those accuracy measures when the outcome is not binary?
- How to estimate the optimal cut-off for computing the accuracy measures, in case of non-binary data?

Business Exit in Europe: layout of the analysis

A competing-risks approach is considered for investigating the determinants of corporate financial distress.

- A comparative analysis of three European markets - France, Italy and Spain - is performed.
- The effects of micro-economic indicators and firm-specific variables on the different states have been estimate via three different models.
    - A competing-risks model for each country.
    - A pooled-state (single risk) model in which all financial distress states are considered at the same time.
    - A pooled-country model in which all countries are pooled together.
- The significant predictors and their sign are compared across the three country models in order to determine the similarities and the differences in the variables that influence the financial distress.

The data

- The sample consists of firms operating in European countries - France, Italy and Spain [2].
- The disease sample is composed of industrial firms that left the market between 2004 and 2010.
- The healthy sample has been randomly selected among the industrial firms according to the following criteria:
  - they were still active at time $t = 2010$;
  - they have not incurred in any kind of exit routes in the period 2004 - 2010;
  - they have provided full information between 2004 and 2010.

[2] The data were collected from AMADEUS - Bureau Van Dijk

The data

- The causes of exiting the market (as available in Amadeus) are:

Table 3: Financial Status

	Italy	France	Spain
Active	12,292	13,102	1898
Bankruptcy	610	264	548
Liquidation	273	37	13

- We concentrate our attention on two mutually exclusive states of exit from the market:
  1. *bankruptcy*: this is a legally declared inability of a company to pay its creditors. The company no longer exists because it has ceased its activities because of the process of bankruptcy;
  2. *liquidation*: the company no longer exists because it has ceased its activities, because of the process of liquidation.
- The reference group is provided by active firms.

The data

- The sample is divided into two parts:
  - *in-sample*: used for the classification ability, in order to determine how accurately a model classifies businesses;
  - *out-of-sample*: used for prediction ability, in order to determine how accurately a model classifies new businesses.
- The time-horizon is 2004-2010.
- The predictions' windows are: 1-year ahead and 2-years ahead.

Financial predictors data-base

- The predictors database was elaborated starting from the financial statements of each firm included in the sample for a total of 197,181 balance sheets.
- We computed nv=72 indicators selected as potential predictors among the most relevant in highlighting current and prospective conditions of operational unbalance (Altman, 2000; Dimitras *et al.*, 1996).
- The financial indicators reflect different aspects of the firms' structure (Table 4).
- Firm-specific variables, such as national legal status, firm size, firm age, publicly quotation, are also considered.

Table 4: Financial Predictors

Area	nv
Liquidity	19
Size and capitalization	11
Profitability	16
Turnover	13
Operating structure	13

Number of variables selected

Table 5: Number of variables selected

	Single-risk model				Bankruptcy				Liquidation			
	Pooled-C.	France	Italy	Spain	Pooled-C.	France	Italy	Spain	Pooled-C.	France	Italy	Spain
Firm-specific variables	4	0	6	4	5	1	5	4	4	1	4	0
Liquidity ratios	6	5	6	7	6	6	7	4	2	1	2	0
Operating structure ratios	5	3	5	4	3	2	1	4	3	1	3	0
Profitability ratios	5	5	8	3	4	4	8	3	3	1	6	0
Size and capitalization ratios	6	2	3	3	4	2	4	3	4	0	3	2
Turnover ratios	7	2	5	6	6	2	5	6	4	2	6	0
Total	33	17	33	27	28	17	30	24	20	6	24	2

The sign of covariates for the single risk model

Code	Variable	Area	Pooled-C.	France	Italy	Spain
IND01	Current assets/Fixed assets	Liquidity			+	
IND03	Cash and cash equivalent/Current liabilities	Liquidity	+			+
IND04	(Current assets - Stock)/Current liabilities	Liquidity	-			
IND06	Working capital/Total assets	Liquidity	-	-	-	+
IND07	Net current assets/Total assets	Liquidity			+	
IND24	Cash flow	Liquidity		-	-	+
IND27	Cash flow/Shareholders funds	Liquidity	-			-
IND30	Current liabilities/Total assets	Liquidity	+	+	+	+
IND31	Current liabilities/(Curr. liab. + Non-Curr. liab.)	Liquidity		-		-
IND33	Cash and cash equivalent/Total assets	Liquidity	-		-	+
IND35	Cash and cash equivalent/Sales	Liquidity		-		
IND46	Financial Expenses/(Curr. liab. + Non-Curr. liab.)	Operating structure	+		+	+
IND47	Financial Expenses/Sales	Operating structure	-		-	-
IND53	EBIT/Operating revenue	Operating structure	-	+	+	-
IND54	Sales	Operating structure	-	-	+	-
IND61	EBIT/Financial Expenses	Operating structure	-	-	-	
IND15	Profit (Loss) for Period/Shareholders funds	Profitability	+	+	+	
IND17	Profit (Loss) for Period/Sales	Profitability			+	-
IND29	Profit (Loss) for Period/(Curr. liab. + Non-Curr. liab.)	Profitability			+	
IND39	EBITDA/Sales	Profitability		-	-	
IND41	EBIT/Fixed Assets	Profitability	+	+		+
IND49	EBIT/Total assets	Profitability	+		+	
IND64	EBIT	Profitability	+		+	
IND71	Standard deviation ROE	Profitability		-	-	
IND72	Standard deviation ROA	Profitability	+	+	+	+
IND10	Shareholders funds/(Curr. liab. + Non-Curr. liab.)	Size and capitalization	+	+	-	
IND11	Shareholders funds/Capital	Size and capitalization	-		-	-
IND13	(Long Term Debt + Loans)/Total assets	Size and capitalization	+		-	
IND38	Current assets/Current liabilities	Size and capitalization	+			
IND50	Current assets/Total assets	Size and capitalization	+	+		-
IND62	Total assets	Size and capitalization	+			+
IND19	Sales/Current assets	Turnover	+		-	
IND20	Debtors/Sales	Turnover	+	+	+	+
IND21	Sales/Shareholders funds	Turnover	+		+	+
IND36	(Current assets - Stock)/Sales	Turnover			+	-
IND37	Working capital/Sales	Turnover	-			-
IND56	Cash and cash equivalent/Depreciation	Turnover	+	-	+	+
IND69	(Debtors/Operating revenue)*360	Turnover	+			
IND70	(Creditors/Operating revenue)*360	Turnover	-			-

The sign of covariates for the the bankruptcy state

Code	Variable	Area	Pooled-C.	France	Italy	Spain
IND03	Cash and cash equivalent/Current liabilities	Liquidity	+		-	+
IND04	(Current assets - Stock)/Current liabilities	Liquidity	-		+	
IND06	Working capital/Total assets	Liquidity	-	-	-	
IND07	Net current assets/Total assets	Liquidity			+	
IND24	Cash flow	Liquidity		-	-	+
IND27	Cash flow/Shareholders funds	Liquidity	-			-
IND30	Current liabilities/Total assets	Liquidity	+	+	+	
IND31	Current liabilities/(Curr. liab. + Non-Curr. liab.)	Liquidity		-		
IND33	Cash and cash equivalent/Total assets	Liquidity	-			
IND34	(Current assets - Stock)/Total assets	Liquidity		-	-	-
IND35	Cash and cash equivalent/Sales	Liquidity		-		
IND46	Financial Expenses/(Curr. liab. + Non-Curr. liab.)	Operating structure				+
IND47	Financial Expenses/Sales	Operating structure	-			-
IND53	EBIT/Operating revenue	Operating structure	-	+		-
IND54	Sales	Operating structure	-			-
IND61	EBIT/Financial Expenses	Operating structure		-	-	
IND15	Profit (Loss) for Period/Shareholders funds	Profitability	+	+	+	
IND17	Profit (Loss) for Period/Sales	Profitability				-
IND29	Profit (Loss) for Period/(Curr. liab. + Non-Curr. liab.)	Profitability			+	
IND39	EBITDA/Sales	Profitability		-	-	
IND41	EBIT/Fixed Assets	Profitability			+	+
IND49	EBIT/Total assets	Profitability	+		+	
IND64	EBIT	Profitability	+		+	
IND71	Standard deviation ROE	Profitability		-	-	
IND72	Standard deviation ROA	Profitability	+	+	+	+
IND08	Shareholders funds/Fixed assets	Size and capitalization			-	
IND10	Shareholders funds/(Curr. liab. + Non-Curr. liab.)	Size and capitalization	+			
IND11	Shareholders funds/Capital	Size and capitalization	-		-	-
IND13	(Long Term Debt + Loans)/Total assets	Size and capitalization	+		-	
IND38	Current assets/Current liabilities	Size and capitalization	+			-
IND50	Current assets/Total assets	Size and capitalization		+	+	
IND62	Total assets	Size and capitalization		-		+
IND19	Sales/Current assets	Turnover			-	
IND20	Debtors/Sales	Turnover	+	+	+	+
IND21	Sales/Shareholders funds	Turnover	+		+	+
IND36	(Current assets - Stock)/Sales	Turnover			+	-
IND37	Working capital/Sales	Turnover	-		-	-
IND56	Cash and cash equivalent/Depreciation	Turnover	+	-		+
IND69	(Debtors/Operating revenue)*360	Turnover	+			
IND70	(Creditors/Operating revenue)*360	Turnover	-			-

The sign of covariates for the the liquidation state

Code	Variable	Area	Pooled-C.	France	Italy	Spain
IND06	Working capital/Total assets	Liquidity	-	-	-	
IND33	Cash and cash equivalent/Total assets	Liquidity	-		-	
IND46	Financial Expenses/(Current liabilities + Non-Current liabilities)	Operating structure			+	
IND53	EBIT/Operating revenue	Operating structure	+		+	
IND54	Sales	Operating structure	-	-		
IND61	EBIT/Financial Expenses	Operating structure	-		-	
IND17	Profit (Loss) for Period/Sales	Profitability			+	
IND29	Profit (Loss) for Period/(Current liabilities + Non-Current liabilities)	Profitability			+	
IND39	EBITDA/Sales	Profitability			-	
IND41	EBIT/Fixed Assets	Profitability	+		+	
IND71	Standard deviation ROE	Profitability	+		+	
IND72	Standard deviation ROA	Profitability	+	+	+	
IND08	Shareholders funds/Fixed assets	Size and capitalization	+			-
IND10	Shareholders funds/(Current liabilities + Non-Current liabilities)	Size and capitalization	-		-	
IND11	Shareholders funds/Capital	Size and capitalization	-		-	
IND13	(Long Term Debt + Loans)/Total assets	Size and capitalization	-		-	
IND62	Total assets	Size and capitalization				-
IND18	Sales/Fixed assets	Turnover		+		
IND19	Sales/Current assets	Turnover	-		-	
IND20	Debtors/Sales	Turnover	+			
IND21	Sales/Shareholders funds	Turnover	+		+	
IND36	(Current assets - Stock)/Sales	Turnover			+	
IND56	Cash and cash equivalent/Depreciation	Turnover			+	
IND58	Non-Current liabilities/Sales	Turnover	-		-	
IND70	(Creditors/Operating revenue)*360	Turnover		-	+	

Selected predictors - single-risk model

Looking at the results of the single-risk model (in which all exit routes are pooled together) for the pooled-country model, it can be noted that

1. The joint-stock companies have a greater probability of failure compare to limited partnership and consortium.
2. The old firms (more than 23 years) have a negative coefficient and their risk decreases while the medium and large size are not significant.
3. Then, high values of IND03, IND30, IND46, IND15, IND41, IND64, IND72, IND10, IND13, IND38, IND50, IND62, IND19, IND20, IND21, IND56, IND69 correspond to increase in the probability of failure.
4. The coefficients of the IND04, IND06, IND27, IND33, IND47, IND54, IND61, IND11, IND37, IND70 are negative and consequently the probability of failure decrease.

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Country Comparison - single-risk model

1. The joint-stock companies have a lower probability of default in Italy and Spain.
2. The medium firms in Italy have a higher risk of failure, while for the very large companies it is lower.
3. In Spain the large and very large firms have a higher probability of being dissolved.
4. There are some financial ratios in common between the three countries, even though the sign of coefficients is different. For example, Working capital/Total assets and Cash flow has a negative coefficient for Italy and France, while it is positive for Spain.

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4. There are some financial ratios in common between the three countries, even though the sign of coefficients is different. For example, Working capital/Total assets and Cash flow has a negative coefficient for Italy and France, while it is positive for Spain.

Country Comparison - single-risk model

1. The joint-stock companies have a lower probability of default in Italy and Spain.
2. The medium firms in Italy have a higher risk of failure, while for the very large companies it is lower.
3. In Spain the large and very large firms have a higher probability of being dissolved.
4. There are some financial ratios in common between the three countries, even though the sign of coefficients is different. For example, Working capital/Total assets and Cash flow has a negative coefficient for Italy and France, while it is positive for Spain.

Selected predictors - competing-risk model

The results of the competing risks framework for the pooled-country model showed that:

1. The joint-stock companies, limited companies, limited parternship have a higher risk of being bankrupted.
2. The consortium and limited cooperative societies have a higher risk of liquidation.
3. As concerns the financial ratios, high values of IND03, IND30, IND15, IND49, IND64, IND72, IND13, IND38, IND20, IND21, IND56, IND69 correspond to high risk of being bankrupt.
4. Moreover, some covariates, such as IND53, IND41, IND71, IND72, IND08, IND20, IND21, have positive coefficients, related to a higher risk of being liquidated.
5. Then, IND04, IND06, IND27, IND33, IND47, IND53, IND54, IND11, IND37, IND70 have negative effect of the bankruptcy, and IND06, IND54, IND61, IND10, IND11, IND13, IND19, IND58 effect negatively the risk of liquidation.

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Country Comparison - competing-risk model

- By checking the results of the competing risks model for each country, it can be observed that only few variable are selected as significant in all the country (ROA s.d and Debtors/Sales for the bankruptcy state).
- The model for Italy needs much more variables for predicting bankruptcy and liquidation than in France and in Spain.
- One possible reason is related to the sample period considered, which included the period 2007-2010 characterized by the *s*tarting of the global financial crisis.
- It seems that the effects of the financial crisis have had a deeper impact in Italy than in France and Spain.

Accuracy measures

Since we have three events, a 3×3 table has been considered:

Table 6: Confusion Matrix

		Actual Class			
		Non-Distressed (code 0)	*Bankruptcy (code 1)*	*Liquidation (code 2)*	
Predicted Class	*Non-Distressed (code 0)*	TP(0)	$E(0\|1)$	$E(0\|2)$	P_0
	Bankruptcy (code 1)	$E(1\|0)$	TP(1)	$E(1\|2)$	P_1
	Liquidation (code 2)	$E(2\|0)$	$E(2\|1)$	TP(2)	P_2
		T_0	T_1	T_2	$T_0 + T_1 + T_2$

Accuracy measures

Based on the table, the accuracy measures for three competing risks can be computed.

- Accuracy, given by $\frac{TP(0)+TP(1)+TP(2)}{T_0+T_1+T_2}$;

Table 7: Confusion Matrix

		Actual Class			
		Non-Distressed (code 0)	*Bankruptcy (code 1)*	*Liquidation (code 2)*	
Predicted Class	*Non-Distressed (code 0)*	TP(0)	$E(0\|1)$	$E(0\|2)$	P_0
	Bankruptcy (code 1)	$E(1\|0)$	TP(1)	$E(1\|2)$	P_1
	Liquidation (code 2)	$E(2\|0)$	$E(2\|1)$	TP(2)	P_2
		T_0	T_1	T_2	$T_0+T_1+T_2$

Accuracy measures

Based on the table, the accuracy measures for three competing risks can be computed.

- Accuracy;
- FP rate (Type I error), given by:

$$FPrate = \frac{E(0|1) + E(0|2)}{T_1 + T_2} \tag{9}$$

Table 8: Confusion Matrix

		Actual Class			
		Non-Distressed (code 0)	*Bankruptcy (code 1)*	*Liquidation (code 2)*	
Predicted Class	*Non-Distressed (code 0)*	TP(0)	$E(0\|1)$	$E(0\|2)$	P_0
	Bankruptcy (code 1)	$E(1\|0)$	TP(1)	$E(1\|2)$	P_1
	Liquidation (code 2)	$E(2\|0)$	$E(2\|1)$	TP(2)	P_2
		T_0	T_1	T_2	$T_0 + T_1 + T_2$

Accuracy measures

Based on the table, the accuracy measures for three competing risks can be computed.

- Accuracy;
- FP rate (Type I error);
- FN rate (Type II error);

$$FNrate = \frac{E(1|0) + E(2|0)}{T_0} \quad (10)$$

Table 9: Confusion Matrix

		Actual Class			
		Non-Distressed (code 0)	*Bankruptcy (code 1)*	*Liquidation (code 2)*	
Predicted Class	*Non-Distressed (code 0)*	TP(0)	$E(0\|1)$	$E(0\|2)$	P_0
	Bankruptcy (code 1)	$E(1\|0)$	TP(1)	$E(1\|2)$	P_1
	Liquidation (code 2)	$E(2\|0)$	$E(2\|1)$	TP(2)	P_2
		T_0	T_1	T_2	$T_0 + T_1 + T_2$

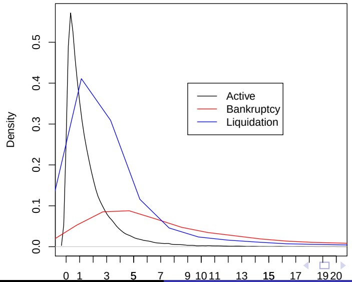
Optimal threshold

- An important task in investigating the failure prediction is the determination of the optimal threshold point used to classify firms into groups (bankruptcy, liquidation, etc.).
- In case of binary outcome (Failed vs. Non failed), the most used methods for computing the threshold are:
  - looking at the empirical distribution of two groups and checking where they intersect;
  - minimizing the total number of misclassifications;
  - minimizing the expected cost of misclassifications;
  - ....
- Since now, no studies are available when there are more than two categories.

Optimal threshold in Competing Risk Models

- In this case we need to determine a threshold point which discriminate between the three groups.
- A possible approach is to looking at the empirical distribution of the risk rate of the three groups.
- Starting from the values at which the curves intersect, we select a range of value to be optimized in terms of accuracy measure.

Figure 1: Risk score for Italy and for 1-year ahead.



Optimal threshold in Competing Risk Models

- The range we choose are:
  - Italy, 1-year ahead: cut1: 0.7-1.7; cut2: 5.0-7.0 (combinations=231);
  - Italy, 2-year ahead: cut1: 0.5-1.5; cut2: 4.0-6.0 (combinations=231);
  - France, 1-year ahead: cut1: 0.8-1.8; cut2: 4.0-7.0 (combinations=341);
  - France, 2-year ahead: cut1: 0.8-1.8; cut2: 3.0-6.0 (combinations=341).[3]
- We compute the accuracy measures.
- We choose as best thresholds those values that maximize the accuracy.

[3] Since only few information on bankruptcy are available for Spain, we excluded this country from the further analysis.

Accuracy measures for the optimal threshold - Italy

Cutoff	Accuracy	FP rate	FN rate	TP rate	TN rate	FP rate (B)	FP rate (L)	FN rate (B)	FN rate (L)
Italy - insample 1-year									
1.7-5.0	0.6135	0.6591	0.3485	0.6515	0.3409	0.4448	0.2143	0.0817	0.2668
Italy - outsample 1-year									
1.7-5.0	0.7572	0.7833	0.2368	0.7632	0.2167	0.0833	0.7000	0.0336	0.2032
Italy - insample 2-year									
1.5-4.0	0.5762	0.6060	0.3862	0.6138	0.3940	0.4095	0.1965	0.1112	0.2750
Italy - outsample 2-year									
1.5-4.0	0.7219	0.7086	0.2684	0.7316	0.2914	0.2028	0.5058	0.0505	0.2179

Accuracy measures for the optimal threshold - France

Cutoff	Accuracy	FP rate	FN rate	TP rate	TN rate	FP rate (B)	FP rate (L)	FN rate (B)	FN rate (L)
France - insample 1-year									
1.8-4.0	0.6556	0.6588	0.3265	0.6735	0.3412	0.6036	0.0553	0.1148	0.2117
France - outsample 1-year									
1.8-4.0	0.7315	0.6552	0.2657	0.7343	0.3448	0.4483	0.2069	0.0906	0.1751
France - insample 2-year									
1.8-3.0	0.6503	0.6531	0.3321	0.6679	0.3469	0.6029	0.0501	0.1790	0.1531
France - outsample 2-year									
1.8-3.0	0.7548	0.7846	0.2392	0.7608	0.2154	0.6057	0.1789	0.1210	0.1182

Variable Selection Techniques - The data

- The disease sample is composed of those industrial firms that had left the market in Italy between 2004 and 2009.
- The healthy sample was randomly selected among the Italian industrial firms according to the following criteria:
  - * were still active at time $t = 2009$;
  - * have not incurred in any kind of exit routes in the period 2004 - 2009;
  - * had provided full information between 2004 and 2009 [4].

	%
Active Firms	73.45
Firms in Bankruptcy	8.07
Firms in Liquidation	9.28
Inactive Firms	9.20

[4] The information were collected from AMADEUS - Bureau Van Dijk

The data

- The sample is divided into two parts:
  - *in-sample*: used for the classification ability, in order to determine how accurately a model classified businesses;
  - *out-of-sample*: used for prediction ability, in order to determine how accurately a model classified new businesses.
- The time-horizon considered is 2004-2009.
- The predictions' windows considered are: 1-year ahead and 2-years ahead.

Financial predictors data-base

- The predictors database was elaborated starting from the financial statements of each firm included in the sample for a total of 8030 balance sheets.
- We computed nv=20 indicators selected as potential predictors among the most relevant in highlighting current and prospective conditions of operational unbalance (Altman, 2000; Dimitras *et al.*, 1996).
- The selected indicators reflect different aspects of the firms' structure:

Table 10: Predictors

Area	nv
Profitability Ratio	10
Operational Ratio	5
Structure Ratio	5

Accuracy measures: 1-year ahead

Table 11: In Sample

Italy - in sample (1-year ahead)				
Stepwise method				
	Bankruptcy	Inactive	Liquidation	Single-Risk
Correct Class Rate	0.76185	0.78819	0.78262	0.74881
Type I Error	0.40930	0.58025	0.47353	0.61441
Type II Error	0.22173	0.17134	0.19132	0.11071
Lasso method				
	Bankruptcy	Inactive	Liquidation	Single-Risk
Correct Class Rate	0.83557	0.81371	0.84399	0.75234
Type I Error	0.73798	0.66392	0.65147	0.72103
Type II Error	0.10938	0.13382	0.10561	0.06458

Accuracy measures: 1-year ahead

Table 12: Out-of-Sample

Italy - out-of-sample (1-year ahead)				
Stepwise method				
	Bankruptcy	Inactive	Liquidation	Single-Risk
Correct Class Rate	0.82707	0.85263	0.84211	0.87218
Type I Error	0.66667	0.56250	0.44068	0.52564
Type II Error	0.17069	0.13713	0.13036	0.07496
Lasso method				
	Bankruptcy	Inactive	Liquidation	Single-Risk
Correct Class Rate	0.92632	0.89774	0.90526	0.88421
Type I Error	0.33333	0.62500	0.54237	0.67949
Type II Error	0.07251	0.08937	0.05116	0.04089

Accuracy measures: 2-years ahead

Table 13: In Sample

Italy - in sample (2-year ahead)				
Stepwise method				
	Bankruptcy	Inactive	Liquidation	Single-Risk
Correct Class Rate	0.76445	0.77875	0.77530	0.73571
Type I Error	0.42384	0.56832	0.49301	0.61593
Type II Error	0.21481	0.18020	0.19688	0.11434
Lasso method				
	Bankruptcy	Inactive	Liquidation	Single-Risk
Correct Class Rate	0.77595	0.79780	0.84116	0.73374
Type I Error	0.53974	0.63354	0.67133	0.71484
Type II Error	0.18974	0.15118	0.10569	0.07498

Accuracy measures: 2-years ahead

Table 14: Out-of-Sample

Italy - out-of-sample (2-year ahead)				
Stepwise method				
	Bankruptcy	Inactive	Liquidation	Single-Risk
Correct Class Rate	0.82350	0.85118	0.83522	0.83677
Type I Error	0.11765	0.65347	0.48503	0.59615
Type II Error	0.17756	0.12113	0.13465	0.08037
Lasso method				
	Bankruptcy	Inactive	Liquidation	Single-Risk
Correct Class Rate	0.83265	0.87848	0.89392	0.84655
Type I Error	0.47727	0.68317	0.65868	0.73397
Type II Error	0.16017	0.09071	0.05408	0.04233

Number of selections

		1-years ahead		2-years ahead	
	ID	Stepwise	Lasso	Stepwise	Lasso
	Profitability ratio				
V1	Return on shareholder funds	1	1	1	1
V2	Return on capital employed	1	2	2	2
V3	Return on total assets	1		1	
V4	Profit margin	1		1	
V5	EBITDA			3	2
V6	EBIT	3	3	2	1
V7	Cash flow/Operating revenue	2	3	2	2
V8	ROE	1	2	2	2
V9	ROA	4	2	1	
V10	ROCE	2	2	2	2
	Operational ratio				
V11	Net assets turnover	4	3	4	3
V12	Interest cover	2	2	2	2
V13	Stock turnover	1	2	2	3
V14	Collection period	3	3	3	3
V15	Credit period	4	4	4	4
	Structure ratio				
V16	Current ratio	2	3	2	1
V17	Liquidity ratio	1	1	2	1
V18	Shareholders liquidity ratio		2	1	1
V19	Solvency ratio	4	4	5	5
V20	Gearing	2	2	2	2

Sign of coefficients estimates - 1-year ahead

Italy in-sample 2004-2008 (1-years ahead)									
	ID	Bankruptcy		Inactive		Liquidation		Single-Risk	
		Stepwise	Lasso	Stepwise	Lasso	Stepwise	Lasso	Stepwise	Lasso
Profitability ratio									
V1	Return on shareholder funds			−	−				
V2	Return on capital employed		+	−	−				
V3	Return on total assets	+							
V4	Profit margin	−							
V5	EBITDA								
V6	EBIT			+	+	+	+	+	+
V7	Cash flow/Operating revenue		−	+	+		−	+	
V8	ROE						−	−	−
V9	ROA	−	−	+		−	−	−	
V10	ROCE	+	+					+	+
Operational ratio									
V11	Net assets turnover	+	+	+		+	+	+	+
V12	Interest cover	−	−					−	−
V13	Stock turnover						−	−	−
V14	Collection period			−	−	−	−	−	−
V15	Credit period	−	−	−	−	−	−	−	−
Structure ratio									
V16	Current ratio	−	−	+	+		−		
V17	Liquidity ratio							+	+
V18	Shareholders liquidity ratio		−		+				
V19	Solvency ratio	−	−	+	−	−	−	−	−
V20	Gearing	+	+	−	+				

Sign of coefficients estimates - 2-years ahead

	Italy - in-sample 2004-2007 (2-years ahead)								
	ID	Bankruptcy		Inactive		Liquidation		Single-Risk	
		Stepwise	Lasso	Stepwise	Lasso	Stepwise	Lasso	Stepwise	Lasso
	Profitability ratio								
V1	Return on shareholder funds			−	−		−		−
V2	Return on capital employed			−			−	−	
V3	Return on total assets	+							
V4	Profit margin	−							
V5	EBITDA	+				+	+	+	+
V6	EBIT			+	+			+	
V7	Cash flow/Operating revenue			+	+		−	+	
V8	ROE					−	−	−	−
V9	ROA	−							
V10	ROCE	+	+					+	+
	Operational ratio								
V11	Net assets turnover	+	+	+		+	+	+	+
V12	Interest cover	−	−					−	−
V13	Stock turnover	−			−		−	−	−
V14	Collection period			−	−	−	−	−	−
V15	Credit period	−	−	−	−	−	−	−	−
	Structure ratio								
V16	Current ratio	−		+	+				
V17	Liquidity ratio	+						+	+
V18	Shareholders liquidity ratio			+	+				
V19	Solvency ratio	−	−	−	−	−	−	−	−
V20	Gearing	+	+	−	−				

Final remarks

- The performance of the Competing Risks approach and the Single-risk model for corporate failure in three European markets - France, Italy and Spain - have been evaluated based on firm-specific variables.
- The use of different variables selection procedures in CR and SR models have been evaluate.
- A possible approach to determine the accuracy measure for multiple exit is also proposed.
- The reached results are very sensible to the data set and to the range used for evaluation purpose.

We would like to...

- Consider different source of information, such as macro-economic variables and market related indicators.
- Extend the data base and the time horizon.
- Evaluate the opportunity to use non-parametric procedure for variable selection that allows for non linearity.

Thanks for your attention