Working paper

A THEORY OF CONTRACTS WITH LIMITED ENFORCEMENT[1]

September 29, 2015

David Martimort, Aggey Semenov and Lars Stole

Abstract. We develop a Theory of Contracts with limited enforcement in the context of a dynamic relationship. The seller is privately informed on his persistent cost while the buyer remains uninformed. Public enforcement relies on remedies for breaches. Private enforcement comes from terminating the relationship. We first characterize *enforcement constraints* under asymmetric information. Those constraints ensure that parties never breach contracts. In particular, a high-cost seller may be tempted to trade high volumes at high prices at the beginning of the relationship before breaching the contract later on. Such *"take-the-money-and-run"* strategy becomes less attractive as time passes. It can thus be prevented by backloading payments and increasing volumes over a *transitory phase*. In a *mature phase*, enforcement constraints are slack and the optimal contract, although keeping memory of the shadow cost of enforcement constraints binding earlier on, looks stationary. Second-best distortions depend on a *modified virtual cost* that encapsulates this shadow cost of enforcement.

Keywords. Asymmetric information, enforcement, breach of contracts, dynamic contracts.

JEL codes. D82.

1. INTRODUCTION

Motivation. The *Theory of Contracts* is a major building block of our current understanding of how markets and organizations perform. Its most impressive contribution is to offer a full characterization of trading possibilities in environments where privately informed parties might have conflicting interests. Equipped with a description of the set of *incentive feasible* allocations, modelers look for institutions, organizations or contractual forms that optimally balance efficiency and rent extraction. If any frictions impede contractual performances and prevent efficient trades, those frictions are supposed to come from asymmetric information.

Although quite successful, this methodology remains somewhat at odds with the view of contracts that is instead cherished by Law scholars. Authors in this field actually devote much effort in studying how contracts are enforced (Macaulay (1963)). Two key

[1]For helpful comments on earlier versions of this work, we thank seminar and conference participants at the *Canadian Economic Theory 2011 Meeting* in Toronto, the *Canadian Economic Association 2012* in Calgary, the 2013 *Workshop on Conflict and Cooperation* in Bergamo, the 2013 conference on *"Contracts, Procurement, and Public-Private Arrangements"* in Florence, Sciences Po in Paris, The *2013 IO Workshop* in Berlin, CREST-ENSAE in Paris, RECODE in Ottawa, the *First Brazilian Conference Series on Public Procurement and Concession Design* in Sao Paulo, the 2014 *Workshop on Public Procurement* in Mannheim, Oslo University, and the 2014 *Latin-American Econometric Society Meeting* in Rio de Janeiro. All remaining errors are our own. Special thanks to Sandeep Baliga, Sergei Guriev, Lucas Maestri, Marco Pagnozzi, Nicola Pavoni and Peter Vida for helpful discussions on earlier versions. We also thank the Editor Marco Ottavianni and three referees whose comments have lead to substantive changes and improvements to this manuscript. The usual disclaimer applies.

[a]Paris School of Economics-EHESS, david.martimort@parisschoolofeconomics.eu.

[b]University of Ottawa, aggey.semenov@uottawa.ca.

[c]University of Chicago Booth School of Business,lars.stole@chicagobooth.edu.

concerns are whether legal disputes and remedies arise to fill contractual loopholes and whether private enforcement, often referred to as *relational contracting*, could be an efficient substitute for more formal agreements.[1] By overlooking enforcement constraints in comparison with more traditional incentive compatibility constraints, the *Theory of Contracts* as it currently stands might bias recommendations on organizational choices in some systematic way and neglect important features of contracting. Without further inquiry, it is *a priori* unclear whether the lessons of this theory remain with limited enforcement. Indeed, a richer set of comparative statics may emerge from a theory that addresses how standard agency distortions depends on the quality of enforcement.

To bridge the aforementioned gap, this paper develops a theory of contracts under limited enforcement. Parties may renege on their contractual duties at any point in time in the course of their relationship. To illustrate, observe that although the quantity that parties want to trade in any period may be easily verifiable and so subject to formal agreements, quality may hardly be so. The seller may pocket advance payments and shirk by providing a lower quality widget later on. The buyer may also delay or not fulfill her payment obligations following delivery. Such opportunistic behaviors can be jointly prevented by means of remedies enforced by Courts (the public side of enforcement) but also by the threat of terminating the relationship (its private side).[2] Facilitating enforcement may then call for reduced volumes and decreased prices so as to lower the opportunity cost of breaches.

A first goal of our analysis is to assess how the distortions induced by limits on enforcement modify the screening responses which are now well-known from asymmetric information models. We delineate circumstances in which limits on enforcement impact on the rent/efficiency trade-off highlighted by this literature and investigate how those limits shape trading patterns. A second issue at the core of our analysis is to understand how to enforce the actual play of a contract when Courts of Law will only impose limited damages following a breach. Although parties can include in their contract an explicit statement about the damages that should be paid by a breaching party (*"liquidated"* or *"stipulated"* damages), Courts routinely strike down contractual provisions stipulating damages for breach of contract when those damages appear excessive in terms of actual or anticipated damages or otherwise appear as a penalty rather than as a compensation. It is hard to articulate an internally consistent theory for this *"penalty doctrine."* (See

[1]This hiatus between the views on Contract Theory held by economists and Law scholars is probably best summarized by Masten (1999) who wrote: *"..., the literatures on contract design and contract enforcement have largely developed independently of one another. Economic theories of contracting, for the most part, give little explicit attention to enforcement issues, the presumption being that courts will see to it (subject only to verifiability constraints) that whatever terms contracting parties arrive at are fulfilled. Indeed, enforcing contracts as written is the court's only function in mainstream contract theory [...] This judicial deference to contracts in economic theory contrasts with the far more intrusive role of courts in economic analyses of Contract Law, in which courts are called on to adjudicate disputes, fill gaps, and devise and implement default rules."* See also Kornhauser and MacLeod (2012) for a recent account.

[2]In his text on Contract Law, Atiyah (1995, p.6) stresses the joint use of public remedies and private devices to enforce contract when he writes: *"there are many sanctions against promise-breakers and law is not needed. The simplest sanction is not to deal with that person again."* Nevertheless, *"most systems of law have established rules which will impose sanctions on those who break their contracts."* Johnson et. al. (2002) have also pointed out the joint use of relational contracting and legal remedies in transition economies where various enforcement costs make it difficult to rely exclusively on the judicial system.

Farnsworth (1982) and Posner (2011, Chapter 8).) The practical upshot is that in any given trading environment, there is an upper bound to the amount of damages that a court will enforce following a breach, even if the damages were agreed to by both parties at an early time. Accepting this limitation on the *public* enforcement of contracts, we ask how parties will optimally design their contract to provide complementary *private* incentives for fulfilling their contractual obligations.

Set-up and main results. Consider a highly stylized model of a trading relationship between an uninformed buyer (the principal, or *she* in the sequel) and a seller (the agent, *he*) who repeatedly trade for delivery of a good or service. The seller has private information on his cost function. This type is persistent over the whole relationship. The buyer, whose preferences are common knowledge, has all bargaining power in designing a long-term contract which specifies prices and quantities over the course of the relationship.

Enforcement constraints. This relationship is subject to bilateral opportunism. The buyer may fail to trade a widget with the requested quality; the seller may not fulfill payment obligations. Contracts should now provide safeguards against opportunism so as to ensure that, any point of time, parties comply with their obligations. Even when taken in tandem, the private and public sides of enforcement may not ensure enforceability. This is so when the perspective of future trading does not suffice to motivate parties to abide to their current obligations or when Court-enforceable remedies prove inadequate. A first step of our analysis thus consists in deriving enforcement constraints ensuring that both the seller and the buyer abide to the contract. Because parties have quasi-linear payoffs, individual enforcement constraints can be pooled into a single forward-looking *enforcement constraint*. The foregone benefits of future trades plus the total remedies paid following a breach by either party must be large enough to ensure joint compliance.

"Take-the-money-and-run" strategy. Enforcement constraints require that the value of continuing trades exceeds the benefits that parties may withdraw from not fulfilling their contractual obligations. Under asymmetric information, the value of trade is reduced because part of it is left as information rent to the privately-informed seller. Enforcement constraints are thus hardened.

Indeed, new strategic possibilities arise under asymmetric information. To induce a low-cost seller to reveal his private information at the start of the relationship, the buyer raises the price paid to this type. A high-cost seller may thus find it attractive to adopt the behavior of a low-cost one at the beginning of the relationship, pocketing large payments for a while before breaching. This *"take-the-money-and-run"* strategy shapes intertemporal incentives.[3] Making such strategies less attractive requires the principal to backload the low-cost seller's payments and reduce outputs below the optimal levels achieved had enforcement costs been null. As a consequence, the optimal dynamic contract goes through two different phases. In the first *transitory phase*, trading volumes and prices increase over time as the high-cost seller's incentives to breach diminish. Indeed, the *"take-the-money-and-run"* strategy becomes less attractive as time passes. In the limit, a high-cost

[3]This phrase is familiar from models with short-term contracting (see for instance, Laffont and Tirole (1993-Chapter 9), and Rey and Salanié (1996)) although we consider a different sort of limit on commitment as it will become clearer below.

seller always mimicking a low-cost type would just violate incentive compatibility. After enough periods, the enforcement problem then looks very much as if it was taking place under complete information. In this more *mature phase*, outputs and prices become stationary. The low-cost seller produces the first-best level of output while that of the high-cost seller remains distorted below the static optimal second-best level. This new distortion is captured by a *modified virtual cost* that accounts for the overall shadow cost of binding enforcement constraints over the transitory phase. Even in the long run, the optimal contract thus keeps memory of the cost of enforcement. Output distortions are exacerbated in comparison with the static optimal second-best contract. Those distortions are magnified when the quality of enforcement (be it public or private) deteriorates.

APPLICATION: CONSTRUCTION CONTRACTS. Construction contracts offer a particularly relevant application for our framework since they exhibit features that closely replicate elements in our model. (Dynamic lending relationships, sovereign debt agreements and trade with developing countries, represent other well-suited applications.) Construction projects, especially those involving large-scale infrastructure, are inherently multistage production processes. For each stage, the contract not only specifies an expected time for delivery but also requires that various indicators are met to check the adequacy between the buyer's needs and what the contractor delivers. Late completions, failures to meet adequate specifications or standards and missed payments in due time are pervasive phenomenon. To respond to those contractual hazards, practitioners have developed practices whose goal is to deal with liquidated damages and extensions of time in a way that can be compatible with the needs of both contractors and clients (Eggleston (2009)). Most standard forms of contracts require that parties establish in advance remedies for possible breaches and those remedies are genuine estimates of possible losses incurred by the party on the other side of the transaction. Enforcement relies on a mixture of public and private devices. Parties often rely on relational contracting and trust to continue relationships even following unforeseen events that could have triggered legal disputes (Johnson and Sohi (2015)). Lastly, a major issue faced by practitioners is that *"take-the-money-and-run"* strategies might put contracts at risk. To illustrate, a client often agrees to make an advance payment (so called *"down payment"*) to a supplier so as to provide the latter enough liquidity to pay start-up costs, hire subcontractors or access key resources and equipment. In such circumstances, the client also seeks to secure his payment against default by the contractor through so-called *"advance payment bonds"*. These bonds protect the client in case the contractor fails to fulfill its obligations, perhaps due to insolvency. Alternatively, backloaded payments as predicted by our model are often found in construction contracts. In response, contractors usually negotiate to obtain the bulk of payments at early stages of the project with the risk of exacerbating incentives to *"take the money and run"*. The *Channel Tunnel* is a famous example: one of the main criticisms of the original arrangement came when observers recognized the hazards associated with excessive frontloaded payments (Vinter and Price (2006, p. 100)).

ORGANIZATION. Section 2 reviews the relevant literature. Section 3 presents the model. Section 4 describes the set of allocations that are incentive feasible and enforceable under asymmetric information. Section 5 characterizes the optimal contract and provides conditions for a pattern of growing trades. Section 6 discusses the results of our findings, allowing either one-sided opportunism or renegotiation. Section 7 draws some implications from our findings for organization theory. Proofs are relegated to an Appendix.

2. LITERATURE REVIEW

Our analysis argues that, even though contract enforcement might rely on public remedies, the threat of terminating relationships following a breach also plays a useful disciplinary role. This threat not only improves enforcement but it also shapes the intertemporal design of incentives and trades. This aspect of our modeling is reminiscent of the relational contracting literature although the scope and domains of applications differ.[4] This literature, starting with the seminal works of Macaulay (1963), Bull (1987), MacLeod and Malcomson (1989, 1998) has already highlighted how the benefits of a continued relationship may be considered as a substitute for missing contracts. To model contractual incompleteness, this stream of research assumes that the seller's effort/output, although observable, remains non-verifiable so that the buyer cannot rely on explicit incentives. Instead, the buyer may use *ex post* discretionary bonuses to boost the seller's incentives. A relational contract is then viewed as a repeated game equilibrium where the buyer keeps on rewarding the seller's continued effort. We differ from the relational contracting literature and from other dynamic contracting models on several grounds that we now describe in more details.

Formal and Relational Contracts. While the relational contracting literature leaves little room for Courts beyond their ability to enforce base payments,[5] this view is somewhat extreme. Relational contracts are rarely established in a vacuum and Courts have often enough information to specify some aspects of trade.[6] This basic framework of the relational contracting literature has thus been extended to study how implicit and explicit incentives may interact (Baker et al. (1994), Bernheim and Whinston (1998), Peace and Stachetti (1998), Schmidt and Schnitzer (1995), Iossa and Spagnolo (2009), Li and Matouschek (2013), Itoh and Morita (2015)). Explicit contracts may crowd out implicit incentives, either because the *status quo* becomes more attractive following breaches or because actions that can be contracted upon and those that can only be incentivized through relational contracts are substitutes.

Explicit contracts might not be signed *beside* more implicit relationships as in this literature but they might instead delineate the repeated game to be played through relational agreements. This calls for embedding insights from the relational contracting literature into a mechanism design framework where opportunistic behaviors are jointly controlled through explicit remedies and self-enforcing agreements. This is the path we follow thereafter. The constraints imposed by the enforceability of contracts, be it through public remedies following breaches or by the perspective of a continued relationship, are then added to the more familiar incentive and participation constraints from mechanism design to fully characterize the set of feasible allocations.

Mechanism Design under Full and Limited commitment. We suppose that the

[4]See the important surveys by Malcomson (2012) for general results and by Gibbons (2005a) for implications of this paradigm for organizational design.

[5]Malcomson (2012) reports that *"Relational contracts are concerned with agreements that can be enforced without resort to Courts. The spirit of much of the theory discussed here is that, although an effective legal system exists, important elements of the relationship cannot be enforced legally because courts do not have the information to do that."*

[6]In this paper, we do not model the role of contract enforcer involved into contractual relationship as a player. For such issue, see Maskin and Tirole (2004) and Rahman (2012).

buyer (who has all bargaining power) commits to a long-term contract with the seller. This assumption again stands in sharp contrast with the relational contracting literature that describes settings with no commitment whatsoever. Our paper thus lies closer to the dynamic mechanism design literature and shares with it a common focus on the intertemporal design of incentives. Starting with Baron and Besanko (1984),[7] this literature has assumed that drafting a new contract at any point of time is infinitely costly while transactions in each period are costlessly enforced. By instead assuming that transactions are subject to opportunistic behavior and breaches and that remedies are finite, our model offers a better account of the contractual environment depicted by Law scholars.[8]

Turning to the nature of output distortions, the mechanism design literature emphasizes that distortions never arise for the most efficient type, as in Battaglini (2005), but distortions which decrease over time arise for the inefficient type. In our paper, the interesting dynamics are instead reversed: the need to prevent the high-cost seller from *"take the money and run"* now leads to decreasing distortions for the efficient, low-cost seller. From a formal point of view, those distortions are determined by the Lagrange multipliers of the enforcement constraints which form a sequence converging towards zero as the *"take-the-money-and-run"* strategy becomes less relevant over time.[9]

By assuming full commitment, we also leave aside the issue of contract renegotiation which is well known from the existing literature following works by Dewatripont (1989), Hart and Tirole (1988) and Rey and Salanié (1996) among others. There, a long-term contract can be breached at no cost to reach a Pareto-improving new agreement as information is revealed through earlier performance. This issue is orthogonal to the enforcement of transactions in any given period that is instead our focus. Although there is no consensus on the most practical notion, renegotiation has also been a concern in repeated games and as such this concern has percolated to the analysis of relational contracting. In Levin (2003), an optimal relational contract is renegotiation-proof assuming that renegotiation is only possible before payments. Goldlücke and Kranz (2013) and Fong and Surti (2009) also investigate renegotiation before actions are taken and payment are made. Section 6.2 shows that our results are robust to renegotiation.

Non-Stationarity, Relational Contracting and Asymmetric Information. We assume the seller has persistent private information on his cost function before contracting. Under full commitment and costless enforcement, Baron and Besanko (1984) demonstrate that the optimal long-term contract could be implemented by the infinite replica of the optimal short-term contract, leading thus to stationary trades. On the other side of the spectrum, assuming that only relational contracts are feasible, Levin (2003) shows that the optimal relational contract is again stationary when types are either common knowledge or private information but independently drawn over time. In our setting, a high-cost seller may adopt the same behavior as a low-cost one in earlier periods of the

[7]See also Battaglini (2005), Pavan, Segal and Toikka (2011) and Eskobar and Toikka (2012).

[8]Enforcement has nevertheless received some attention in models that stress the limited ability of Courts to enforce obligations (Schwartz and Watson (2004), Doornik (2010) and Kvaløy and Olsen (2009), Laffont and Martimort (2002, Chapter 9), Guasch, Laffont and Straub (2003)).

[9]In passing, this analysis requires to write down the optimal contract as the solution to an optimization problem with an infinite number of enforcement constraints which necessitates a careful use of duality theory in infinite dimensional spaces, an approach based on the work of Dechert (1982).

relationship before breaching the contract. Such a *"take-the-money-and-run"* strategy is optimally addressed through growing trades. Building also on Levin (2003), Kwon (2013) derives the optimal relational contract with persistent shocks and shows that it is no longer stationary.[10]

Horner (2002), Fong and Li (2010) and Halac (2012) have also addressed how types persistence affects relational contracts. Although the information structures there differ from ours, they all ask how private information is revealed over time and how it determines time-varying stakes.[11] Halac (2012) shows how a principal may want to signal that the relationship has value to induce higher-powered incentives and then renege on the agreement. Malcomson (2015) considers Levin (2003)'s model but with persistent private information. He shows that full separation is not possible at any date. Respecting enforcement constraints forces pooling of some types. This is similar to patterns from the dynamic contracting literature with short-term contracts (see Laffont and Tirole (1993, Chapter 9) among others). There, the *"ratchet effect"* calls for bonuses to reward earlier information revelation but it also exacerbates the *"take-the-money-and-run"* strategy which is found attractive by high-cost seller. With relational contracts, this tension is solved by pooling allocations. In our setting, commitment allows the buyer to better control this *"take-the-money-and-run"* strategy by means of growing stakes while still inducing information revelation upfront as in standard mechanism design environments under full commitment. The canonical model of relational contracting with persistent types in Malcomson (2015) and our model thus sharply differ in terms of how information is revealed over time. An important consequence is that the optimal mechanism in our asymmetric information context cannot be implemented with relational contracts.

The growing phase of the optimal contract in our setting is reminiscent of the reputation literature (Sobel (1985), Ghosh and Ray (1996), Watson (1999, 2002), Halac (2012)). There, relationships might start *"small"* to ease reputations building when there is uncertainty on traders' degree of opportunism. Watson (2002) considers a partnership game between two players with two-sided uncertainty about motives. Players, to encourage cooperation, start with small stakes which are growing over time. There is no such private information on behavioral types in our setting. Finally, our paper is also somewhat related to Board (2011)'s findings that, to prevent agent's opportunism, a principal must give to his agent a sufficiently large rent which comes both in terms of payments today and promised future payments. Delayed payments prevents future incidences of *"hold-up."*

3. THE MODEL

3.1. *Basics*

PREFERENCES. We consider an infinitely-repeated relationship between a buyer (the principal or *she*) and a seller (the agent, *he*) who provides a service or good on her behalf. Time is indexed by $\tau \geq 0$ and we denote by $\delta < 1$ the common discount factor.

[10] There is also a related literature on dynamic moral hazard problems with persistent shocks. Kwon (2015) shows that the optimal contract entails a probationary period without payments before the contract implements the first-best action in every period. Fuchs (2007) relaxes the assumption that output is common knowledge and shows that the threat of termination provides incentives.

[11] The non-stationary of relational contracts may also come from learning persistent types as in a model of the labor market proposed by Yang (2012) or intertemporal insurance concerns as in Hemsley (2013).

A *trade profile* is an infinite array of payments and (non-negative) outputs $(\mathbf{t}, \mathbf{q}) \equiv \{(t_\tau, q_\tau)\}_{\tau=0,..,\infty}$ over this long-term relationship. Both the buyer and the seller have quasi-linear utility functions defined over trade profiles. Their discounted payoffs are respectively given by:

$$(1-\delta)\sum_{\tau=0}^{\infty}\delta^\tau(S(q_\tau)-t_\tau) \text{ and } (1-\delta)\sum_{\tau=0}^{\infty}\delta^\tau(t_\tau-\theta q_\tau).$$

The buyer's benefit function S is differentiable, increasing and concave ($S' > 0 > S''$) with $S(0) = 0$. To ensure positive outputs under all circumstances below, we require that $S'(0)$ is sufficiently large, though bounded. The set of feasible outputs is an interval $\mathcal{Q} = [0, \overline{\mathcal{Q}}]$ with $\overline{\mathcal{Q}}$ sufficiently large enough to ensure interior solutions.

Information. The seller has private information about his cost parameter θ which takes values in $\Theta = \{\underline{\theta}, \overline{\theta}\}$ (with $\Delta\theta > 0$). This parameter is persistent over the whole relationship and drawn once for all before contracting. Let $\nu > 0$ (resp. $1-\nu > 0$) be the probability that the seller has a low (resp. high) cost. Let also $\mathbb{E}_\theta(\cdot)$ denote the expectation operator.

3.2. *Costless Enforcement*

As benchmarks, we consider the simple case of costless enforcement, first with complete and then with asymmetric information.

Complete information. Under complete information, it is routine to show that the first-best outcome is implemented by means of a stationary contract $(t^{fb}(\theta), q^{fb}(\theta))$ (or a stationary rent-output allocation $(U^{fb}(\theta), q^{fb}(\theta))$) which is defined as:

$$S'(q^{fb}(\theta)) = \theta \text{ and } U^{fb}(\theta) = t^{fb}(\theta) - \theta q^{fb}(\theta) = 0 \quad \forall \theta \in \Theta.$$

At the first best, the buyer's marginal benefit must equal the seller's marginal cost. Given that the buyer has all bargaining power, she can fully extract the seller's rent.This allocation is infinitely repeated over time.

Asymmetric information. The *Revelation Principle* applies in a dynamic trading environment when parties commit to a long-term contract (Baron and Besanko (1984)). There is no loss of generality in restricting the analysis to direct and truthful revelation mechanisms that stipulate the seller's payments and outputs in each period as a function of his upfront report on cost. Such a contract is an infinite sequence $\mathcal{C} = \{(\mathbf{t}(\theta), \mathbf{q}(\theta))\}_{\theta\in\Theta}$.

A second important insight due to Baron and Besanko (1984) is that, among all possible payments profiles that may implement the optimal allocation, one possibility is to rely on the infinite replica of the optimal static contract. This contract entails first-best production for the low-cost seller and the usual *Baron-Myerson distortion* for the high-cost seller's output $q^{bm}(\overline{\theta})$ (which remains positive provided that $S'(0)$ is large enough):

$$S'(q^{bm}(\overline{\theta})) = \overline{\theta} + \frac{\nu}{1-\nu}\Delta\theta.$$

Under asymmetric information, the buyer's marginal benefit must equal the seller's *virtual cost* $\overline{\theta}+\frac{\nu}{1-\nu}\Delta\theta$. A downward output distortion for the high-cost seller reduces the information rent left to the low-cost type. As familiar in such screening environments, the rents for both types are expressed as:

$$U^{bm}(\underline{\theta})=\Delta\theta q^{bm}(\overline{\theta})>U^{bm}(\overline{\theta})=0.$$

Among other possible payments profiles that may implement this allocation, the following stationary prices offer a convenient benchmark for the rest of the analysis:

$$t^{bm}(\underline{\theta})=\underline{\theta}q^{fb}(\underline{\theta})+\Delta\theta q^{bm}(\overline{\theta})\text{ and }t^{bm}(\overline{\theta})=\overline{\theta}q^{bm}(\overline{\theta}).$$

3.3. *Costly Enforcement: Setting the Stage*

Parties can fully commit to a long-term contract stipulating a trade profile. Yet, at any date, the mere play of this mechanism may be subject to opportunistic behaviors. A party breaches the contract if his current benefit of doing so exceeds the cost. This cost includes the foregone opportunities for future trades (the private side of enforcement) but also the legal remedies that this party has to pay for not fulfilling obligations (the public side). A contract stipulates prices and quantities in all periods, but trade itself is at risk.

To control such opportunistic behaviors, the parties will find it useful to contract on two prices for each trading period τ – a *pre-production* payment (required by the seller before producing the good) and a *post-delivery* payment, required by the seller after the delivery of the good. We denote these pre-production and post-delivery payments by $t_{1,\tau}$ and $t_{2,\tau}$, respectively. The pre-production payment by the buyer helps control her incentives for breach; the post-trade payment helps control the seller's incentives not to deliver the good as required. [12] A direct mechanism is now an infinite triplet $\mathcal{C}=\{(\mathbf{t}_1(\theta),\mathbf{t}_2(\theta),\mathbf{q}(\theta))\}_{\theta\in\Theta}$ stipulating pre- and post-delivery payments as well as outputs in each period. The total payment to a seller reporting type θ is denoted $t_\tau(\theta)=t_{1,\tau}(\theta)+t_{2,\tau}(\theta)$ at date τ.

Equipped with those notations, we explore the incentives for parties to deviate from the expected play of the mechanism and opportunistically breach the contract.

Seller's breach. After having pocketed the pre-delivery payment $t_{1,\tau}(\theta)$, the seller may not deliver the quantity $q_\tau(\theta)$. This deviation is attractive if $t_{1,\tau}(\theta)$ is large enough.

Buyer's breach. After the delivery of a quantity $q_\tau(\theta)$, the buyer may not pay the post-delivery price $t_{2,\tau}(\theta)$ if this payment is now too large.

Denote by $K\geq 0$ (resp. $L\geq 0$) the remedy paid by the the buyer (resp. seller) in case she (resp. he) breaches the agreement.[13] Those penalties are exogenously specified by the Court. When remedies are infinite ($K=L=+\infty$), enforcement is perfect and the optimal contract is the infinite replica of the Baron-Myerson allocation. To focus on less

[12]For simplicity, we assume that there is no discounting between those sub-periods.

[13]Those remedies be viewed as expected remedies that incorporate the probability that Law is just not enforced by Courts, maybe because of a *"speculative loss"* doctrine.

trivial cases, we will thus assume finite penalties ($K, L < +\infty$).[14] As we will show below, the availability of both pre-production and post-delivery payments allows the parties to shift the force of a penalty from one side of the transaction to the other. As a consequence, the critical constraint on public enforcement is the sum of the available penalties, that we define as $M = K + L$.

TIMING. The contracting game unfolds as follows.

1. Prior to any trading, at date $\tau = 0^-$, the seller privately learns his cost parameter θ. The buyer then offers a mechanism $\mathcal{C}$. The seller, in turn, accepts or rejects the offer. If he accepts, the seller reports a type $\hat{\theta}$; if he rejects, both parties receive their reservation values, normalized to zero.
2. Each trading period at date $\tau \geq 0$ is as follows.
   - The buyer offers the pre-delivery payment $t_{1,\tau}(\hat{\theta})$.
   - The seller produces $q_\tau(\hat{\theta})$ or breaches the contract and pays the remedy L.
   - If $q_\tau(\hat{\theta})$ is produced as required, the post-delivery payment $t_{2,\tau}(\hat{\theta})$ is paid by the buyer or she breaches the contract and pays the remedy K. Following breach on either side of the transaction, the contract is terminated.[15]

NOTATION. In order to express various feasibility constraints in compact form, we now define the per-period value of the seller's output, his rent and the buyer's payoff going *forward* from date τ on as, respectively,

$$q_\tau^+(\theta) = (1-\delta)\sum_{s=0}^{\infty}\delta^s q_{\tau+s}(\theta),$$

$$U_\tau^+(\theta) = (1-\delta)\sum_{s=0}^{\infty}\delta^s(t_{\tau+s}(\theta)-\theta q_{\tau+s}(\theta)), \quad V_\tau^+(\theta) = (1-\delta)\sum_{s=0}^{\infty}\delta^s\left(S(q_{\tau+s}(\theta)) - t_{\tau+s}(\theta)\right).$$

We may also define the seller's *backward* output and per-period average rent leading up to date τ, assuming the seller breaches the contract at τ after accepting the pre-production payment, $t_{1,\tau}$. These *backward* output and rent functions are, respectively,[16]

$$q_\tau^-(\theta) = (1-\delta)\sum_{s=0}^{\tau-1}\delta^s q_s(\theta),$$

$$U_\tau^-(\theta) = (1-\delta)\sum_{s=0}^{\tau-1}\delta^s(t_s(\theta) - \theta q_s(\theta)) + \delta^\tau(1-\delta)(t_{1,\tau}(\theta) - L).$$ [17]

[14] Our formulation and results below work the same way if penalties are liquidated damages paid by the breaching party to its partner, or instead are paid to a third party (or destroyed).

[15] We thus follow here Abreu (1988) and the literature on relational contracts (Levin (2003) and Halac (2012) among others) in specifying that the worst equilibrium is played following a breach.

[16] The following identities hold: $U_\infty^-(\theta) \equiv U_0^+(\theta)$ and $q_\infty^-(\theta) \equiv q_0^+(\theta)$ $\forall \theta \in \Theta$ where $U_0^+(\theta)$ (resp. $q_0^+(\theta)$) is the seller's intertemporal rent (resp. output) over the whole relationship.

[17] We adopt the convention that $\sum_{s=0}^{-1} y_s \equiv 0$ for any sequence $\mathbf{y}$.

To simplify the formulation of the enforcement constraints, we also define the *enforcement surplus* (an expression that we will make clear below) as

$$\Psi(\theta, \mathbf{q}_\tau(\theta)) = (1-\delta)\left(\sum_{s=1}^{\infty} \delta^s (S(q_{\tau+s}(\theta) - \theta q_{\tau+s}(\theta)) - \theta q_\tau(\theta) + M\right), \tag{3.1}$$

where $\mathbf{q}_\tau(\theta) = \{q_s(\theta)\}_{s \geq \tau}$ is an intertemporal trade profile starting at date τ.

3.4. *Reinterpretation of the Model/Discussions of our Assumptions*

Instead of viewing quantity and price as being imperfectly enforceable, we could have assumed that it is costless to write a contract stipulating how much quantity is traded and at what price providing quality is unverifiable. Suppose, for example, that the good may come in two possible quality levels and quality, although observable, remains non-verifiable unless the Court is called upon for inspection. A high quality good yields benefits to the buyer and is costly (but efficient) to produce; a worthless quality good is costless to produce. The seller now breaches the contract when providing the low quality. The buyer breaches the agreement when not paying following his claim that the good has low quality. Disputes arise whenever the buyer and the seller disagree on their assessments of quality. The Court is called upon to solve this dispute. Following the Court's inspection, the seller pays a remedy if the quality is low and the buyer does so if the quality is high but he pretended the opposite to avoid due payment.

This interpretation of the model is much in lines with the existing relational contracting literature. As in this literature, the seller's incentives to produce and the buyer's incentives to pay for high quality cannot be contractually specified. It must be equilibrium behavior for the seller to offer a high quality and for the buyer not to argue about quality. Again, the continuation value of the relationship helps to curb bilateral opportunism; the private side of enforcement matters.

Compared with the relational contracting literature, two novel ingredients are added when quantities are contractible. First, since a Court of Law is present to enforce such contracts, we posit that it can also assess a disputed quality by inspecting. Remedies are now also useful tools to curb opportunism; the public side of enforcement also matters.[18] Second, the possibility of choosing contractual stakes in each period allows parties to *ex ante* control the game they will be playing. This new feature of the modeling transforms the study of relational contracts into a mechanism design exercise.

In the spirit of mechanism design, the most natural assumption to start with is to assume that parties can fully commit to a long-term contract stipulating quantities and prices. Renegotiation is precluded. In other words, penalties for voiding an existing contract and signing a new one are infinite although remedies for not providing high quality or not fulfilling payment obligations remain finite. Commitment is a standard assumption in the Law and Economics literature. For instance, Edlin (1998) argues that *"...renegotiation is impossible prior to breach decision,"* while Shavell (2004, p.315) stresses that renegotiation may fail because of time constraints or asymmetric information.

[18] There is a vast literature on remedies and their role on litigation (Shavell (1980), Bebchuck (1984), Reinganum and Wilde (1986) among others). In contrast, we view dispute resolution as a *black-box*.

Competition on both sides of the market also provides an effective commitment to terminate a relationship once it has been breached, thereby hindering renegotiation. To illustrate, suppose that the seller has not complied and that remedies have been paid; the buyer could immediately turn to another producer to complete the project in a market context, a situation illustrated by Macchiavello and Morjaria (2014) in their analysis of the Kenyan market for roses. In construction contracts, competition prevails among both contractors and subcontractors. When a subcontractor fails to deliver, renegotiation does not occur. Instead, the contractor may hire another subcontractor to complete the work.[19] In both cases, contractual breach induces agents to turn to alternative relationships, effectively taking a new draw from the distribution. One can imagine that switching partners is an attractive option relative to renegotiation if departures from equilibrium play reflect negatively on some underlying, unobserved character of the breaching party.

From a more theoretical viewpoint, the full commitment assumption could be criticized, but certainly not more than in other dynamic contexts where optimal contracts are well known not to be time consistent.[20] Making this assumption allows us to describe an upper bound on the possible gains from trade that can be achieved under asymmetric information and costly enforcement.

Finally, our assumption of bounded penalties for breach is consistent with the legal doctrine and with existing evidence. Our goal is not to endogenize the levels of remedies. We take those levels as fixed and derive the consequences of maybe badly designed remedies on contracting patterns.

4. IMPLEMENTABLE ALLOCATIONS

This section characterizes the set of feasible allocations when enforcement is costly. This set is constrained by the usual agent's incentive compatibility and participation constraints but also, and this is the novelty of our framework compared with more standard mechanism design environments, by a new set of dynamic enforcement constraints.

Seller's participation constraints. A seller with type θ finds the mechanism C *individually rational* when the following *interim* participation constraint holds:

$$U_0^+(\theta) \geq 0, \quad \forall \theta \in \Theta. \tag{4.1}$$

[19]In *RDP Royal Palm Hotel vs. Clark Construction Group* (reported in Callahan (2009, p.199-200)), *RDP* hired another contractor and sued *Clark* for breaching the contract. In *Saxon Construction vs. Masterclean of North Carolina* (reported by Hinze (2001, p. 246)), a subcontractor, *Masterclean*, failed to complete the work which led *Saxon Construction* to hire another subcontractor. Sometimes, switching contractor may be forbidden by Court. To illustrate, in *Abbey Development vs. PP Brickwork Ltd.* (2003), the contract had a convenience termination clause that was used by *Abbey* to switch to another contractor. However, the Court did not allow *Abbey* to do so. http://www.out-law.com/en/topics/projects–construction/construction-contracts/termination-and-suspension-of-construction-contracts.This is under such circumstances that we may expect renegotiation to occur. For completeness, Section 6.2 below discusses the possibility of renegotiation.

[20]See Baron and Besanko (1984) and Pavan et al. (2014) for other dynamic environments with optimal contracts which are not renegotiation-proof. Battaglini (2005) presents cases of renegotiation-proofness when types are not perfectly correlated.

Buyer's enforcement constraints. The buyer should pay in each period her due payment to the seller. If she deviates and does not pay for delivery, she incurs a penalty K (public side of enforcement) and the relationship ends (the private side).

Definition 1 *The mechanism $\mathcal{C}$ is* ***buyer-enforceable*** *if and only if:*

$$\delta V^{+}_{\tau+1}(\theta) \geq (1-\delta)(t_{2,\tau}(\theta) - K) \quad \forall \theta \in \Theta \quad \forall \tau. \tag{4.2}$$

The lefthand side represents the buyer's discounted payoff from period $\tau + 1$ onwards on the equilibrium path. The righthand side is her deviation payoff for the current period. It takes into account the fact that trade never occurs from date τ on following breach by the buyer.

Seller's enforcement constraints. The seller's enforcement constraints are complex because they interact with incentive compatibility requirements. Indeed, the possibility for non-compliance now affects how incentive constraints should be written.

Definition 2 *The mechanism $\mathcal{C}$ is* ***seller-enforceable*** *if and only if:*

$$U^{+}_{0}(\theta) \geq \max_{\hat{\theta}\in\Theta} U^{-}_{\tau}(\hat{\theta}) + (\hat{\theta} - \theta) q^{-}_{\tau}(\hat{\theta}) \quad \forall \theta \in \Theta, \quad \forall \tau. \tag{4.3}$$

The enforcement constraints (4.3) say that a seller with type θ prefers to choose his targeted contract rather than adopting a *"take-the-money-and-run"* strategy. This strategy consists in mimicking a type $\hat{\theta}$ at all dates $0 \ldots . \, \tau - 1$, delivering the corresponding output, but breaching the contract at date τ, being only punished from that date onwards.

Incentive Compatibility. Taking $\tau = \infty$, the enforceability conditions (4.3) imply standard incentive compatibility:

$$U^{+}_{0}(\theta) = \max_{\hat{\theta}\in\Theta} U^{+}_{0}(\hat{\theta}) + (\hat{\theta} - \theta) q^{+}_{0}(\hat{\theta}), \quad \forall \theta \in \Theta. \tag{4.4}$$

In turn, a well known consequence of incentive compatibility is that the discounted output over the whole relationship satisfies a familiar monotonicity condition:

$$q^{+}_{0}(\theta) \text{ weakly decreasing.} \tag{4.5}$$

Pooling enforcement constraints. The enforcement surplus (3.1) represents the parties' net gain from enforcing the contract from date τ on. It takes into account future gains from trade but also the foregone penalties from not deviating at date τ.

Definition 3 *The mechanism $\mathcal{C}$ is* ***enforceable*** *if and only if it is both buyer- and seller-enforceable.*

Pooling altogether the individual enforcement constraints (4.2) and (4.3), we obtain a new set of feasibility conditions.

LEMMA 1 *An incentive compatible and individually rational mechanism $\mathcal{C}$ is* ***enforceable*** *if and only if*

$$\Psi(\theta, \mathbf{q}_\tau(\theta)) \geq \delta^{-\tau} \max_{\hat{\theta}\in\Theta} \left\{ U_0^+(\theta) - U_0^+(\hat{\theta}) + (\theta - \hat{\theta}) q_\tau^-(\theta) \right\}, \quad \forall \theta \in \Theta, \quad \forall \tau \geq 0. \tag{4.6}$$

Note that Ψ in (4.6) depends only on the aggregate penalty, $M = K + L$, and thus an immediate consequence of Lemma 1 is that the distribution of remedies is irrelevant. The buyer, who has all bargaining power, can undo any initial distribution and structure payments so as to fully internalize the consequences of a breach even when it might originate from the seller. Everything thus happens as if the only threat was a breach of contract by the buyer herself and the remedy that would apply for such breach is M.

Although the procedure of pooling enforcement constraints on both sides of the market is reminiscent of Levin (2003)'s approach in his study of bilateral opportunism in relational contracts, the details differ in interesting ways. Contrary to Levin (2003), our pooling procedure must account for the fact that parties are asymmetrically informed. This explains the presence on the righthand side of (4.6) of the possible manipulation of the seller's reports on his type.

"TAKE-THE-MONEY-AND-RUN". It should be intuitive that the most salient enforcement constraints on the seller's side are those of a high-cost type. Indeed, a low-cost seller, if he chooses to lie on his type and produce a quantity $q_\tau(\overline{\theta})$ at a low marginal cost gets an information rent $\Delta\theta q_\tau(\overline{\theta})$ in period τ. This seller has no incentives for early breaches if he wants to pocket these rents over the whole relationship.

Instead, because asymmetric information requires an increase in the price paid to a low-cost type to induce information revelation, a high-cost seller may now find a *"take-the-money-and-run"* strategy particularly attractive. That type may prefer to mimic a low-cost seller for a few initial periods so as to pocket these large prices before breaching the contract. Mimicking the low-cost seller forever is not profitable, however, given the standard incentive compatibility condition, so the incentives for breach culminate early in the relationship.

ENFORCEMENT UNDER COMPLETE INFORMATION. To deepen our understanding of condition (4.6), suppose for the moment that the seller's cost is common knowledge and, thus, there is no possibility of non-truthful reports on the righthand side of (4.6). In this complete information setting, the enforcement constraints would thus become

$$\Psi(\theta, \mathbf{q}_\tau(\theta)) \geq 0, \quad \forall \theta \in \Theta, \quad \forall \tau \geq 0. \tag{4.7}$$

Feasibility would now require simply that the discounted value of future trades covers the sum of the individual costs for breaching the contract.

Condition (4.6) is similar to (4.7) with the only difference being that, under asymmetric information, the prices paid to the low-cost seller have also to account for his information rent. The enforceability constraint is hardened under asymmetric information.

TIME-DEPENDENCE. The existence of a *"take-the-money-and-run"* strategy evoked above already stressed the difference between the earlier periods and the rest of the relationship. To sharpen intuition on this issue and analyze its consequences on contractual dynamics, consider the aggregate enforcement constraint (4.6) in state $\underline{\theta}$. Suppose also that the incentive compatibility constraint (4.4) that prevents a low-cost seller from pretending being a high cost one is binding, a property that will hold at the optimal contract:

$$U_0^+(\underline{\theta}) = U_0^+(\overline{\theta}) + \Delta\theta q_0^+(\overline{\theta}). \tag{4.8}$$

The enforcement constraint (4.6) written in state $\underline{\theta}$ then becomes:

$$\Psi(\underline{\theta}, \mathbf{q}_\tau(\underline{\theta})) \geq \delta^{-\tau} \max\left\{\Delta\theta(q_0^+(\overline{\theta}) - q_\tau^-(\underline{\theta})), 0\right\}. \tag{4.9}$$

By construction, $q_\tau^-(\underline{\theta})$ converges towards $q_0^+(\underline{\theta})$ as τ grows large. If the monotonicity condition (4.5) is strict, the maximum on the righthand side of (4.9) is thus zero. The enforcement constraint (4.9) then boils down to its complete information expression (4.7). In the long run, asymmetric information has no impact on enforcement constraints.

In the short run, however, asymmetric information impacts enforcement. Taking instead $\tau = 0$, the enforcement constraint (4.9) becomes more stringent than (4.7):

$$\Psi(\underline{\theta}, \mathbf{q}_0^+(\underline{\theta})) \geq \Delta\theta q_0^+(\overline{\theta}). \tag{4.10}$$

Reinforcing an intuition given above, the *"take-the-money-and-run"* strategy only matters early in the relationship. In contrast with the case of complete information, enforcement constraints now depend explicitly on time. The beginning and the tail of the relationship do not look the same for the high-cost seller. This feature explains that the optimal contract may not be stationary.

5. OPTIMAL DYNAMIC CONTRACT

The buyer's objective is to maximize the discounted net surplus he obtains from trade, subject to the seller's participation, incentive compatibility and the new enforcement constraints:

$$(\mathcal{P}): \max_{(\mathbf{q}(\theta), U_0^+(\theta))} \mathbb{E}_\theta \left((1-\delta)\sum_{\tau=0}^{\infty} \delta^\tau \left(S(q_\tau(\theta)) - \theta q_\tau(\theta)\right) - U_0^+(\theta) \right)$$

subject to (4.1), (4.4), and (4.6).

5.1. *Complete Information Benchmark*

To build intuition and provide further comparison with the case of asymmetric information, let us briefly analyze the case of complete information. To this end, suppose *a priori* that the buyer offers a stationary output profile $\mathbf{q} = (q, q, ...)$ in state θ. The enforcement surplus would become:

$$\psi(\theta, q) = \Psi(\theta, \mathbf{q}) = \delta S(q) - \theta q + (1-\delta)M.$$

Observe that ψ is strictly concave in q, achieves a maximum at $q^{db}(\theta) < q^{fb}(\theta)$ (which is defined as $\delta S'(q^{db}(\theta)) = \theta$) and admits a zero at some positive $q^e(\theta) > q^{db}(\theta)$, providing M is not too large (an assumption that we make from now on):

$$\delta S(q^e(\theta)) - \theta q^e(\theta) + (1-\delta)M = 0. \tag{5.1}$$

It follows that $q^e(\theta)$ is the largest output that could be enforced under complete information.

The optimal contract under complete information and limited enforcement can be shown to be stationary but not necessarily first-best.[21] The intuition for stationarity is straightforward. The enforcement constraints (4.7) look the same from any date on so that distortions are the same in every period. The corresponding optimal output under complete information $q^{ci}(\theta)$ is thus given by the following expression:

$$q^{ci}(\theta) = \min\{q^e(\theta), q^{fb}(\theta)\}.$$

Observe that, for a fixed level of M, the inequality $q^e(\theta) \geq q^{fb}(\theta)$ always holds when δ is close to 1. Enforcement is not an issue if parties care enough about the future, even with zero penalties. In contrast, when the discount factor and the available penalties are sufficiently small, the enforcement constraint is binding. The optimal output in any period is then the greatest output compatible with enforcement, namely $q^e(\theta)$. Production must be reduced below the first best to reduce payments and incentives for breaches.

The optimal contract under complete information can be implemented with a pure relational contract. To see how and in a slight extension of the relational contracting literature, suppose that parties can include into this stage game a *"no trade"* action that would trigger liquidated damages. Doing so expands punishments beyond the foregone value of trade. Following Levin (2003), we can derive the optimal relational contract in such a context. This contract again implements the stationary output $q^{ci}(\theta)$ as above. In other words, the ability to parties to commit does not matter under complete information. Relational contracts suffice. As the analysis below will show us, this is no longer the case under asymmetric information.

5.2. *Implementing the Baron-Myerson Outcome*

Turning now to the characterization of the optimal contract under asymmetric information, we first highlight simple conditions ensuring that the infinite replica of the Baron-Myerson's outcome can still be enforced.

Assumption 1

$$\Delta\theta q^{bm}(\bar{\theta}) \leq \psi(\underline{\theta}, q^{fb}(\underline{\theta})) \text{ and } 0 \leq \psi(\bar{\theta}, q^{bm}(\bar{\theta})).$$

Proposition 1 *At the optimal contract, the Baron-Myerson outputs $(q^{fb}(\underline{\theta}), q^{bm}(\theta))$ are implemented in each period if and only if Assumption 1 holds.*

[21] The proof of existence and stationarity of an optimal contract is available upon request.

When the gains from trade suffice to prevent breaches by both parties, the stationary Baron-Myerson allocation is feasible even under limited enforcement. This is so even if the price paid to this low-cost seller is greater than under complete information so as to pay for his information rent. To give more intuition, observe that the first condition in Assumption 1 implies that the enforcement constraint (4.9) holds at date 0 if all payments to the low-cost seller are paid upfront. The most attractive deviation for the buyer consists of not paying that amount at date 0 and immediately breaching the contract, but Assumption 1 ensures that such deviation is not valuable. The second condition in Assumption 1 says that the Baron-Myerson output could be enforced if the seller was known to have a high cost parameter.

5.3. *Stationary Contracts*

When Assumption 1 fails, the Baron-Myerson outcome can no longer be implemented under asymmetric information. In this case, a useful starting point is to consider the optimal stationary contract $\mathbf{q}(\theta) = (q(\theta), q(\theta), ...)$, but with modified outputs that satisfy the enforcement constraints.[22] The enforcement constraint in state $\underline{\theta}$, namely (4.9), reduces to

$$\psi(\underline{\theta}, q(\underline{\theta})) \geq \delta^{-\tau} \max \left\{ \Delta\theta(q(\overline{\theta}) - (1-\delta^{\tau})\, q(\underline{\theta})), 0 \right\} \quad \forall \tau \geq 0.$$

For any weakly decreasing output profile, this constraint holds at $\tau \geq 1$ if it already holds at $\tau = 0$. Intuitively, with a stationary contract, if any breach were to happen, it should arise as soon as possible. This leads to the simpler requirement:

$$\psi(\underline{\theta}, q(\underline{\theta})) \geq \Delta\theta q(\overline{\theta}). \tag{5.2}$$

Let denote by Λ the non-negative Lagrange multiplier of the binding enforcement constraint (5.2). This multiplier measures the shadow cost of enforcement. It is important to stress that, because this multiplier determines the optimal screening distortions, the quality of enforcement and contract performances are now linked altogether. Indeed, maximizing the buyer's payoff under the feasibility constraints (5.2) and the restriction to stationary contracts yields the following expressions of the downward output distortions:

$$S'(q^{st}(\underline{\theta})) = \underline{\theta} + \frac{(1-\delta)\Lambda}{\nu + \Lambda\delta}\Delta\theta \text{ and } S'(q^{st}(\overline{\theta})) = \overline{\theta} + \frac{\nu + \Lambda}{1-\nu}\Delta\theta.$$

Both outputs are reduced below the Baron-Myerson levels when the multiplier Λ is positive. To relax the binding enforceability constraint (5.2), the buyer would like to reduce the price paid to a low-cost seller so as to make breaches less attractive. Two instruments are used in tandem. First, the buyer procures even less from a high-cost seller than in the Baron-Myerson scenario. This reduces the low-cost seller's information rent and thus his payment. Second, the buyer also asks for less output from a low-cost seller which also reduces his payment.

With stationary contracts, reducing the low-cost seller's output has nevertheless two conflicting effects. First, as just claimed, it decreases the benefits of a current breach. Second, it also reduces surpluses in future trading rounds which, on the contrary, harms enforceability. Those two conflicting roles are disentangled with non-stationary contracts.

[22] A rationale for this restriction in the set of feasible contracts is that the buyer is involved in a series of bilateral relationships, facing a population of sellers whose arrivals follow a Poisson process and bilateral contracts remain anonymous and thus independent on the first date at which such bilateral trade occurs.

5.4. *Growing Dynamics*

To characterize the dynamics of optimal contracts, we first introduce some conditions.

ASSUMPTION 2

$$0 < \psi(\underline{\theta}, q^{fb}(\underline{\theta})) < \Delta\theta q^{bm}(\overline{\theta}) \text{ and } 0 \leq \psi(\overline{\theta}, q^{bm}(\overline{\theta})).$$

Those conditions prevent from reaching the positive results of Proposition 1. From the first inequality, enforcement would not be an issue if the seller was known to have a low cost, i.e., $q^{fb}(\underline{\theta}) < q^{e}(\underline{\theta})$. Instead, the second inequality implies that the enforcement surplus in state $\underline{\theta}$ does not suffice to ensure enforceability if the buyer has to pay for the extra rent $\Delta\theta q^{bm}(\overline{\theta})$ that a low-cost seller earns under asymmetric information. Finally, the third inequality means that there would be enforcement problem if the high-cost seller was asked to produce the Baron-Myerson output even under complete information. Taken together those conditions ensure that only the enforcement constraint (4.9) in state $\underline{\theta}$ may matter at the optimum.

Remember that pooling enforcement constraints for both the seller and the buyer requires us to take into account asymmetric information. In particular, the enforcement constraint (4.9) is obtained by summing up the buyer's enforcement constraint for state $\underline{\theta}$ with the seller's enforcement constraint (*"take-the-money-and-run"* strategy) in state $\overline{\theta}$. The first of these constraints is certainly relaxed by reducing the price paid to a low-cost seller. As with stationary contracts, it *a priori* means reducing not only the output of a low-cost seller but also, by incentive compatibility, the output of a high-cost one. Relaxing the second of these constraints also calls for making the *"take-the-money-and-run"* strategy less attractive. This also requires us to not only reduce outputs but also a subtle design of payments over time.

Equipped with Assumption 2, we can now characterize contractual dynamics. Theorem 1 shows that the pattern of trades with a low-cost seller entails two distinct phases. In the earlier periods, output continuously increases while remaining below efficiency for both types. Later, in a more mature phase, trade with a low-cost seller entails first-best production and the sole distortion concerns the high-cost seller's output. The optimal contract in the long run exhibits features which are similar to those found under the standard Baron-Myerson scenario *modulo* a modification of the virtual cost that now reflects the magnitude of the enforcement problem.

THEOREM 1 *Suppose that Assumption 2 holds. There exists $\tau^* \geq 1$ such that the optimal contract passes through two different phases.*

1. TRANSITORY PHASE. *For $\tau \leq \tau^*$, the optimal output $q^{sb}_{\tau}(\underline{\theta})$ of the low-cost seller strictly increases over time but remains below its first-best value:*

   (5.3) $\quad q^{e}(\overline{\theta}) < q^{sb}_{\tau}(\underline{\theta}) \leq q^{fb}(\underline{\theta}).$

   The enforcement constraint (4.9) is binding at all dates $\tau \leq \tau^*$ and the sequence $q^{sb}_{\tau}(\underline{\theta})$ obeys the recursive condition $q_{\tau+1}(\underline{\theta}) = \Phi(q_{\tau}(\underline{\theta}))$ where the function $\Phi(q) =$

$S^{-1}\left(\frac{1}{\delta}\left(\overline{\theta}q-(1-\delta)M\right)\right)$ defined over the interval $\left[\frac{(1-\delta)M}{\overline{\theta}},+\infty\right)$, is increasing, convex and has a unique fixed point $q^e(\overline{\theta})$.

2. Mature phase. *For $\tau>\tau^*$, the optimal output $q^{sb}_\tau(\underline{\theta})$ of a low-cost seller is set at its first-best level:*

$$q^{sb}_\tau(\underline{\theta})=q^{fb}(\underline{\theta}). \tag{5.4}$$

The enforcement constraint (4.9) is slack.
The high-cost seller always produces the same quantity $q^{sb}(\overline{\theta})$ which remains below the Baron-Myerson level. Specifically, there exists $\Lambda_\infty>0$ such that

$$S'(q^{sb}(\overline{\theta}))=\overline{\theta}+\frac{\nu+\Lambda_\infty}{1-\nu}\Delta\theta. \tag{5.5}$$

Only the low-cost seller receives a positive information rent:

$$U_0^{+sb}(\underline{\theta})=\Delta\theta q^{sb}(\overline{\theta})>0=U_0^{+sb}(\overline{\theta}).$$

The brief analysis of the stationary contracts made in Section 5.3 showed how the buyer is torn between two objectives when he wants to ease contract enforcement. On the one hand, he would like to compress payments and reduce production today, especially from a low-cost seller. On the other hand, keeping a high output from this seller also increases future gains from trade, making it more attractive not to breach the relationship which relaxes the current enforcement constraint. With non-stationary contracts, the buyer benefits from the fact that the *"take-the-money-and-run"* strategy becomes less attractive over time to separate those two objectives. In the early periods, the buyer distorts production for the low-cost seller much as what was needed with stationary contracts. This compresses current payments and eases earlier enforcement constraints. Yet, the buyer can also use his commitment power to delay payments for the low-cost seller's rent. There is less need to distort production, up to the point where efficient quantities are traded in the mature phase of contracting.

Intuition for the Dynamics of Output Distortions. To better understand output distortions, we now construct a *"third-best"* contract which also exhibits a growing dynamics as the optimal contract of Theorem 1 but in a crude way using only two output steps. This construction conveys the main intuition behind the existence of two different contracting phases. Consider the following output profile:[23]

$$\mathbf{q}_{\hat{\tau}^*}(\underline{\theta})=(\underbrace{q^e(\overline{\theta}),....,q^e(\overline{\theta})}_{\text{for dates }\tau\leq\hat{\tau}^*},\ \underbrace{q^{fb}(\underline{\theta}),....}_{\text{for dates }\tau>\hat{\tau}^*})\text{ and }\mathbf{q}_{\hat{\tau}^*}(\overline{\theta})=(q^{bm}(\overline{\theta}),...,q^{bm}(\overline{\theta})). \tag{5.6}$$

Although the high-cost seller's output is stationary and fixed at the Baron-Myerson level, the low-cost seller's production goes through two phases. For the first $\hat{\tau}^*+1$ periods, the

[23] Payments are determined by the binding incentive and participation constraints for the low- and high-cost seller respectively.

low-cost seller produces the maximal quantity that can be enforced had the seller been known of a high cost, namely $q^e(\bar\theta)$. After that, trade is efficient.

For the sake of the argument, we strengthen the first condition in Assumption 2 as:

$$0 < \psi(\underline\theta, q^{fb}(\underline\theta)) + (1-\delta)\underline\theta(q^{fb}(\underline\theta) - q^e(\bar\theta)) < \Delta\theta q^{bm}(\bar\theta).$$

The righthand side inequality ensures that offering $\mathbf{q}_0(\underline\theta) = (\underbrace{q^e(\bar\theta)}_{\text{at } \tau=0}, \underbrace{q^{fb}(\underline\theta), \dots}_{\text{for dates } \tau>0})$ together with $\mathbf{q}_0(\bar\theta) = (q^{bm}(\bar\theta), \dots, q^{bm}(\bar\theta))$ would violate the enforcement constraint (4.9) at date 0. In other words, a contract of the form (5.6) is necessarily such that the transitory phase has more than one period, i.e., $\hat\tau^* > 0$. As we will see below, $\hat\tau^*$ is constructed so that the buyer can still enjoy efficient trades with a low-cost seller from date $\hat\tau^*+1$ onwards if he is ready to maintain a low output over the first τ^* periods of the transitory phase. Such a distortion allows one to keep the low-cost seller's payments small and make the *"take-the-money-and-run"* strategy of the high-cost seller less tempting.

To make this simple point more formally, observe that Assumption 2 (and thus its strengthening) also implies that $q^{bm}(\bar\theta) < q^e(\bar\theta)$ so that the sequence

$$\delta^{-\tau}\Delta\theta\left(q^{bm}(\bar\theta) - (1-\delta^\tau)q^e(\bar\theta)\right)$$

is actually decreasing. This monotonicity implies that there exists a first date $\hat\tau^* \geq 1$ at which the enforcement constraint (4.9) holds for the profile $(\mathbf{q}_{\hat\tau^*}(\underline\theta), \mathbf{q}_{\hat\tau^*}(\bar\theta))$. This τ^* is the first integer such that:

$$\delta(S(q^{fb}(\underline\theta)) - \underline\theta q^{fb}(\underline\theta)) - (1-\delta)\underline\theta q^e(\bar\theta) + (1-\delta)M \geq \delta^{-\hat\tau^*}\Delta\theta\left(q^{bm}(\bar\theta) - (1-\delta^{\hat\tau^*})q^e(\bar\theta)\right). \tag{5.7}$$

Intuitively, if the buyer waits long enough before requesting efficient trades, he will eventually prevent the *"take-the-money-and-run"* strategy. Delaying efficient trades till the end of the transitory phase is thus a first cost of limited enforcement. The second cost of limited enforcement is that output is inefficiently low over this transitory phase.

With this *"third-best"* contract, this transitory phase is constructed so that the output profile (5.6) also satisfies the enforcement constraints (4.9) at all dates $\tau \leq \hat\tau^*$.[24] Intuitively, a contract that offers $q^e(\bar\theta)$ to a low-cost seller in the earlier periods is certainly immune to the possibility that the high-cost seller adopts the *"take-the-money-and-run"* strategy at those dates. Indeed, even if the seller was known to be a high-cost type, the benefits of breaching a contract that would request $q^e(\bar\theta)$ from such seller would just cover the cost of the breach.

[24]To see how, just multiply (5.7) by $\delta^{\hat\tau^*-\tau}$ and (5.1) (taken for $\theta = \bar\theta$) by $1-\delta^{\hat\tau^*-\tau}$ and sum the two conditions so obtained to get, after simplifications, that the enforcement constraint (4.9) at all dates τ such that $\tau \leq hat\tau^*$ also holds:

$$(1-\delta)\left(\sum_{s=1}^{\hat\tau^*-\tau}\delta^s(S(q^e(\bar\theta)) - \theta q^e(\bar\theta)) + \sum_{\hat\tau^*-\tau+1}^{\infty}\delta^s(S(q^{fb}(\underline\theta)) - \underline\theta q^{fb}(\underline\theta))\right) - (1-\delta)\underline\theta q^e(\bar\theta) + (1-\delta)M$$
$$\geq \delta^{-\tau}\Delta\theta\left(q^{bm}(\bar\theta) - (1-\delta^\tau)q^e(\bar\theta)\right).$$

Output Distortions. The optimal contract differs from the *"third-best"* contract just constructed along two dimensions. First, the buyer compresses payments to the low-cost seller by reducing his information rent. This means implementing an output $q^{sb}(\overline{\theta})$ lower than the Baron-Myerson level. Since, under Assumption 2, the only concern is to prevent the high-cost seller's *"take-the-money-and-run"* strategy, and since all periods are the same, an equal incentive distortion is imposed on his own output at all dates. By imposing downward distortions below the Baron-Myerson level, the buyer facilitates enforcement in earlier periods and shortens the transitory phase. This minimizes the distortions of not implementing efficient trades earlier on.

Second, the buyer can also relax enforcement constraints over the transitory phase by being less extreme in the downward distortion of the low-cost seller's output. At the optimum, the buyer implements outputs $q^{sb}_{\tau}(\overline{\theta})$ which might be close to $q^{e}(\overline{\theta})$ for the transitory phase but remain greater. As times passes over the transitory phase, those distortions are less significant in response to an enforcement problem of a lower magnitude.

Compared with the case of stationary contracts, optimal distortions are spread over the first $\hat{\tau}^*$ periods. Indeed, an optimal stationary contract would cause the enforcement constraint (4.9) to bind with a very high shadow cost at date $\tau = 0$, but to be slack for all other periods. It is more profitable for the buyer to spread the cost of enforceability across the first periods, but this requires a non-stationary allocation. The construction thus resembles the output profile (5.6). There also, enforcement constraints are binding at all dates $\tau \leq \hat{\tau}^*$.

In the mature phase of contracting, output distortions with the low-cost seller are no longer needed. The contract becomes stationary. In contrast with what arises when restricting *a priori* to stationary contracts, a low-cost seller now produces efficiently. Our model thus predicts an increasing dispersion of outputs over time, contrary to what happens with stationary contracts.

The parameter Λ_∞ that characterizes output distortions is actually the sum of the Lagrange multipliers for all binding enforcement constraints over the transitory phase. Echoing our findings with stationary contracts, this parameter again links altogether the nature of the screening distortions and the quality of enforcement. Remarkably, the output distortion for a high-cost seller obeys a *modified Baron-Myerson* formula (5.5) that illustrates how virtual costs must now be modified with limited enforcement. A greater value of Λ_∞ translates into greater output distortions.

Backloaded payments. To ease enforceability, the buyer supplements output distortions with payments that make the *"take-the-money-and-run"* strategy less attractive. To this end, the buyer backloads payments to the low-cost seller while still keeping an overall price large enough to pay for the latter's information rent.

That the enforcement constraints (4.9) bind over the transitory phase puts some structure on the intertemporal profile of payments. This stands in sharp contrast with the case of costless enforcement (Baron and Besanko (1984)). There, only the values of the overall intertemporal payments to both types are known from the binding incentive and participation constraints and, although payments can be chosen to be stationary, there is much leeway beyond that specific choice. The next Proposition characterizes payments that implement the optimal allocation described in Theorem 1.

PROPOSITION 2 *Suppose that Assumption 2 holds. The following payments implement the optimal contract.*

1. *Pre-delivery payments cover the seller's penalty for breach:*

$$t^{sb}_{1,\tau}(\theta) = L \quad \forall \theta \in \Theta, \quad \forall \tau \geq 0. \tag{5.8}$$

2. *The high-cost seller's payment is constant over time:*

$$t^{sb}_{\tau}(\overline{\theta}) = \overline{\theta} q^{sb}(\overline{\theta}) \quad \forall \tau \geq 0. \tag{5.9}$$

3. *The low-cost seller's payment is increasing over the transitory phase and constant over the mature phase:*

$$t^{sb}_{\tau}(\underline{\theta}) = \begin{cases} \overline{\theta} q_{\tau}(\underline{\theta}) & \forall \tau \leq \tau^* - 1, \\ \underline{\theta} q_{\tau}(\underline{\theta}) + \frac{\delta^{-\tau}\Delta\theta}{1-\delta}\left(q^{sb}(\overline{\theta}) - q^{-}_{\tau}(\underline{\theta})\right) & \tau = \tau^*, \\ \underline{\theta} q^{fb}(\underline{\theta}) & \forall \tau > \tau^*. \end{cases} \tag{5.10}$$

Choosing a pre-delivery payment that just covers the seller's penalty for breach (condition (5.8)) is akin to redistributing remedies between the buyer and the seller. Everything happens as if the former now pays all remedies ($K' = M$) in case she himself breaches the agreement while the former pays nothing ($L' = 0$) and is only subject to the private side of enforcement if he breaches. In other words, the buyer who has all bargaining power can undo any initial allocation of remedies through a convenient design of payments without modifying the nature of the enforcement constraints. With such choice, the post-delivery price becomes:

$$t^{sb}_{2,\tau}(\theta) = -L + t^{sb}_{\tau}(\theta) \quad \forall \theta \in \Theta, \quad \forall \tau.$$

The contract must also extract the high-cost seller's rent so that his interim participation constraint is binding at the optimum. The payments (5.9) achieve this goal by imposing the stricter requirement of a binding participation constraint in every single period. With such scheme, a high-cost seller who revealed his type at the start of the relationship certainly won't breach the contract at any future date because he is just indifferent between abiding to the terms of the contract and breaching in each period.

Preventing the high-cost seller's *"take-the-money-and-run"* strategy requires one to keep the high-cost type indifferent between telling the truth (and making zero profit each period) and pretending to be more efficient. In particular, the payment for the low-cost seller must be adjusted to cover the high-type's cost of producing the high output over the transitory phase. In other words, the high-cost seller makes also zero profit in each period of the transitory phase if he pretends to be efficient. Countervailing incentives are neutralized with this scheme.[25]

[25] Contrary to the earlier literature (Lewis and Sappington (1989)), countervailing incentives here apply to different types and full separation remains possible.

The final step consists in checking that the payments in (5.10) offered over the mature phase, together with those given over the transitory phase, ensure that the low-cost seller receives enough information rent to reveal his type truthfully at date 0.

From Proposition 2, the dynamics of the low-type seller's *current* payoff are described by

$$(5.11)\quad U_\tau(\underline{\theta}) = t_\tau(\underline{\theta}) - \underline{\theta} q_\tau(\underline{\theta}) = \begin{cases} \Delta\theta q_\tau(\underline{\theta}) & \forall \tau \le \tau^* - 1, \\ \frac{\delta^{-\tau}\Delta\theta}{1-\delta}\left(q^{sb}(\overline{\theta}) - q_\tau^-(\underline{\theta})\right) & \tau = \tau^*, \\ 0 & \forall \tau > \tau^*. \end{cases}$$

Because output increases over the transitory phase, the low-cost seller's per-period payoff thus also grows over this phase. The main purpose of reducing payoffs for this type is indeed to prevent the high-cost type from *"taking the money and run."* Because of discounting, it is more efficient to reduce payments to a low-cost seller earlier on. Later, that payoff must increase to provide enough rent over the whole relationship to induce information revelation. The last non-zero payoff $U_{\tau^*}(\underline{\theta})$ corrects for this effect. After date τ^*, both types enjoy no rent in each period.

Length of the transitory phase. The optimal length of the transitory phase trades off two competing effects that can be best seen by coming back on the crude contract that uses two output steps which we discussed above. First, increasing the number of periods where the enforcement constraints (4.9) are binding allows to keep the high-cost seller's output close to the Baron-Myerson level. However doing so also forces to keep inefficient trades with a low-cost seller for too long. We now provide bounds on the length of this transitory phase.

Proposition 3 *Suppose that Assumption 2 holds. The length of the transitory phase τ^* satisfies the following bounds:*

$$(5.12)\quad \frac{ln\left(\frac{\Delta\theta\left(q^{bm}(\overline{\theta})-q^e(\overline{\theta})\right)}{\psi(\underline{\theta},q^{fb}(\underline{\theta}))-\Delta\theta q^e(\overline{\theta})}\right)}{ln(\delta)} > \tau^* \ge \frac{ln\left(\frac{\Delta\theta\left(q^{bm}(\overline{\theta})-q^{fb}(\overline{\theta})\right)}{\psi(\underline{\theta},q^{fb}(\underline{\theta}))-\Delta\theta q^{fb}(\overline{\theta})}\right)}{ln(\delta)} - 1.$$

When $q^{fb}(\underline{\theta}) - q^e(\overline{\theta})$ is small enough, both the righthand and the lefthand sides of (5.12) grow large. Intuitively, when $q^{fb}(\underline{\theta})$ is *"almost"* enforceable, the optimal contract remains close to $q^e(\overline{\theta})$ for a very long time before moving towards the nearby first-best level.

When δ goes to 0, only the public side of enforcement matters. Future gains from trade are of no help to prevent opportunism. In that limiting case, the numerators on both sides of (5.12) remain bounded while the denominator goes to infinity. The transitory phase lasts only one period when Assumption 2 holds. The enforcement constraint (4.9) at date 0 then almost reduces to a simple constraint on current output and forward rent, namely,

$$M - \underline{\theta} q_0(\underline{\theta}) \approx_{\delta\approx 0} \Delta\theta q^{sb}(\overline{\theta}).$$

When the future no longer matters, the relationship is almost static with the sole possibilities for enforcement coming from public remedies. Output distortions for the low-cost seller are then concentrated on that first period of the relationship.

When instead δ goes to 1, the private side of enforcement becomes *de facto* the best vehicle to sustain the relationship. The bounds in (5.12) give much less information. This is precisely the scenario where enforcement constraints might be slack. For δ close to 1, Assumption 1 simplifies to:

$$S(q^{fb}(\underline{\theta})) - \underline{\theta} q^{fb}(\underline{\theta}) \geq \Delta\theta q^{bm}(\overline{\theta}), \tag{5.13}$$

implying that the gains from trade with a low-cost seller have to only exceed the rent left to that type to ensure enforcement.

6. ROBUSTNESS

6.1. *One-Sided Breach*

When opportunism is only one sided, there exist pre- and post-delivery payments that implement the Baron-Myerson outcome at no extra cost. To illustrate this point, we first consider the case where only the buyer is opportunistic. In each period, pre- and post-delivery payments can be constructed so as to induce the buyer to perform. More precisely, consider the following payments:

$$t_{2,\tau}(\theta) = K + \frac{\delta}{1-\delta}(S(q^{bm}(\theta)) - t^{bm}(\theta)) \text{ and } t_{1,\tau}(\theta) + t_{2,\tau}(\theta) = t^{bm}(\theta), \quad \forall \tau \geq 0, \quad \forall \theta \in \Theta.$$

Post-delivery payments are such that the buyer is always indifferent between performance and breaching, in the latter case paying the corresponding remedies and losing his future gains from trade at the Baron-Myerson allocation. In other words, the mechanism so constructed is buyer-enforceable and constraints (4.2) always hold. Pre-delivery payments are such that the seller receives the corresponding Baron-Myerson payments.

The analysis of the case where the informed seller misbehaves is left to the Appendix. Details differ but the main idea remains. With one-sided opportunism, there is enough freedom in designing an intertemporal profile of payments to prevent one-sided breaches at no cost. We can thus conclude:

THEOREM 2 *One-sided opportunism is costless.*

6.2. *Renegotiation*

In our main analysis, we assumed that parties might not be able to perfectly enforce a transaction in any given period. At the same time, those parties can instead commit not to renegotiate their *ex ante* agreement, an assumption that was also discussed at length in Section 3.4. Commitment is a way to avoid reputation and legal costs or to mitigate the consequences of inefficient *ex ante* investments that may be caused by the threat of renegotiation.

In some circumstances, a commitment not to renegotiate may be less reasonable. To illustrate, large infrastructure projects often require specific investments so that parties are *ex post* locked into bilateral monopoly relationships. A contract when breached might still be renegotiated towards another deal with the same partner if completing the project has a significant joint value.[26] With such renegotiation, the Court also takes a more active stance, not only enforcing remedies for breaches of the old agreement but also enforcing the new contract.

To model a simple renegotiation protocol, we suppose that parties can write any long-term contract they wish, making full use of the Court's penalties L and K when drafting a new agreement. In this scenario, the Court has a simple role. It will enforce any long-term contract using penalties up to L and K, but it will allow parties to tear up their initial contract when there is mutual agreement to do. Parties may also continue their relationship following an earlier breach with a pure relational contract; the Court being irrelevant in this case. A last option is for parties to continue to perform under the terms of the original contract.

Malcomson (2015) observes that standard notions of renegotiation-proofness that were developed for infinite horizon games only apply under complete information.[27] In our asymmetric information context, we shall thus impose a renegotiation-proofness requirement only for continuation equilibria that follow full separation of the seller's type at an earlier stage. We thus consider Perfect Bayesian equilibria in pure strategies for which, following full information revelation at stage 0, continuations payoffs lie on the Pareto frontier of subgame-perfect equilibria of the complete information continuation so defined.

To characterize such complete information continuation, we recall our findings of Section 5.1. We showed there that the optimal enforceable contract under complete information was stationary and could be implemented with relational contracting. The stationary contract with outputs $q_\tau(\theta) = q^{ci}(\theta) = \min\{q^e(\theta), q^{fb}(\theta)\}$ for all $\tau \geq 1$ and payments $t_\tau(\theta) = \theta q^{ci}(\theta)$ thus lies on the Pareto frontier of subgame-perfect equilibria of the complete information continuation game.

To get a characterization of the optimal renegotiation-proof contract which is comparable with that obtained in Theorem 1, we still suppose that Assumption 2 holds. In particular, this assumption implies $q^{ci}(\underline{\theta}) = q^{fb}(\underline{\theta})$ and $q^{ci}(\overline{\theta}) \geq q^{bm}(\overline{\theta})$.

We now want to characterize date 0-outputs at the optimal renegotiation-proof contract. To this end, we first observe that, being given the expression of the outputs implemented in the complete information continuation just described, the seller's intertemporal information rents at an optimal contract (which are obtained when the usual incentive and participation constraints are binding[28]) can be expressed as:

$$U_0^+(\underline{\theta}) = \Delta\theta\left((1-\delta)q_0(\overline{\theta}) + \delta q^{ci}(\overline{\theta})\right) \text{ and } U_0^+(\overline{\theta}) = 0.$$

[26]Other circumstances, referred to as *"change orders"* in the parlance of construction contracts, may also require that parties agree to tear up their initial agreement and change it for another deal which might be more responsive to changing conditions if any unexpected contingency arises.

[27]See Bergin and MacLeod (1993) for a synthesis of the literature.

[28]The proof is standard and thus omitted.

Taking into account the expressions of those rent and output profiles, date 0-enforcement constraint (4.6) in state $\underline{\theta}$ becomes:

$$\delta\left(S(q^{fb}(\underline{\theta}))-\underline{\theta}q^{fb}(\underline{\theta})\right)-(1-\delta)\underline{\theta}q_0(\underline{\theta})+(1-\delta)M\geq\Delta\theta((1-\delta)q_0(\overline{\theta})+\delta q^{ci}(\overline{\theta})). \tag{6.1}$$

The optimal date 0-outputs $\left(q_0^r(\underline{\theta}),q_0^r(\overline{\theta})\right)$ maximizes the buyer's expected profit subject to constraint (6.1).

PROPOSITION 4 *Suppose that the Assumption 2 holds. The output profiles* $\mathbf{q}_0(\underline{\theta})=(q_0^r(\underline{\theta}),q^{fb}(\underline{\theta}),...,q^{fb}(\underline{\theta}),...)$ *and* $\mathbf{q}_0(\overline{\theta})=(q_0^r(\overline{\theta}),q^{ci}(\overline{\theta}),...,q^{ci}(\overline{\theta}),...)$ *implemented at the optimal renegotiation-proof contract are such that* $q_0^r(\underline{\theta})<q^{fb}(\underline{\theta})$ *and* $q_0^r(\overline{\theta})<q^{ci}(\overline{\theta})$ *with:*

$$S'(q_0^r(\underline{\theta}))=\left(1+\frac{\lambda_r}{\nu}\right)\underline{\theta}\ \text{and}\ S'(q_0^r(\overline{\theta}))=\overline{\theta}+\frac{\nu+\lambda_r}{1-\nu}\Delta\theta, \tag{6.2}$$

where $\lambda_r>0$ *is the Lagrange multiplier for date 0-enforcement constraint (6.1).*

Thus, the renegotiation-proof output profiles exhibit growing dynamics which bear some similarities to the commitment case. This shows the robustness of our earlier findings to the possibility of renegotiation. However, some differences remain. Renegotiation-proofness imposes requires that payments and outputs from date 1 onwards are the best ones that can be implemented with relational contracts under complete information. In particular, the seller makes zero profit in all those periods. To induce information revelation, the buyer is thus forced to pay all the low-cost seller's information rent at date 0. This makes the high-cost seller's *"take-the-money-and-run"* strategy particularly attractive at that date. Unfortunately, the buyer can no longer push the cost of date 0-enforcement to future periods as she would do under full commitment. Date 0-enforcement constraint (6.1) can now only be relaxed by reducing payments and outputs at this date. This explains that date 0-outputs are set below their Baron-Myerson levels for both types.

7. APPLICATIONS

Our model is useful to address a number of important questions in organization theory.

7.1. *Asset Specificity and Contract Enforcement*

Our contractual setting can be viewed as a stylized modeling of an ongoing relationship between a contractor and his long-term supplier for an essential input. *Transaction Costs Economics* has already discussed at length how opportunism and asset specificity shape such relationships, especially in terms of their impacts on the optimal degree of vertical integration and more generally on contract duration.[29] In our context, and using the language of *Transaction Costs Economics*, parties must build safeguards against bilateral opportunism over the growing phase whenever enforcement constraints are binding. Instead, in the long run, more mature relationships are no longer subject to such threat.

Even though specific investments are not present *per se* in our baseline model, our framework can readily be extended to link asset specificity and the quality of enforcement. Making assets more specific to the relationship may thus act as a safeguard against

[29] Joskow (1987,1988), Crocker and Masten (1988, 1996), Ramey and Watson (2001) and Halac (2015).

the possibility of breaches. As a simple extension along these lines, suppose that, prior to contracting, the buyer makes a relation-specific investment whose cost is i. This investment enhances surplus in each period τ by an amount $B(i)$ (with $B'(i) > 0$ and $B''(i) < 0$) so that the net gains from trade in period τ can be written as:

$$S(q_\tau) - \theta q_\tau + B(i) - i.$$

Assuming, for simplicity, an interior solution, the efficient level of investment i^{fb} satisfies:

$$B'(i^{fb}) = 1.$$

Denote by $C(i)$ the buyer's opportunity cost for the foregone use of dedicated assets if the contract is breached. This cost also increases with asset specificity ($C'(i) > 0$ with $C''(i) > 0$). This cost can be counted as an implicit penalty for breach that would be borne by the buyer if she does not fulfill her obligations. Keeping the same expression as above for the enforcement surplus $\Psi(\underline{\theta}, \overline{\mathbf{q}}_\tau(\underline{\theta}))$, the enforcement constraints become:

$$\Psi(\underline{\theta}, \overline{\mathbf{q}}_\tau(\underline{\theta})) + \delta B(i) + (1-\delta)C(i) \geq \delta^{-\tau} \max\left\{\Delta\theta(q_0^+(\overline{\theta}) - q_\tau^-(\underline{\theta})), 0\right\} \quad \forall \tau. \tag{7.1}$$

Assuming that $C''(i)$ is small enough so that the lefthand side of (7.1) remains concave in i, the optimal investment level i^e must now take into account the impact of such investment on contracts enforceability. Increasing investment has a first direct effect in relaxing enforcement constraints and second, an indirect effect, that comes from changing optimal trade profiles in response to the fact that enforcement constraints are easier to satisfy. By the Envelope Theorem, this indirect effect vanishes and i^e simply solves

$$B'(i^e) = 1 - \Lambda_\infty \left(\delta B'(i^e) + (1-\delta)C'(i^e)\right) < 1.$$

From this, we immediately conclude that $i^e > i^{fb}$. The buyer is now eager to increase her investment as a commitment device to facilitate enforcement. Moreover, those incentives are more pronounced as Λ_∞ is bigger, i.e., when enforcement is more difficult.

7.2. *Relational Contracting and Firm's Boundaries*

Gibbons (2005b) and Baker et al. (2001) argue that one of the key research questions in the Property Rights literature is to understand how relational contracts are affected by firm's boundaries. Our paper contributes to this important debate. Suppose now that the informed seller may perform some specific investment i_s prior to contracting. That investment improves the value of trade by increasing the probability $\nu(i_s)$ of being efficient. In a vertically integrated firm owned by the buyer, the seller becomes an employee of the firm and, at any point in time, this employee has the right to leave the firm.[30] Following Riordan (1990), we may also assume that ownership gives access to information. The seller thus enjoys no information rent and has no incentives to make any ex ante investment; $i_s = 0$.

Under vertical separation instead, the seller owns the assets, retains private information and enjoys an expected informational rent worth $\nu(i_s)q^{sb}(\overline{\theta})$. This rent acts as an engine for investment but it also hardens the enforcement problem. Our model first predicts that *"take-the-money-and-run"* strategy will only arise in market relationships between firms

[30] Presumably, $L = 0$ in the case of non-alienable employment relationships.

that remain vertically separated. Second, market relationships come with greater volumes over time whereas intrafirm exchanges may exhibit more stable patterns. Finally, output distortions being greater when enforcement problems are more acute, vertical integration becomes more attractive when enforcement is more difficult.

REFERENCES

Abreu, D. (1988), "On the Theory of Infinitely Repeated Games with Discounting," *Econometrica,* 56: 383-396.

Atyiah, P. (1995), *An Introduction to the Law of Contract*, Princeton University Press, Princeton.

Baker, G., R. Gibbons and K. Murphy (1994), "Subjective Performance Measures in Optimal Incentive Contracts," *Quarterly Journal of Economics,* 109: 1125-56.

Baker, G., R. Gibbons and K. Murphy (2001), "Bringing the Market Inside the Firm?" *American Economic Review*, 91: 212-218.

Baron, D. and D. Besanko (1984), "Regulation and Information in a Continuing Relationship," *Information Economics and Policy,* 1: 267-302.

Baron, D. and R. Myerson (1982), "Regulating a Monopolist with Unknown Costs," *Econometrica*, 50: 911-930.

Battaglini, M. (2005), "Long-Term Contracting with Markovian Consumers," *American Economic Review,* 95: 637-658.

Bebchuk, L. (1984), "Litigation and Settlement under Imperfect Information," *The RAND Journal of Economics,* 15: 404-415.

Bergin, J. and W. MacLeod (1993), "Efficiency and Renegotiation in Repeated Games," *Journal of Economic Theory,* 61: 42-73.

Bernheim, B. and M. Whinston (1998), "Incomplete Contracts and Strategic Ambiguity," *American Economic Review,* 88: 902-932.

Board, S. (2011), "Relational Contracts and the Value of Loyalty," *American Economic Review,* 101: 3349-3367.

Bull, C. (1987), "The Existence of Self-Enforcing Implicit Contracts," *Quarterly Journal of Economics,* 102: 147-59.

Callahan, M. (2009) *Termination of Construction and Design Contracts,* Aspen Law and Business.

Crocker, K. and S. Masten (1988), "Mitigating Contractual Hazards: Unilateral Options and Contract Length," *The RAND Journal of Economics,* 19: 327-343.

Crocker, K. and S. Masten (1996), "Regulation and Administered Contracts Revisited: Lessons from Transaction-Cost Economics for Public Utility Regulation," *Journal of Regulatory Economics,* 9: 5-39.

Dechert, W. (1982), "Lagrange Multipliers in Infinite Horizon Discrete Time Optimal Control Models," *Journal of Mathematical Economics,* 9: 285-302.

Dewatripont, D. (1989), "Renegotiation and Information Revelation over Time: The Case of Optimal Labor Contracts," *Quarterly Journal of Economics,* 104: 589-619.

Doornik, K. (2010), "Incentive Contracts with Enforcement Costs," *Journal of Law, Economics, and Organization,* 26: 115-143.

Edlin, A. (1998), Breach Remedies. In *The New Palgrave Dictionary of Economics and the Law*, ed. P. Newman, London: Macmillan, 1: 174-179.

Eggleston, B., (2009), *Liquidated Damages and Extensions of Time in Construction Contracts*, Third Edition, Wiley-Blackwell.

Escobar, J.F. and J Toikka (2013) "Efficiency in Games with Markovian Private Information," *Econometrica*, 81: 1887-1934.

Farnsworth, E. A., (1982), *Contracts*, Little Brown Publishers.

Fong, Y.-F. and J. Li (2010), "Information Revelation in Relational Contracts," Mimeo Northwestern University, Kellogg School of Management.

Fong, Y.-F. and J. Surti (2009), "On the Optimal Degree of Cooperation in the Repeated Prisoner's Dilemma with Side-Payments," *Games and Economic Behavior*, 67: 277-291.

Fuchs, W. (2007), "Contracting with Repeated Moral Hazard and Private Evaluations," *American Economic Review,* 97: 1432-1448.

Goldlücke, S. and S. Kranz, S. (2013), "Renegotiation-Proof Relational Contracts," *Games and Economic Behavior,* 80: 157-178.

Guasch, J. L., J.J. Laffont and S. Straub (2003), "Renegotiation of Concession Contracts in Latin America," (Vol. 3011), World Bank Publications.

Gibbons, R. (2005a), "Four Formal(izable) Theories of the Firm," *Journal of Economic Behavior and Organization,* 58: 200-245.

Gibbons, R. (2005b), "Incentives Between Firms (and Within)," *Management Science,* 51: 2-17.

Ghosh, P., and D. Ray (1996), "Cooperation in Community Interaction without Information Flows," *Review of Economic Studies,* 63: 491-519.

Halac, M. (2012), "Relational Contracts and the Value of Relationships," *American Economic Review,* 102: 750–79.

Halac, M. (2015), "Investing in a Relationship," *The RAND Journal of Economics*, 46: 165-185.

Hart, O. and J. Tirole (1988), "Contract Renegotiation and Coasian Dynamics," *Review of Economic Studies*, 55: 509-540.

Hemsley, P. (2013), "Dynamic Moral Hazard with Self-Enforceable Incentive Payments," Mimeo Toulouse School of Economics.

Hinze, J. (2001), *Construction Contracts,* 2 ed. McGraw-Hill.

Horner, J. (2002), "Reputation and Competition," *American Economic Review*, 92: 644-663.

Iossa, E. and G. Spagnolo (2009), "Contracts as Threats: On a Rationale for Rewarding A while Hoping for B," Mimeo Brunel University.

Itoh, H. and H. Morita (2015), "Formal Contracts, Relational Contracts and the Threat-Point Effect," *American Economic Journal: Microeconomics*, 7: 318-346.

Johnson, J. and R. Sohi (2015), "Understanding and Resolving Major Contractual Breaches in Buyer-Seller Relationships: A Grounded Theory Approach," *Journal of the Academy of Marketing Science,* 1-21.

Johnson, S., J. McMillan and C. Woodruff (2002), "Courts and Relational Contracts," *Journal of Law, Economics, and Organization,* 18: 221-277.

Joskow, P. (1987), "Contract Duration and Relationship and Relationship-Specific Investments: Evidence from Coal Markets," *American Economic Review*, 77: 168-185.

Joskow, P. (1988), "Asset Specificity and the Structure of Vertical Relationships: Empirical Evidence," *Journal of Law, Economics, and Organization*, 4: 98-115.

Kornhauser, L. and B. MacLeod (2012), "Contracts Between Legal Persons," *The Handbook of Organizational Economics*, R. Gibbons and J. Roberts eds., North-Holland.

Kvaløy, O. and T. Olsen (2009), "Endogenous Verifiability and Relational Contracting," *American Economic Review*, 99: 2193-2208.

Kwon, S. (2013), "Relational Contracts in a Persistent Environment," Working paper University College London.

Kwon, S. (2015), "Dynamic Moral Hazard with Persistent States," Working paper University College London.

Laffont, J.J. and D. Martimort (2002), *The Theory of Incentives: The Principal-Agent Model*, Princeton University Press.

Laffont, J.J. and J. Tirole (1993), *A Theory of Incentives in Procurement and Regulation*, MIT Press.

Levin, J. (2003), "Relational Incentive Contracts," *American Economic Review,* 93: 835-857.

Lewis, T., and D. Sappington (1989), "Countervailing Incentives in Agency Problems," *Journal of Economic Theory*, 49: 294-313.

Li, J., and N. Matouschek (2013), "Managing Conflicts in Relational Contracts," *American Economic Review,* 103: 2328-51.

Macaulay, S. (1963), "Non-Contractual Relations in Business: A Preliminary Study," *American Sociological Review*, 28: 55-67.

Macciavello, R. and A. Morjaria (2014), "The Value of Relationships: Evidence from a Supply Shock to Kenya Rose Exports," *American Economic Review,* forthcoming

MacLeod, B. and J. Malcomson (1989), "Implicit Contracts, Incentive Compatibility, and Involuntary Unemployment," *Econometrica,* 57: 447-480.

MacLeod, B. and J. Malcomson (1998), "Motivation and Markets," *American Economic Review,* 88: 388-411.

Malcomson, J. (2012), "Relational Incentive Contracts," in *Handbook of Organizational Economics*, R. Gibbons and J. Roberts eds. Princeton University Press.

Malcomson, J. (2015), "Relational Incentive Contracts with Persistent Private Information," *Econometrica,* forthcoming.

Maskin, E. and J. Tirole (2004), "The Politician and the Judge: Accountability in Government", *American Economic Review,* 94: 1034-1054.

Masten, S. (1999), "Contractual Choice," *Encyclopedia of Law and Economics.* B. Boukaert, G. DeGeest, eds. Edward Elgar Publishing.

Pavan, A., Segal, I., and J. Toikka, (2014), "Dynamic Mechanism Design: A Myersonian Approach," *Econometrica,* 82: 601-653.

Pearce, D. and E. Stacchetti (1998), "The Interaction of Implicit and Explicit Contracts in Repeated Agency," *Games and Economic Behavior*, 23: 75-96.

Posner, E. A. (2011). *Contract Law and Theory.* Aspen Student Treatise Series. Wolters Kluwer Legal Publications.

Rahman, D. (2012), "But Who Will Monitor the Monitor?" *American Economic Review,* 102: 2767-2797.

Ramey, G. and J. Watson (2001), "Bilateral Trade and Opportunism in a Matching Market," *The B.E. Journal of Theoretical Economics*, 1(1): 1-35.

Reinganum, J. and L. Wilde (1986), "Settlement, Litigation, and the Allocation of Litigation Costs," *The RAND Journal of Economics,* 17: 557-566.

Rey, P. and B. Salanié (1996), "On the Value of Commitment with Asymmetric Information," *Econometrica*, 64: 1395-1414.

Riordan, M. (1990), "What Is Vertical Integration," in M. Aoki, B. Gustaffson and O. Williamson, eds., *The Firm as a Nexus of Treaties,* Sage Publications.

Schmidt, K. and M. Schnitzer (1995), "The Interaction of Explicit and Implicit Contracts," *Economics Letters*, 48: 193-199.

Schwartz, A. and J. Watson (2004), "The Law and Economics of Costly Contracting," *Journal of Law, Economics, and Organization*, 20: 2-31.

Shavell, S. (1980), "Damage Measures for Breach of Contract," *Bell Journal of Economics,* 11: 466-499.
Shavell, S. (2004), *Foundations of Economic Analysis of Law*, Harvard University Press.
Sobel, J. (1985), "A Theory of Credibility," *Review of Economic Studies,* 52: 557-573.
Vinter, G. and G. Price (2006), *Project Finance: A Legal Guide,* Sweet & Maxwell.
Watson, J. (1999), "Starting Small and Renegotiation," *Journal of Economic Theory,* 85: 52-90.
Watson, J. (2002), "Starting Small and Commitment," *Games and Economic Behavior,* 38: 176-199.
Yang, H. (2012), "Nonstationary Relational Contracts with Adverse Selection," *International Economic Review*, 54: 525-547.

APPENDIX

Proof of Lemma 1: Necessity. Observe that

$$U_0^+(\hat{\theta}) = U_\tau^-(\hat{\theta}) - \delta^\tau(1-\delta)(t_{1,\tau}(\hat{\theta}) - L) + \delta^\tau U_\tau^+(\hat{\theta}) \quad \forall \hat{\theta} \in \Theta.$$

Using this condition, we rewrite (4.3) as:

$$U_0^+(\theta) \geq U_0^+(\hat{\theta}) + \delta^\tau(1-\delta)(t_{1,\tau}(\hat{\theta}) - L) - \delta^\tau U_\tau^+(\hat{\theta}) + (\hat{\theta} - \theta)q_\tau^-(\hat{\theta}) \quad \forall (\theta, \hat{\theta})^2 \in \Theta^2, \quad \forall \tau \geq 0.$$

Permuting the roles of θ and $\hat{\theta}$ and manipulating the latter condition yields:

(A.1)
$$\delta^\tau \left(U_\tau^+(\theta) - (1-\delta)(t_{1,\tau}(\theta) - L)\right) \geq U_0^+(\theta) - U_0^+(\hat{\theta}) + (\theta - \hat{\theta})q_\tau^-(\theta) \quad \forall (\theta, \hat{\theta})^2 \in \Theta^2, \quad \forall \tau \geq 0.$$

We can now rewrite (4.2) in a more explicit form as:

$$\sum_{s=0}^{\infty} \delta^s (S(q_{\tau+s}(\theta)) - t_{\tau+s}(\theta)) \geq S(q_\tau(\theta)) - t_{1,\tau}(\theta) - K \quad \forall \tau \geq 0.$$

Developing, we get:

$$(1-\delta)\sum_{s=1}^{\infty} \delta^s (S(q_{\tau+s}(\theta)) - \theta q_{\tau+s}(\theta)) \geq (1-\delta)\theta q_\tau(\theta) + U_\tau^+(\theta) - (1-\delta)(t_{1,\tau}(\theta) + K) \quad \forall \tau \geq 0.$$

Multiplying by δ^τ yields:

(A.2)
$$(1-\delta)\delta^\tau \left(\sum_{s=1}^{\infty} \delta^s (S(q_{\tau+s}(\theta)) - \theta q_{\tau+s}(\theta)) - \theta q_\tau(\theta) + M\right) \geq \delta^\tau \left(U_\tau^+(\theta) - (1-\delta)(t_{1,\tau}(\theta) - L)\right).$$

Taken together, (A.1) and (A.2) are compatible if and only if:

$$(1-\delta)\delta^\tau \left(\sum_{s=1}^{\infty} \delta^s (S(q_{\tau+s}(\theta)) - \theta q_{\tau+s}(\theta)) - \theta q_\tau(\theta) + M\right) \geq \max_{\hat{\theta} \in \Theta}\{U_0^+(\theta) - U_0^+(\hat{\theta}) + (\theta - \hat{\theta})q_\tau^-(\theta)\}$$

(where (4.4) holds) which can be rewritten as (4.6).

SUFFICIENCY. Suppose that (4.6) holds for a quantity profile $\{\mathbf{q}(\theta)\}_{\theta\in\Theta}$. Consider the profile of payments $\{\mathbf{t}(\theta)\}_{\theta\in\Theta}$ (and thus forward rents $U_\tau^+(\theta)$) defined as:

$$\delta^\tau\left(U_\tau^+(\theta)-(1-\delta)(t_{1,\tau}(\theta)-L)\right)=\max_{\hat\theta\in\Theta}\{U_0^+(\theta)-U_0^+(\hat\theta)+(\theta-\hat\theta)q_\tau^-(\theta)\}. \tag{A.3}$$

By construction, both (A.1) and (A.2) hold with those payments. In particular, (A.1) is an equality. If we add the requirement

$$t_{1,\tau}(\theta)=L \quad \forall\theta\in\Theta,\forall\tau\geq 0 \tag{A.4}$$

then (A.3) fully determines the profile of forward looking rents for the given values of $U_0^+(\overline{\theta})$ and $U_0^+(\underline{\theta})$ that respect (4.4).

Consider an allocation such that the incentive compatibility of a low-cost seller and the participation constraint of a high-cost one are both binding. These conditions altogether determine the values of $U_0^+(\underline{\theta})$ and $U_0^+(\overline{\theta})$ as:

$$U_0^+(\underline{\theta})=\Delta\theta q_0^+(\overline{\theta}) \text{ and } U_0^+(\overline{\theta})=0. \tag{A.5}$$

From this, we obtain:

$$\max_{\hat\theta\in\Theta}\{U_0^+(\underline{\theta})-U_0^+(\hat\theta)+(\underline{\theta}-\hat\theta)q_\tau^-(\underline{\theta})\}=\max\left\{\Delta\theta(q_0^+(\overline{\theta})-q_\tau^-(\underline{\theta})),0\right\} \tag{A.6}$$

and

$$\max_{\hat\theta\in\Theta}\{U_0^+(\overline{\theta})-U_0^+(\hat\theta)+(\overline{\theta}-\hat\theta)q_\tau^-(\overline{\theta})\}=\max\left\{\Delta\theta(-q_0^+(\underline{\theta})+q_\tau^-(\overline{\theta})),0\right\}=0, \tag{A.7}$$

where the last inequality follows from $q_\tau^-(\overline{\theta})\leq q_0^+(\overline{\theta})\leq q_0^+(\underline{\theta})$ (since (4.5) necessarily holds from incentive compatibility).

From (A.3) taken for $\theta=\overline{\theta}$, (A.4) and (A.6), we deduce that:

$$U_\tau^+(\overline{\theta})=0 \quad \forall\tau \tag{A.8}$$

and thus,

$$t_\tau(\overline{\theta})-\overline{\theta}q_\tau(\overline{\theta})=0 \quad \forall\tau. \tag{A.9}$$

From (A.3) taken for $\theta=\underline{\theta}$, (A.4) and (A.6), we also deduce that:

$$U_\tau^+(\underline{\theta})=\delta^{-\tau}\max\left\{\Delta\theta(q_0^+(\overline{\theta})-q_\tau^-(\underline{\theta})),0\right\} \quad \forall\tau. \tag{A.10}$$

Q.E.D.

For future references, we may rewrite Lemma 1 by developing the enforcement constraints (4.6) as:

LEMMA A.1 *An incentive compatible mechanism $\mathcal{C}$ is* ***enforceable*** *if and only if the following enforcement constraints hold at all dates $\tau\geq 0$:*

$$\Psi(\underline{\theta},\mathbf{q}_\tau^+(\underline{\theta}))\geq\delta^{-\tau}\max\left\{\Delta\theta(q_0^+(\overline{\theta})-q_\tau^-(\underline{\theta})),0\right\}, \tag{A.11}$$

$$\Psi(\overline{\theta}, \mathbf{q}_\tau^+(\overline{\theta})) \geq \delta^{-\tau} \max\left\{\Delta\theta(q_\tau^-(\overline{\theta}) - q_0^+(\underline{\theta})), 0\right\}, \tag{A.12}$$

$$\min\left\{\delta^\tau \Psi(\underline{\theta}, \mathbf{q}_\tau^+(\underline{\theta})) + \Delta\theta q_\tau^-(\underline{\theta}); \Delta\theta q_0^+(\underline{\theta})\right\} \geq \max\left\{-\delta^\tau \Psi(\overline{\theta}, \mathbf{q}_\tau^+(\overline{\theta})) + \Delta\theta q_\tau^-(\overline{\theta}); \Delta\theta q_0^+(\overline{\theta})\right\}. \tag{A.13}$$

PROOF OF LEMMA A.1: The incentive compatibility conditions (4.4) imply:

$$\Delta\theta q_0^+(\underline{\theta}) \geq U_0^+(\underline{\theta}) - U_0^+(\overline{\theta}) \geq \Delta\theta q_0^+(\overline{\theta}). \tag{A.14}$$

Inserting the second (resp. first) of these inequalities into (4.6) taken for $\theta = \underline{\theta}$ (resp. taken for $\theta = \overline{\theta}$) yields (A.11) (resp. (A.12)).

There exist values of $U_0^+(\underline{\theta}) - U_0^+(\overline{\theta})$ that satisfy (A.14) and (4.6) if and only if the following condition holds:

$$\min\left\{\delta^\tau \Psi(\underline{\theta}, \mathbf{q}_\tau^+(\underline{\theta})) + \Delta\theta q_\tau^-(\underline{\theta}); \Delta\theta q_0^+(\underline{\theta})\right\} \geq U_0^+(\underline{\theta}) - U_0^+(\overline{\theta}) \geq \max\left\{-\delta^\tau \Psi(\overline{\theta}, \mathbf{q}_\tau^+(\overline{\theta})) + \Delta\theta q_\tau^-(\overline{\theta}); \Delta\theta q_0^+(\overline{\theta})\right\}. \tag{A.15}$$

This finally gives us condition (A.13). Observe that the participation constraints (4.1) can be satisfied for both types by conveniently choosing non-negative values for $U_0^+(\underline{\theta})$ and $U_0^+(\overline{\theta})$ while keeping $U_0^+(\underline{\theta}) - U_0^+(\overline{\theta})$ that satisfies (A.15).

Q.E.D.

PROOF OF PROPOSITION 1: Consider problem $(\mathcal{P})$ written with the enforcement constraints (A.11), (A.12) and (A.13). We first neglect these constraints and consider the participation constraint (4.1) for type $\overline{\theta}$ and the incentive constraint (4.4) for type $\underline{\theta}$. Notice that the enforcement constraints no longer contain $U_0^+(\underline{\theta})$ and $U_0^+(\overline{\theta})$ thanks to Lemma A.1. Thus, (4.1) and (4.4) are both binding at the optimum of the so relaxed problem. The corresponding optimal outputs are stationary and respectively given by $q^{fb}(\underline{\theta})$ and $q^{bm}(\overline{\theta}) < q^{fb}(\underline{\theta})$. It is routine to check the remaining participation and incentive constraints. Turning now to (A.11) written with those stationary outputs, it becomes:

$$\psi(\underline{\theta}, q^{fb}(\underline{\theta})) \geq \delta^{-\tau} \max\left\{\Delta\theta(q^{bm}(\overline{\theta}) - (1-\delta^\tau)q^{fb}(\underline{\theta})); 0\right\}. \tag{A.16}$$

Because $q^{bm}(\overline{\theta}) < q^{fb}(\underline{\theta})$, the righthand side of (A.16) is maximum at $\tau = 0$. Manipulating leads to the first inequality in Assumption 1.

Observe that (A.12) now becomes:

$$\psi(\overline{\theta}, q^{bm}(\overline{\theta})) \geq \delta^{-\tau} \max\left\{\Delta\theta((1-\delta^\tau)q^{bm}(\overline{\theta}) - q^{fb}(\underline{\theta})); 0\right\}. \tag{A.17}$$

Since $(1-\delta^\tau)q^{bm}(\overline{\theta}) < q^{bm}(\overline{\theta}) < q^{fb}(\underline{\theta})$, the righthand side above is 0 giving us the second inequality in Assumption 1.

Turning now to (A.13), this condition becomes:

$$\min\left\{\delta^\tau \psi(\underline{\theta}, q^{fb}(\underline{\theta})) + \Delta\theta(1-\delta^\tau)q^{fb}(\underline{\theta}); \Delta\theta q^{fb}(\underline{\theta})\right\} \geq \max\left\{-\delta^\tau \psi(\overline{\theta}, q^{bm}(\overline{\theta})) + \Delta\theta(1-\delta^\tau)q^{bm}(\overline{\theta}); \Delta\theta q^{bm}(\overline{\theta})\right\}.$$

The latter condition immediately follows from (A.16), (A.17) and $q^{fb}(\underline{\theta}) \geq q^{bm}(\overline{\theta})$. *Q.E.D.*

PROOF OF THEOREM 1: PRELIMINARIES. Denote by l_∞ the Banach space of all bounded sequences $\mathbf{x}$ such that $\|\mathbf{x}\|_\infty \equiv \sup|x_\tau| < \infty$. Given that $\mathbf{q}$ is a bounded interval, the set $\mathbf{Q}_\infty$ of all non-negative output sequences $\mathbf{x} = \{x_\tau\}_{\tau=0}^\infty$ on $\mathbf{q}$ is a subset of l_∞. Let also l_1 denote the space of all sequences $\mathbf{x}$ such that $\|\mathbf{x}\| \equiv \sum_{\tau=0}^\infty |x_\tau| < \infty$. The dual space of l_∞ is $l_\infty^* = l_1 \bigoplus l_s$ where l_s is the set of bounded linear functional generated by the purely additive measures on the integers. (Theorem A.1 below shows that the sequence of Lagrange multipliers that characterizes optimal contracts belongs in fact to l_1.) We denote by $\mathcal{A}$ the closed and convex subset of $\mathbf{Q}_\infty \times \mathbf{Q}_\infty \subseteq l_\infty \times l_\infty$ such that the monotonicity condition (4.5) holds.

SIMPLIFYING THE OBJECTIVE FUNCTION. We first consider a relaxed problem $(\mathcal{P})$ with, on top of the enforcement constraints, only the incentive compatibility constraint (4.4) of type $\underline{\theta}$ and the participation constraint (4.1) of type $\theta = \overline{\theta}$ which are binding at the optimum. We are thus neglecting the incentive compatibility constraint (4.4) for type $\overline{\theta}$ and the participation constraint (4.1) for type $\theta = \underline{\theta}$. (Notice again that the enforcement constraints do not contain $U_0^+(\theta)$ and thus $U_0^+(\theta)$ can be decreased without affecting these constraints.) Constraint (4.4) for type $\overline{\theta}$ holds when (4.4) for type $\theta = \underline{\theta}$ is binding and the allocation satisfies (4.5). Constraint (4.1) holds for type $\theta = \underline{\theta}$ if 4.1) for $\theta = \overline{\theta}$ and (4.4) for type $\theta = \underline{\theta}$ are both binding since output is non-negative.

Second, we neglect (A.12) and (A.13) which are both checked ex post.) Inserting $\overline{U}_0(\underline{\theta}) = \Delta\theta q_0^+(\overline{\theta})$ and $\overline{U}_0(\overline{\theta}) = 0$ into the maximand simplifies the objective function that becomes:

$$f(\mathbf{q}) = E_\theta\left((1-\delta)\sum_{\tau=0}^{\infty}\delta^\tau\left(S(q_\tau(\theta)) - m(\theta)q_\tau(\theta)\right)\right) \text{ where } m(\theta) = \begin{cases}\underline{\theta} & \text{if } \theta = \underline{\theta},\\ \overline{\theta} + \frac{\nu}{1-\nu}\Delta\theta & \text{if } \theta = \overline{\theta}\end{cases}.$$

The function $f(\mathbf{q})$ maps $\mathcal{A}$ into $\mathbb{R}$ and is strictly concave. It thus admits a (single-valued) superdifferential $\partial f(\mathbf{q})$ given by:

$$\partial f(\mathbf{q}) = (1-\delta)\delta^\tau\left\{\left(\nu\left(S'(q_\tau(\underline{\theta})) - \underline{\theta}\right), (1-\nu)\left(S'(q_\tau(\overline{\theta})) - \overline{\theta}\right) - \frac{\nu}{1-\nu}\Delta\theta\right)\right\}_{\tau\geq 0}.$$

It can be easily checked that $\partial f(\mathbf{q})$ belongs to $l_1 \times l_1$.

CONSTRAINED SET. We rewrite (4.9) as:

$$\text{(A.18)} \quad g_\tau(\mathbf{q}) = \delta^\tau\Psi(\underline{\theta}, \overline{\mathbf{q}}_\tau(\underline{\theta})) - \max\left\{\Delta\theta(q_0^+(\overline{\theta}) - q_\tau^-(\underline{\theta})), 0\right\} \geq 0.$$

The function $g_\tau(\mathbf{q})$ maps $\mathcal{A}$ into $\mathbb{R}$ and is strictly concave in $\mathbf{q}$ for all $\tau \geq 0$. It thus admits a (single-valued) superdifferential $\partial g_\tau(\mathbf{q})$. Let also denote $g(\mathbf{q}) = \{g_\tau(\mathbf{q})\}_{\tau\geq 0}$.

FORMULATION. We rewrite the maximization problem as:

$$(\mathcal{P}): \quad \max_{\mathbf{q}\in\mathcal{A}} f(\mathbf{q}) \text{ subject to } g(\mathbf{q}) \geq 0.$$

$(\mathcal{P})$ is an optimization problems with infinitely many constraints, a feature that requires careful use of duality arguments. The corresponding Lagrangian can be written as:

$$\mathcal{L}(\mathbf{q}, \lambda) = f(\mathbf{q}) + \lambda g(\mathbf{q})$$

$$= E_\theta\left((1-\delta)\left(\sum_{\tau=0}^{\infty}\delta^\tau(S(q_\tau(\theta)) - m(\theta)q_\tau(\theta))\right)\right) + \sum_{\tau=0}^{\infty}\lambda_\tau\left(\delta^\tau\Psi(\underline{\theta}, \mathbf{q}_\tau^+(\underline{\theta})) - \max\left\{\Delta\theta(q_0^+(\overline{\theta}) - q_\tau^-(\underline{\theta})), 0\right\}\right).$$

Next Theorem reminds an important result due to Dechert (1982) that ensures the existence of a sequence of non-negative Lagrange multipliers $\lambda = \{\lambda_\tau\}_{\tau=0}^\infty \in l_1$ for this problem.

THEOREM A.1 *(Dechert, 1982) Suppose f and g are concave and Fréchet differentiable with $\partial f(\mathbf{q}) \in l_1 \times l_1$. Let $\mathbf{q}^*$ be a solution to $(\mathcal{P})$. Suppose that the following conditions hold.*

1. *There exists $\tilde{\mathbf{q}} \in \mathcal{A}$ such that (Slater condition):*

$$\text{(A.19)} \quad \sup_{\tau} g_\tau(\tilde{\mathbf{q}}) > 0.$$

2. *g is asymptotically insensitive if $\forall \mathbf{x} \in \mathcal{A}$, and $\mathbf{y}$ such that $y_\tau \neq 0$ for finitely many τ and $\mathbf{x} + \mathbf{y} \in \mathcal{A}$:*

$$\text{(A.20)} \quad \textit{(AI)} \lim_{\tau \to +\infty} g_\tau(\mathbf{x} + \mathbf{y}) - g_\tau(\mathbf{x}) = 0.$$

3. *g is asymptotically non-anticipatory if $\forall \mathbf{x} \in \mathcal{A}$, and $\mathbf{y}$ such that $\mathbf{x} + \mathbf{y} \in \mathcal{A}$ and $y^T_\tau = \begin{cases} 0 & \textit{if } \tau \leq T \\ y_\tau & \textit{if } \tau > T \end{cases}$:*

$$\text{(A.21)} \quad \textit{(ANA)} \lim_{T \to +\infty} g_\tau(\mathbf{x} + \mathbf{y}^T) = g_\tau(\mathbf{x}), \quad \forall \tau \geq 0.$$

Then there exists $\lambda \in l_1$ such that:

$$\text{(A.22)} \quad \lambda g(\mathbf{q}^*) = 0,\text{[31]}$$

$$\text{(A.23)} \quad \mathcal{L}(\mathbf{q}^*, \lambda) \geq \mathcal{L}(\mathbf{q}, \lambda) \quad \forall \mathbf{q} \in \mathcal{A}.$$

We already noticed that $\partial f(\mathbf{q}) \in l_1 \times l_1$. It remains to check that Conditions (A.19), (A.20) and (A.21) hold. First, the Slater condition (A.19) is satisfied when $M > 0$ by $\tilde{\mathbf{q}} \equiv 0$. Second, by a remark in the text, for τ large enough, $\max\left\{\Delta\theta(q^+_0(\overline{\theta}) - q^-_\tau(\underline{\theta})), 0\right\} = 0$ for any $\mathbf{q} \in \mathcal{A}$. Thus, we get:

$$\text{(A.24)} \quad |g_\tau(\mathbf{x} + \mathbf{y}) - g_\tau(\mathbf{x})| = \delta^\tau |\Psi(\underline{\theta}, \mathbf{q}^+_\tau(\underline{\theta}) + \mathbf{y}^+_\tau(\underline{\theta})) - \Psi(\underline{\theta}, \mathbf{q}^+_\tau(\underline{\theta}))| \leq K\delta^\tau \to_{\tau \to +\infty} 0$$

for some $K > 0$ since $\Psi(\underline{\theta}, \mathbf{q}^+)$ is continuous and $\mathcal{Q}$ is bounded. Henceforth, condition (A.21) holds. Similarly, condition (A.21) trivially holds.

OPTIMIZATION. First, we rewrite the optimality condition by means of superdifferentials, assuming that the optimal output profile is in the interior of $\mathcal{A}$ (i.e. the monotonicity condition (4.5) is strict). This gives us:

$$\text{(A.25)} \quad 0 \in \partial f(\mathbf{q}^*) + \lambda \partial g(\mathbf{q}^*).$$

We can now now explore the implications of the optimality conditions (A.25).

1. Optimality w.r.t. $q_\tau(\underline{\theta})$:

$$\text{(A.26)} \quad S'(q_\tau(\underline{\theta})) - \underline{\theta} = \frac{\lambda_\tau \underline{\theta} - \left(\sum_{s=\tau+1}^{\infty} \lambda_s 1_s\right) \Delta\theta}{\nu + \sum_{s=0}^{\tau-1} \lambda_s},$$

where $1_s = \begin{cases} 1 & \text{if } \Delta\theta(q^+_0(\overline{\theta}) - q^-_s(\underline{\theta})) > 0 \\ \in [0,1] & \text{if } \Delta\theta(q^+_0(\overline{\theta}) - q^-_s(\underline{\theta})) = 0 \\ 0 & \text{if } \Delta\theta(q^+_0(\overline{\theta}) - q^-_s(\underline{\theta})) < 0 \end{cases}.$

[31] We also use the convention that the product equality $\mathbf{xy} = 0$ should be understood coordinate wise as $x_\tau y_\tau = 0$ for all $\tau \geq 0$.

2. Optimality w.r.t. $q_\tau(\overline{\theta})$:

$$\text{(A.27)} \quad S'(q_\tau(\overline{\theta})) - \overline{\theta} = \frac{\nu + \sum_{s=0}^{\infty} \lambda_s 1_s}{1-\nu} \Delta\theta,$$

where the assumption $S'(0)$ sufficiently large ensures that $q_\tau(\overline{\theta})$ remains positive.

Output distortions. From (A.27), we necessarily have $q_\tau(\overline{\theta}) \leq q^{bm}(\overline{\theta})$ and the inequality is strict provided one multiplier at least is positive, a fact which is known to be true when Assumption 2 holds since this assumption means that the Baron-Myerson allocation (obtained when all multipliers are zero) is no longer implementable.

Turning now to the low-cost seller's output, the optimal output of the low-cost seller is first-best far enough in the future. To show that, we first prove a first Lemma.

Lemma A.2 *Suppose that Assumption 2 holds. There exists $\tau^* \geq 0$ such that:*

$$\text{(A.28)} \quad \lambda_\tau = 0 \quad \forall \tau \geq \tau^*,$$

and

$$\text{(A.29)} \quad S'(q_\tau(\underline{\theta})) = \underline{\theta} \quad \forall \tau > \tau^*.$$

Proof of Lemma A.2: Because $\lambda_\tau \geq 0$ and $\lambda \in l_1$ (i.e., $\sum_{s=0}^{\infty} \lambda_s < +\infty$), we have $\lim_{\tau \to +\infty} \lambda_\tau = 0$ and $\sum_{s=0}^{\infty} \lambda_s 1_s < +\infty$. Inserting into (A.26), yields:

$$\text{(A.30)} \quad \lim_{\tau \to +\infty} S'(q_\tau(\underline{\theta})) = \underline{\theta}.$$

The first-best output for a low-cost seller is always implemented in the limit. For τ large enough, an allocation in the interior of $\mathcal{C}$ is strictly monotonic and thus (4.9) writes as in (4.7). But passing to the limit and using (A.30), (4.7) becomes:

$$\psi(\underline{\theta}, q^{fb}(\underline{\theta})) \geq 0.$$

By Assumption 2, this latter inequality is actually strict and thus (4.9) cannot be binding for τ large enough so that (A.28) holds. *Q.E.D.*

Binding enforcement constraints. Because $(\mathcal{P})$ is a concave problem, the necessary conditions for optimality (A.22) and (A.23) are also sufficient. From Lemma A.2, the solution is such that (A.18) is binding at all dates $\tau \leq \tau^*$. In that case, we conjecture that $\Delta\theta(q_0^+(\overline{\theta}) - q_\tau^-(\underline{\theta})) > 0$ for all such dates.

Let us now define the sequence $\mathbf{\Lambda}$ of cumulative multipliers as:

$$\Lambda_\tau = \sum_{s=0}^{\tau-1} \lambda_s$$

with the convention $\Lambda_0 = 0$. Because all multipliers λ_s are non-negative, $\mathbf{\Lambda}$ is a non-decreasing and non-negative sequence with terminal value $\Lambda_{\tau^*+1} = \Lambda_\infty$. From the optimality condition (A.26) and given our conjecture, the sequence $\mathbf{\Lambda}$ satisfies the recursive equation:

$$\left(S'(q_\tau(\underline{\theta})) - \underline{\theta}\right)(\nu + \Lambda_\tau) = (\Lambda_{\tau+1} - \Lambda_\tau)\underline{\theta} - (\Lambda_\infty - \Lambda_{\tau+1})\,\Delta\theta.$$

After manipulations, we get:

$$\bar{\theta}\Lambda_{\tau+1} - \Delta\theta\Lambda_\infty = \nu(S'(q_\tau(\underline{\theta})) - \underline{\theta}) + S'(q_\tau)\Lambda_\tau. \tag{A.31}$$

Observe also that (A.26) implies

$$S'(q_{\tau^*}(\underline{\theta})) = \underline{\theta} + \frac{\underline{\theta}\lambda_{\tau^*}}{\nu + \Lambda_\infty - \lambda_{\tau^*}} \geq \underline{\theta} \Rightarrow q_{\tau^*}(\underline{\theta}) \leq q^{fb}(\underline{\theta}). \tag{A.32}$$

Consider thus a non-decreasing sequence $q_\tau(\underline{\theta})$ (and strictly so for $\tau \leq \tau^*$) such that $q_{\tau^*}(\underline{\theta}) \leq q^{fb}(\underline{\theta})$. (An argument below will show that the optimal outputs satisfy this monotonicity property.) We can rewrite (A.31) as:

$$\Lambda_{\tau+1} = \alpha_\tau \Lambda_\tau + \beta_\tau \tag{A.33}$$

with

$$\alpha_\tau = \frac{S'(q_\tau(\underline{\theta}))}{\bar{\theta}} \text{ and } \beta_\tau = \frac{\Delta\theta\Lambda_\infty + \nu(S'(q_\tau(\underline{\theta})) - \underline{\theta})}{\bar{\theta}}. \tag{A.34}$$

Lemma A.3 *Suppose that Assumption 2 holds. There exists* $\tau^* \geq 0$ *such that:*

$$\lambda_\tau > 0 \quad \forall \tau \leq \tau^* \text{ and } \lambda_\tau = 0 \quad \forall \tau > \tau^*. \tag{A.35}$$

Proof of Lemma A.3: From Lemma A.2, we know that there exists a maximal date $\tau^* \geq 0$ such that the multiplier λ_τ is positive only for $\tau \leq \tau^*$. We want to show that indeed $\lambda_\tau > 0$ for all $\tau \leq \tau^*$. To this end, observe that the sequence $\mathbf{\Lambda}$ is increasing at all $\tau \leq \tau^* - 1$ (so that all corresponding multipliers λ_τ remain positive). We have:

$$\Lambda_{\tau+1} > \Lambda_\tau \Leftrightarrow \Lambda_\tau < \frac{\beta_\tau}{1-\alpha_\tau} = \frac{\Delta\theta\Lambda_\infty + \nu(S'(q_\tau(\underline{\theta})) - \underline{\theta})}{\bar{\theta} - S'(q_\tau(\underline{\theta}))}. \tag{A.36}$$

Since the righthand side of (A.36) is decreasing in $q_\tau(\underline{\theta})$, the sequence $\frac{\beta_\tau}{1-\alpha_\tau}$ is itself decreasing. Moreover, the following string of conditions holds:

$$\Lambda_{\tau^*} = \Lambda_\infty - \lambda_{\tau^*} \leq \Lambda_\infty \leq \frac{\beta_{\tau^*}}{1-\alpha_{\tau^*}} = \frac{\Delta\theta\Lambda_\infty + \nu(S'(q_{\tau^*}(\underline{\theta})) - \underline{\theta})}{\bar{\theta} - S'(q_{\tau^*}(\underline{\theta}))},$$

where the first inequality follows from $\lambda_{\tau^*} \geq 0$ and the last one from (A.32). Now, we can write:

$$\Lambda_{\tau^*-1} = \frac{\Lambda_{\tau^*} - \beta_{\tau^*-1}}{\alpha_{\tau^*-1}} \leq \frac{\frac{\beta_{\tau^*}}{1-\alpha_{\tau^*}} - \beta_{\tau^*-1}}{\alpha_{\tau^*-1}} < \frac{\frac{\beta_{\tau^*-1}}{1-\alpha_{\tau^*-1}} - \beta_{\tau^*-1}}{\alpha_{\tau^*-1}} = \frac{\beta_{\tau^*-1}}{1-\alpha_{\tau^*-1}},$$

where the last righthand side inequality uses the fact that the sequence $\frac{\beta_\tau}{1-\alpha_\tau}$ is decreasing. Proceeding recursively, we obtain:

$$\Lambda_\tau < \frac{\beta_\tau}{1-\alpha_\tau} \quad \forall \tau \leq \tau^* - 1.$$

Hence, $\lambda_\tau = \Lambda_{\tau+1} - \Lambda_\tau > 0$ for all $\tau \leq \tau^*$ and $\lambda_\tau = \Lambda_{\tau+1} - \Lambda_\tau = 0$ for $\tau > \tau^*$. Henceforth, when (A.18) is binding at date τ^*, it is also so at all dates $\tau < \tau^*$. *Q.E.D.*

We now set up the stage for a sharp characterization of contractual dynamics. To this end, consider any sequence $\mathbf{q}$, starting with an arbitrary output level $q_0 \in \left[q^e(\bar{\theta}), q^{fb}(\underline{\theta})\right]$ and constructed recursively as:

$$\text{(A.37)} \quad \begin{cases} q_0 \in \left[q^e(\bar{\theta}), q^{fb}(\underline{\theta})\right] \\ q_{\tau+1} = \Phi(q_\tau). \end{cases}$$

The function $\Phi(q) = S^{-1}\left(\frac{1}{\delta}\left(\bar{\theta}q - (1-\delta)M\right)\right)$ is defined over the interval $\left[\frac{(1-\delta)M}{\bar{\theta}}, +\infty\right)$. It is increasing, convex and has a unique fixed point $q^e(\bar{\theta})$. For future references, we also note that the inverse function $\Gamma(q) = \Phi^{-1}(q) = \frac{1}{\bar{\theta}}(\delta S(q) + (1-\delta)M)$ is increasing and concave.

Equipped with the characterization of such sequences, we now explore the consequences of Lemma A.3 for the optimal outputs produced by a low-cost seller. When $\tau^* \geq 2$, we may indeed rewrite (A.18) when binding at two subsequent dates τ and $\tau+1$ for all τ such that $\tau + 1 \leq \tau^*$ respectively as:

$$\text{(A.38)} \quad \Psi(\underline{\theta}, \mathbf{q}_\tau^+(\underline{\theta})) = \delta^{-\tau}\Delta\theta(q_0^+(\bar{\theta}) - q_\tau^-(\underline{\theta})),$$

$$\text{(A.39)} \quad \delta\Psi(\underline{\theta}, \mathbf{q}_{\tau+1}^+(\underline{\theta})) = \delta^{-\tau}\Delta\theta(q_0^+(\bar{\theta}) - q_{\tau+1}^-(\underline{\theta})).$$

By subtracting (A.38) from (A.39), we obtain:

$$\delta S(q_{\tau+1}(\underline{\theta})) - \underline{\theta}q_\tau(\underline{\theta}) + (1-\delta)M = \Delta\theta q_\tau(\underline{\theta}).$$

Simplifying, the sequence $\mathbf{q}(\underline{\theta}) = \{q_\tau(\underline{\theta})\}_{\tau\geq 0}$ satisfies the recursive condition (A.37) for all $\tau \geq 0$ such that $\tau \leq \tau^* - 1$. Starting thus from $q_{\tau^*}(\underline{\theta}) \in \left[q^e(\bar{\theta}), q^{fb}(\underline{\theta})\right)$, we may then construct the following (backward) recursive sequence of outputs $\Gamma(q_{\tau^*}(\underline{\theta})) = q_{\tau^*-1}(\underline{\theta})$, and thus $\Gamma^s(q_{\tau^*}(\underline{\theta})) = q_{\tau^*-s}(\underline{\theta})$ (or $\Gamma^{\tau^*-s}(q_{\tau^*}(\underline{\theta})) = q_s(\underline{\theta})$) for $s \leq \tau^*$ where Γ^k denotes the k-th iteration of the mapping Γ). By construction, $q_\tau(\underline{\theta})$ for all $\tau \leq \tau^*$ is increasing in τ for all $\tau \leq \tau^*$. Moreover, that $q^e(\bar{\theta}) < q^{fb}(\underline{\theta})$ (which is implied by Assumption 2 since $\psi(\underline{\theta}, q^{fb}(\underline{\theta})) < \Delta\theta q^{bm}(\bar{\theta}) < \Delta\theta q^{fb}(\underline{\theta})$) also implies $q^e(\bar{\theta}) = \Gamma(q^e(\bar{\theta})) \leq \Gamma(q^{fb}(\underline{\theta})) < q^{fb}(\underline{\theta})$. Therefore, we get:

$$\text{(A.40)} \quad q^e(\bar{\theta}) \leq q_\tau(\underline{\theta}) \quad \forall \tau \leq \tau^* - 1$$

and thus (5.3) holds.

CHECKING THE OMITTED CONSTRAINTS. It is routine to check that (4.4) for $\bar{\theta}$ and (4.1) for $\underline{\theta}$ are both satisfied.

We now check that the remaining enforcement constraints hold. First, (A.12) amounts to

$$\text{(A.41)} \quad \delta^\tau\psi(\bar{\theta}, q^{sb}(\bar{\theta})) \geq \max\left\{\Delta\theta((1-\delta^\tau)q^{sb}(\bar{\theta}) - q_0^{+sb}(\underline{\theta})), 0\right\}.$$

Observe that $(1-\delta^\tau)q^{sb}(\bar{\theta}) \leq q^{sb}(\bar{\theta}) < q^{bm}(\bar{\theta}) < q^e(\bar{\theta}) < q_\tau^{sb}(\underline{\theta})$ for all $\tau \geq 0$. Thus, $(1-\delta^\tau)q^{sb}(\bar{\theta}) < q_0^{+sb}(\underline{\theta})$ and (A.41) is implied by $\psi(\bar{\theta}, q^{sb}(\bar{\theta})) > \psi(\bar{\theta}, q^{bm}(\bar{\theta})) \geq 0$ where the last inequality follows from Assumption 2 and $q^{sb}(\bar{\theta}) < q^{bm}(\bar{\theta})$.

Second, (A.13) now amounts to

(A.42)

$$\min\left\{\delta^\tau\Psi(\underline{\theta},\mathbf{q}_\tau^{+sb}(\underline{\theta}))+\Delta\theta q_\tau^-(\underline{\theta});\Delta\theta q_0^{+sb}(\underline{\theta})\right\}\geq\max\left\{-\delta^\tau\psi(\bar{\theta},q^{sb}(\bar{\theta}))+\Delta\theta(1-\delta^\tau)q^{sb}(\bar{\theta});\Delta\theta q^{sb}(\bar{\theta})\right\}.$$

From (A.18) being satisfied at all date τ, the righthand side above can be bounded below by $\min\left\{\Delta\theta q^{sb}(\bar{\theta});\Delta\theta q_0^{+sb}(\underline{\theta})\right\}=\Delta\theta q^{sb}(\bar{\theta})$. Because $q^{sb}(\bar{\theta})<q^{bm}(\bar{\theta})<q^{fb}(\underline{\theta})<q^{e}(\underline{\theta})$ (where the last inequality follows from Assumption 2), $\psi(\bar{\theta},q^{sb}(\bar{\theta}))+\Delta\theta q^{sb}(\bar{\theta})=\psi(\underline{\theta},q^{sb}(\bar{\theta}))>0$ and the the righthand side of (A.42) amounts to $\Delta\theta q^{sb}(\bar{\theta})$ which is thus lower than the lefthand side found above. *Q.E.D.*

Proof of Proposition 2: Condition (A.4) amounts to (5.8). Condition (5.9) follows from the fact that the high-cost seller's output is constant over time and from (A.9). To prove (5.10) we need to show first that $t_\tau^{sb}(\underline{\theta})=\bar{\theta}q_\tau^{sb}(\underline{\theta})$ holds over a transitory phase. Then, Lemma A.4 below shows that this phase coincides with the transitory phase described in Theorem 1. Note that $q_0^+(\bar{\theta})=q^{sb}(\bar{\theta})$ and that $q_0^+(\bar{\theta})-q_\tau^-(\underline{\theta})=q^{sb}(\bar{\theta})-q_\tau^-(\underline{\theta})$ is decreasing in τ (with by definition $q^{sb}(\bar{\theta})-q_0^-(\underline{\theta})=q^{sb}(\bar{\theta})>0$). We have $q_{\tau^*+1}(\underline{\theta})=q_{\tau^*+2}(\underline{\theta})=...=q^{fb}(\underline{\theta})$. Now observe that (5.3) implies that

$$q_\tau^-(\underline{\theta})=(1-\delta)\sum_{s=0}^{\tau-1}\delta^\tau q_s(\underline{\theta})>(1-\delta)\,\frac{1-\delta^\tau}{1-\delta}q^e(\bar{\theta})=(1-\delta^\tau)\,q^e(\bar{\theta}).$$

Since $q^e(\bar{\theta})>q^{bm}(\bar{\theta})$ (from the second condition in Assumption 2) and $q^{bm}(\bar{\theta})\geq q^{sb}(\bar{\theta})$ (from (5.5)), we have $q^e(\bar{\theta})>q^{sb}(\bar{\theta})$. Moreover, $q_\tau^-(\underline{\theta})$ is an increasing sequence, bounded above by $q^{fb}(\underline{\theta})$ (from (5.3)); so it converges towards a finite limit $q_\infty^-(\underline{\theta})$ such that $q_\infty^-(\underline{\theta})\geq q^e(\bar{\theta})>q^{sb}(\bar{\theta})$. Finally, we conclude on the existence of a date τ' such that:

$$q^{sb}(\bar{\theta})-q_\tau^-(\underline{\theta})\geq 0\quad\forall\tau\leq\tau'\text{ and }q^{sb}(\bar{\theta})-q_\tau^-(\underline{\theta})<0\quad\forall\tau>\tau'. \tag{A.43}$$

Inserting into (A.10), we obtain:

$$U_\tau^+(\underline{\theta})=\begin{cases}\delta^{-\tau}\Delta\theta\left(q^{sb}(\bar{\theta})-q_\tau^-(\underline{\theta})\right) & \forall\tau\leq\tau',\\ 0 & \text{otherwise.}\end{cases} \tag{A.44}$$

1. Consider a date τ such that $\tau+1\leq\tau'$. For such τ, we have

$$U_\tau^+(\underline{\theta})=(1-\delta)\left(t_\tau(\underline{\theta})-\underline{\theta}q_\tau(\underline{\theta})\right)+\delta U_{\tau+1}^+(\underline{\theta}).$$

Or, using (A.44),

$$U_\tau^+(\underline{\theta})=(1-\delta)\left(t_\tau(\underline{\theta})-\underline{\theta}q_\tau(\underline{\theta})\right)+\delta\delta^{-\tau-1}\Delta\theta\left(q^{sb}(\bar{\theta})-q_{\tau+1}^-(\underline{\theta})\right).$$

Using again (A.44) to express the lefthand side yields:

$$\delta^{-\tau}\Delta\theta(q^{sb}(\bar{\theta})-q_\tau^-(\underline{\theta}))=(1-\delta)\left(t_\tau(\underline{\theta})-\underline{\theta}q_\tau(\underline{\theta})\right)+\delta^{-\tau}\Delta\theta\left(q^{sb}(\bar{\theta})-q_{\tau+1}^-(\underline{\theta})\right).$$

Simplifying, we can write:

$$0=(1-\delta)\left(t_\tau(\underline{\theta})-\underline{\theta}q_\tau(\underline{\theta})\right)-\delta^{-\tau}\Delta\theta(1-\delta)\delta^\tau q_\tau(\underline{\theta})=(1-\delta)\left(t_\tau(\underline{\theta})-\bar{\theta}q_\tau(\underline{\theta})\right).$$

Henceforth, (5.10) holds for all τ such that $\tau+1\leq\tau'$.

2. At date τ', we use (5.10) ro rewrite (A.44) as:

$$U^+_{\tau'}(\underline{\theta}) = (1-\delta)\left(t_{\tau'}(\underline{\theta}) - \underline{\theta} q_{\tau'}(\underline{\theta})\right) = \delta^{-\tau'}\Delta\theta\left(q^{sb}(\bar{\theta}) - q^-_{\tau'}(\underline{\theta})\right)$$

or, equivalently,

$$t_{\tau'}(\underline{\theta}) = \underline{\theta} q_{\tau'}(\underline{\theta}) + \frac{\delta^{-\tau'}}{1-\delta}\Delta\theta\left(q^{sb}(\bar{\theta}) - q^-_{\tau'}(\underline{\theta})\right).$$

3. Finally, consider $U^+_{\tau}(\underline{\theta})$ and $U^+_{\tau+1}(\underline{\theta})$ for $\forall\tau > \tau'$. We have

$$0 = U^+_{\tau}(\underline{\theta}) = (1-\delta)\left(t_{\tau}(\underline{\theta}) - \underline{\theta} q_{\tau}(\underline{\theta})\right) + \delta U^+_{\tau+1}(\underline{\theta}) = (1-\delta)\left(t_{\tau}(\underline{\theta}) - \underline{\theta} q_{\tau}(\underline{\theta})\right).$$

Summarizing Items **1** to **3**, we obtain:

$$t_\tau(\underline{\theta}) = \begin{cases} \bar{\theta} q_\tau(\underline{\theta}) & \forall\tau \leq \tau' - 1, \\ \underline{\theta} q_\tau(\underline{\theta}) + \frac{\delta^{-\tau}}{1-\delta}\Delta\theta\left(q^{sb}(\bar{\theta}) - q^-_\tau(\underline{\theta})\right) & \tau = \tau', \\ \underline{\theta} q_\tau(\underline{\theta}) & \forall\tau > \tau'. \end{cases} \tag{A.45}$$

We now prove that the transitory phase has length τ^*.

Lemma A.4 $\tau^* = \tau'$.

Proof of Lemma A.4.: Note that the enforcement constraint is binding for $\tau \leq \tau^*$. Thus we have

$$\delta(S(q^{fb}(\underline{\theta})) - \underline{\theta} q^{fb}(\underline{\theta})) - (1-\delta)\underline{\theta} q_{\tau^*}(\underline{\theta}) + (1-\delta)M = \delta^{-\tau^*}\max\left\{\Delta\theta(q^+_0(\bar{\theta}) - q^-_{\tau^*}(\underline{\theta})), 0\right\}.$$

But we can find a lower bound for the lefthand side as:

$$\delta(S(q^{fb}(\underline{\theta})) - \underline{\theta} q^{fb}(\underline{\theta})) - (1-\delta)\underline{\theta} q_{\tau^*}(\underline{\theta}) + (1-\delta)M \geq \psi(\underline{\theta}, q^{fb}(\underline{\theta})) > 0$$

(where the last inequality follows from the first condition in Assumption 2). Thus, we have

$$\max\left\{\Delta\theta(q^+_0(\bar{\theta}) - q^-_{\tau^*}(\underline{\theta})), 0\right\} = \Delta\theta(q^+_0(\bar{\theta}) - q^-_{\tau^*}(\underline{\theta})) > 0$$

and finally

$$\tau^* \leq \tau'.$$

Assume now that $\tau' \geq \tau^* + 1$. Then, both constraints (A.18) for τ' and $\tau' + 1$ are binding so that:

$$\Psi(\underline{\theta}, \mathbf{q}^+_{\tau'}(\underline{\theta})) = \delta^{-\tau}\Delta\theta(q^+_0(\bar{\theta}) - q^-_{\tau'}(\underline{\theta})),$$

and

$$\delta\Psi(\underline{\theta}, \mathbf{q}^+_{\tau'+1}(\underline{\theta})) = \delta^{-\tau}\Delta\theta(q^+_0(\bar{\theta}) - q^-_{\tau'+1}(\underline{\theta})).$$

By subtracting one equation from the other, we obtain:

$$\delta S(q_{\tau'+1}(\underline{\theta})) - \underline{\theta} q_{\tau'}(\underline{\theta}) + (1-\delta)M = \Delta\theta q_{\tau'}(\underline{\theta}).$$

Because $q_{\tau'+1}(\underline{\theta}) = q_{\tau'}(\underline{\theta}) = q^{fb}(\underline{\theta})$ we obtain

$$\psi(\underline{\theta}, q^{fb}(\underline{\theta})) = 0$$

which yields a contradiction with the first condition of Assumption 2. We can thus conclude:

$$\tau^* \geq \tau'$$

which ends the proof of Lemma A.4. *Q.E.D.*

Gathering Lemma A.4 and (A.45) yields (5.10) and ends the proof of the proposition. *Q.E.D.*

PROOF OF PROPOSITION 3: The structure of the solution to $(\mathcal{P})$ given by our earlier findings (with (A.18) being binding at all dates $\tau \leq \tau^*$) implies that the enforcement constraint (A.18) at date τ^* can be written as:

(A.46)

$$\delta^{\tau^*}\left(\delta(S(q^{fb}(\underline{\theta})) - \underline{\theta}q^{fb}(\underline{\theta})) - (1-\delta)\underline{\theta}q_{\tau^*}(\underline{\theta}) + (1-\delta)M\right) \geq \Delta\theta\left(q(\overline{\theta}) - (1-\delta)\sum_{\tau=0}^{\tau^*-1}\delta^\tau\Gamma^{\tau^*-\tau}(q_{\tau^*}(\underline{\theta}))\right).$$

Taking into account this structure of the solution, the optimal contract that solves $(\mathcal{P})$ must also solve the following problem:

$$(\mathcal{R}): \max_{(q_{\tau^*}(\underline{\theta}),q(\overline{\theta}),\tau^*)} (1-\delta)\left(\sum_{\tau=0}^{\tau^*}\delta^\tau(S(\Gamma^{\tau^*-\tau}(q_{\tau^*}(\underline{\theta}))) - \underline{\theta}\Gamma^{\tau^*-\tau}(q_{\tau^*}(\underline{\theta}))) + \sum_{\tau=\tau^*+1}^{\infty}\delta^\tau(S(q^{fb}(\underline{\theta})) - \underline{\theta}q^{fb}(\underline{\theta}))\right)$$
$$+(1-\nu)\left(S(q(\overline{\theta})) - \overline{\theta}q(\overline{\theta})\right) - \nu\Delta\theta q(\overline{\theta})$$

subject to (A.46).

Problem $(\mathcal{R})$ gives deeper results on the nature of the solution to $(\mathcal{P})$. It allows to decompose the optimization into two phases. The first transitory phase over the first τ^* periods has a growing output $\Gamma^{\tau^*-\tau}(q_{\tau^*}(\underline{\theta}))$ for the low-cost seller till one reaches a value $q_{\tau^*}(\underline{\theta}) \leq q^{fb}(\underline{\theta})$ to be found. The second phase has a fixed output $q^{fb}(\underline{\theta})$. In both phases, the high-cost seller's output remains constant. Yet, we already know from Lemma A.3, that the solution to $(\mathcal{R})$ cannot have either $\tau^* = \infty$ or $\tau^* = -1$ (with the convention that $\sum_{\tau=0}^{\tau=-1} y_\tau = 0$) when Assumption 2 holds. Fixing τ^* in the maximand above defines a collection of programs $(\mathcal{R}_{\tau^*})$. Whenever the corresponding constraint (A.46) is slack in $(\mathcal{R}_{\tau^*})$, the solution to $(\mathcal{R}_{\tau^*})$ entails an output profile such that $q_{\tau^*}(\underline{\theta}) = q^{fb}(\underline{\theta})$ and $q(\overline{\theta}) = q^{bm}(\overline{\theta})$. Therefore, it is not the solution to $(\mathcal{R})$ (and thus to $(\mathcal{P})$). Next Lemma shows the existence of a first integer τ^* such that (A.46) taken at that date can no longer be slack. This is at such τ^* that $(\mathcal{R})$ achieves its maximum. Lemma A.5 provides a characterization of τ^*.

LEMMA A.5 *Suppose that Assumption 2 holds. There exists a unique $\tau^* \geq 0$ such that:*

$$\text{(A.47)}\qquad \delta^{\tau^*}\psi(\underline{\theta}, q^{fb}(\underline{\theta})) < \Delta\theta\left(q^{bm}(\overline{\theta}) - (1-\delta)\sum_{\tau=0}^{\tau^*-1}\delta^\tau\Gamma^{\tau^*-\tau}(q^{fb}(\underline{\theta}))\right)$$

$$\text{(A.48)}\qquad \delta^{\tau^*+1}\psi(\underline{\theta}, q^{fb}(\underline{\theta})) \geq \Delta\theta\left(q^{bm}(\overline{\theta}) - (1-\delta)\sum_{\tau=0}^{\tau^*}\delta^\tau\Gamma^{\tau^*+1-\tau}(q^{fb}(\underline{\theta}))\right).$$

PROOF OF LEMMA A.5: Denote

$$\vartheta(\tau) = \delta^{-\tau}\Delta\theta\left(q^{bm}(\overline{\theta}) - (1-\delta)\sum_{s=0}^{\tau-1}\delta^s\Gamma^{\tau-s}(q^{fb}(\underline{\theta}))\right).$$

Actually, $\vartheta(\tau)$ is a decreasing sequence. Indeed, $\vartheta(\tau+1) < \vartheta(\tau)$ amounts to $q^{bm}(\overline{\theta}) < \Gamma^{\tau+1}(q^{fb}(\underline{\theta}))$ which holds since, from Assumption 2, we have $q^{bm}(\overline{\theta}) < q^e(\overline{\theta}) < \Gamma^{\tau-s}(q^{fb}(\underline{\theta})) < q^{fb}(\underline{\theta})$ for all $\tau - 1 \geq s \geq 0$. From Assumption 2, we also know that $\psi(\underline{\theta}, q^{fb}(\underline{\theta})) < \vartheta(0)$. Moreover, we have $\lim_{\tau\to+\infty}(1-\delta)\sum_{s=0}^{\tau-1}\delta^s\Gamma^{\tau-s}(q^{fb}(\underline{\theta})) = q^e(\overline{\theta}) > q^{bm}(\overline{\theta})$ where the last inequality also follows from Assumption 2. Hence, for τ large enough, we also have $\vartheta(\tau) < 0$. Gathering these findings, there exists a unique $\tau^* \geq 0$ such that both inequalities (A.47) and (A.48) hold together. *Q.E.D.*

We can now use (A.47), (A.48) and the inequalities $q^e(\overline{\theta}) < \Gamma^{\tau^*-\tau}(q^{fb}(\underline{\theta})) < q^{fb}(\underline{\theta})$ to find the following bounds on τ^*

$$\delta^{\tau^*}\psi(\underline{\theta}, q^{fb}(\underline{\theta})) < \Delta\theta \left(q^{bm}(\overline{\theta}) - (1-\delta^*)q^e(\overline{\theta})\right)$$

and

$$\delta^{\tau^*+1}\psi(\underline{\theta}, q^{fb}(\underline{\theta})) \geq \Delta\theta \left(q^{bm}(\overline{\theta}) - (1-\delta^{\tau^*+1})q^{fb}(\underline{\theta})\right).$$

Taken together those inequalities give us (5.12).

Q.E.D.

PROOF OF THEOREM 2: Suppose now that only the privately informed seller might behave opportunistically. To construct a mechanism that implements the Baron-Myerson allocation and is seller-enforceable, it must be that (4.3) always holds at any date and for all types. First, the low-cost seller's enforcement constraint is not an issue if

$$U_0^+(\underline{\theta}) = \Delta\theta q^{bm}(\overline{\theta}) \geq U_\tau^-(\overline{\theta}) + (1-\delta^\tau)\Delta\theta q^{bm}(\overline{\theta}), \quad \forall \tau \geq 0.$$

The following stationary payment extracts the high-cost seller's surplus in each period and ensure that the latter constraints always hold:

$$t_{1\tau}(\overline{\theta}) + t_{2\tau}(\overline{\theta}) = t^{bm}(\overline{\theta}) = \overline{\theta}q^{bm}(\overline{\theta}) \text{ with } t_{1,\tau}(\overline{\theta}) = L \quad \forall \tau \geq 0.\text{[32]}$$

Turning now to the payments given to a high-cost seller to prevent the *take-the-money-and-run* strategy, (4.3) implies that backward payoffs $U_\tau^-(\underline{\theta})$ must satisfy:

$$U_0^+(\overline{\theta}) = 0 \geq U_\tau^-(\underline{\theta}) - (1-\delta^\tau)\Delta\theta q^{fb}(\underline{\theta}), \quad \forall \tau \geq 0.$$

Finding such payoffs (and thus the payments to a low-cost seller) is now easy. Define now τ^* as the highest integer such that $(1-\delta^\tau)\Delta\theta q^{fb}(\underline{\theta}) < \Delta\theta q^{bm}(\overline{\theta})$. Such integer exists and is unique because $q^{fb}(\underline{\theta}) > q^{bm}(\overline{\theta})$. Over the first τ^* periods, pre-delivery payments are adjusted so that the high-cost seller remains indifferent between breaching or not in each period:

$$U_\tau^-(\underline{\theta}) = (1-\delta^\tau)\Delta\theta q^{bm}(\overline{\theta}) + \delta^\tau(1-\delta)(t_{1,\tau}(\underline{\theta}) - L) = (1-\delta^\tau)\Delta\theta q^{fb}(\underline{\theta}), \quad \forall \tau < \tau^*.$$

After those τ^* earlier periods, pre-delivery payments implement a constant backward rent equal to the low-cost seller's Baron-Myerson information rent:

$$U_\tau^-(\underline{\theta}) = (1-\delta^\tau)\Delta\theta q^{bm}(\overline{\theta}) + \delta^\tau(1-\delta)(t_{1,\tau}(\underline{\theta}) - L) = \Delta\theta q^{bm}(\overline{\theta}), \quad \forall \tau \geq \tau^*.$$

Post-delivery payments are then adjusted to implement Baron-Myerson payments:

$$t_{1,\tau}(\underline{\theta}) + t_{2,\tau}(\underline{\theta}) = t^{bm}(\underline{\theta}) = \underline{\theta}q^{bm}(\underline{\theta}) + \Delta\theta q^{bm}(\overline{\theta}) \quad \forall \tau \geq 0.$$

Q.E.D.

PROOF OF PROPOSITION 4: Taking into account the expression of the renegotiation-proof output profiles, date 0-enforcement constraint (4.6) in state $\overline{\theta}$ can now be written as:

$$\delta\left(S(q^{ci}(\overline{\theta})) - \overline{\theta}q^{ci}(\overline{\theta})\right) - (1-\delta)\overline{\theta}q_0(\overline{\theta}) + (1-\delta)M \geq 0 \tag{A.49}$$

where $q^{ci}(\overline{\theta}) = \min\{q^e(\overline{\theta}), q^{fb}(\overline{\theta})\}$.

Using previous notations to express virtual costs, the buyer's intertemporal payoff at a renegotiation-proof contract becomes:

$$\mathbb{E}_\theta((1-\delta)(S(q_0(\theta)) - m(\theta)q_0(\theta)) + \delta(S(q^{ci}(\theta)) - m(\theta)q^{ci}(\theta))). \tag{A.50}$$

The optimal renegotiation-proof date-0 outputs are thus obtained by maximizing this expression subject to the enforcement constraints (6.1) and (A.49). We first neglect (A.50) and optimize with (6.1) as the sole constraint. Denoting by λ_r the non-negative Lagrange multiplier pour (6.1), and optimizing yields the first-order conditions (6.2).

Suppose now that $\lambda_r = 0$. Then (6.1) would become:

$$\delta S(q^{fb}(\underline{\theta})) - \underline{\theta}q^{fb}(\underline{\theta}) + (1-\delta)M \geq \Delta\theta((1-\delta)q^{bm}(\overline{\theta}) + \delta q^{ci}(\overline{\theta})) \geq \Delta\theta q^{bm}(\overline{\theta})$$

where the last inequality follows from the second condition in Assumption 2 which amounts to $q^{bm}(\overline{\theta}) \leq q^{ci}(\overline{\theta})$. A contradiction with the first condition in Assumption 2. Hence, $\lambda_r > 0$.

We now check that (A.49) is satisfied for the optimal output $q_0^r(\overline{\theta})$ which means:

$$\delta\left(S(q^{ci}(\overline{\theta})) - \overline{\theta}q^{ci}(\overline{\theta})\right) - (1-\delta)\overline{\theta}q_0^r(\overline{\theta}) + (1-\delta)M \geq 0.$$

When $q^{ci}(\overline{\theta}) = q^e(\overline{\theta}) \leq q^{fb}(\overline{\theta})$, this latter condition holds since $q_0^r(\overline{\theta}) < q^{bm}(\overline{\theta})$, with Assumption 2 holding, implies:

$$\delta\left(S(q^e(\overline{\theta})) - \overline{\theta}q^e(\overline{\theta})\right) - (1-\delta)\overline{\theta}q_0^r(\overline{\theta}) + (1-\delta)M \geq \delta\left(S(q^e(\overline{\theta})) - \overline{\theta}q^e(\overline{\theta})\right) - (1-\delta)\overline{\theta}q^{bm}(\overline{\theta}) + (1-\delta)M$$

$$\geq \psi(\overline{\theta}, q^{bm}(\overline{\theta})) \geq 0.$$

Q.E.D.